ASTR 620: Planetary Processes

Professor Eric Nielsen

Lecture 20: Exoplanets

Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Homework 5 due on Monday, November 6 at 11:59pm
- Jury Duty

Review of the last class

- Rossby waves:
  - (A) — result in lines of clouds on the lee side of a mountain
  - (B) — are longitudinal pressure waves that travel at the speed of sound
  - (C) — cause the jet streams to meander

Review of the last class

- As the jet stream in the northern hemisphere moves north:
  - (A) — it acquires positive relative vorticity because the Coriolis term is increasing
  - (B) — it acquires negative relative vorticity because the Coriolis term is increasing
  - (C) — it acquires positive relative vorticity because the Coriolis term is decreasing
  - (D) — it acquires negative relative vorticity because the Coriolis term is decreasing

Review of the last class

- Which of these would be a direct detection method for finding exoplanets?
  - (A) — Observing the motion of the host star due to the planet's gravity
  - (B) — Observing the host star getting fainter because the planet is blocking some of its light
  - (C) — Observing the motion of another planet due to the planet's gravity
  - (D) — Observing light from the planet that's spatially resolved from the host star's light

Review of the last class

- Without any approximations, if a planet is orbiting a star, which semi-major axis and mass should I use in Kepler's third law?

$$P^2 = \frac{a^3}{M}$$

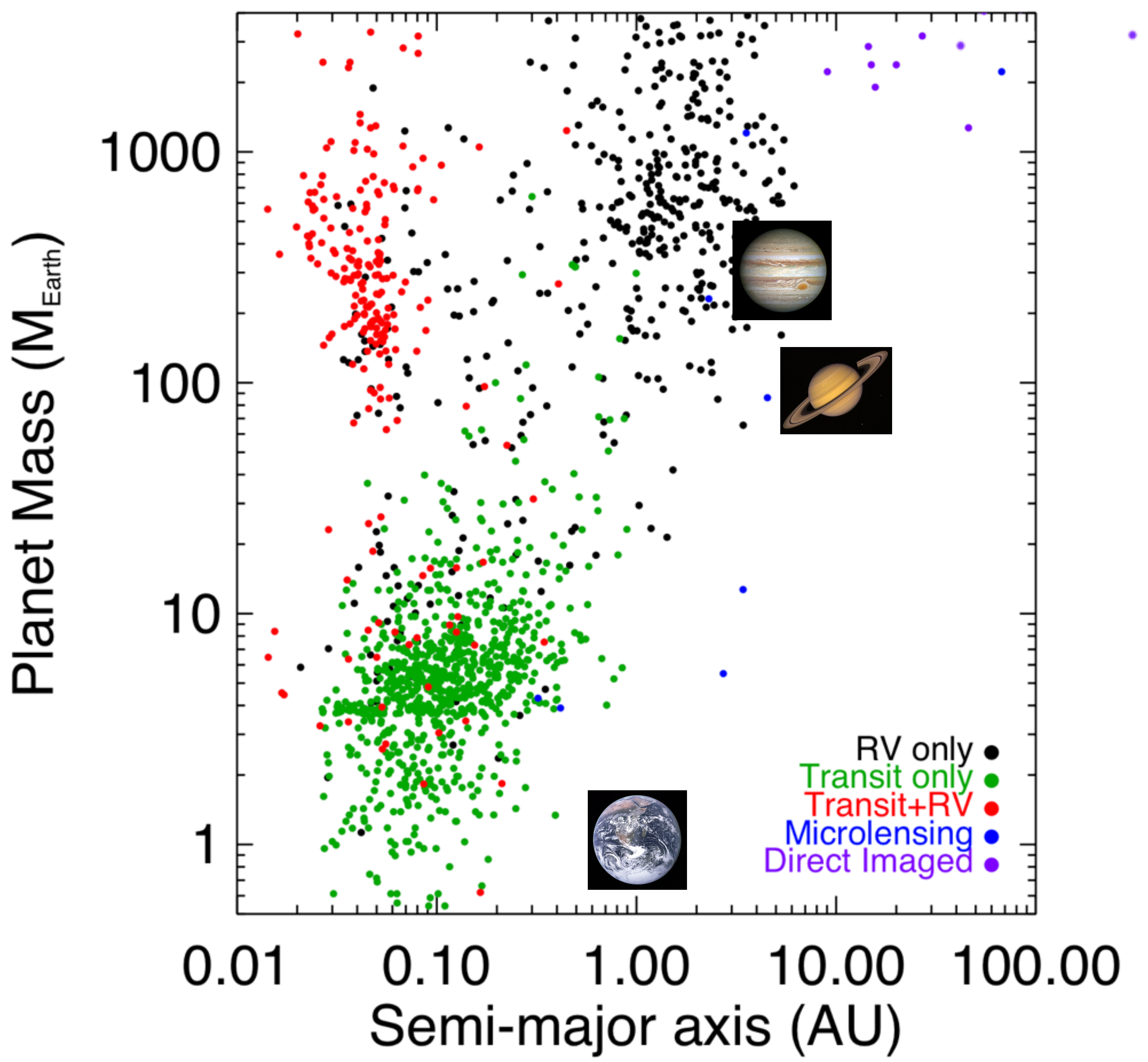
  - (A) — semi-major axis of the star around the center of mass, mass of the planet
  - (B) — semi-major axis of the planet around the center of mass, mass of the star
  - (C) — semi-major axis around the center of mass and mass of the star
  - (D) — semi-major axis around the center of mass and mass of the planet
  - (E) — The sum of the two semi-major axes around the center of mass, and the sum of the two masses

Review of the last class

- In terms of planet mass and semi-major axis, which solar system planets have we found exoplanet analogs (exoplanets with almost identical properties) for?
    - (A) — All planets in our Solar System
    - (B) — Jupiter only
    - (C) — Jupiter, Saturn, Uranus, and Neptune
    - (D) — Jupiter and Earth
    - (E) — Earth only

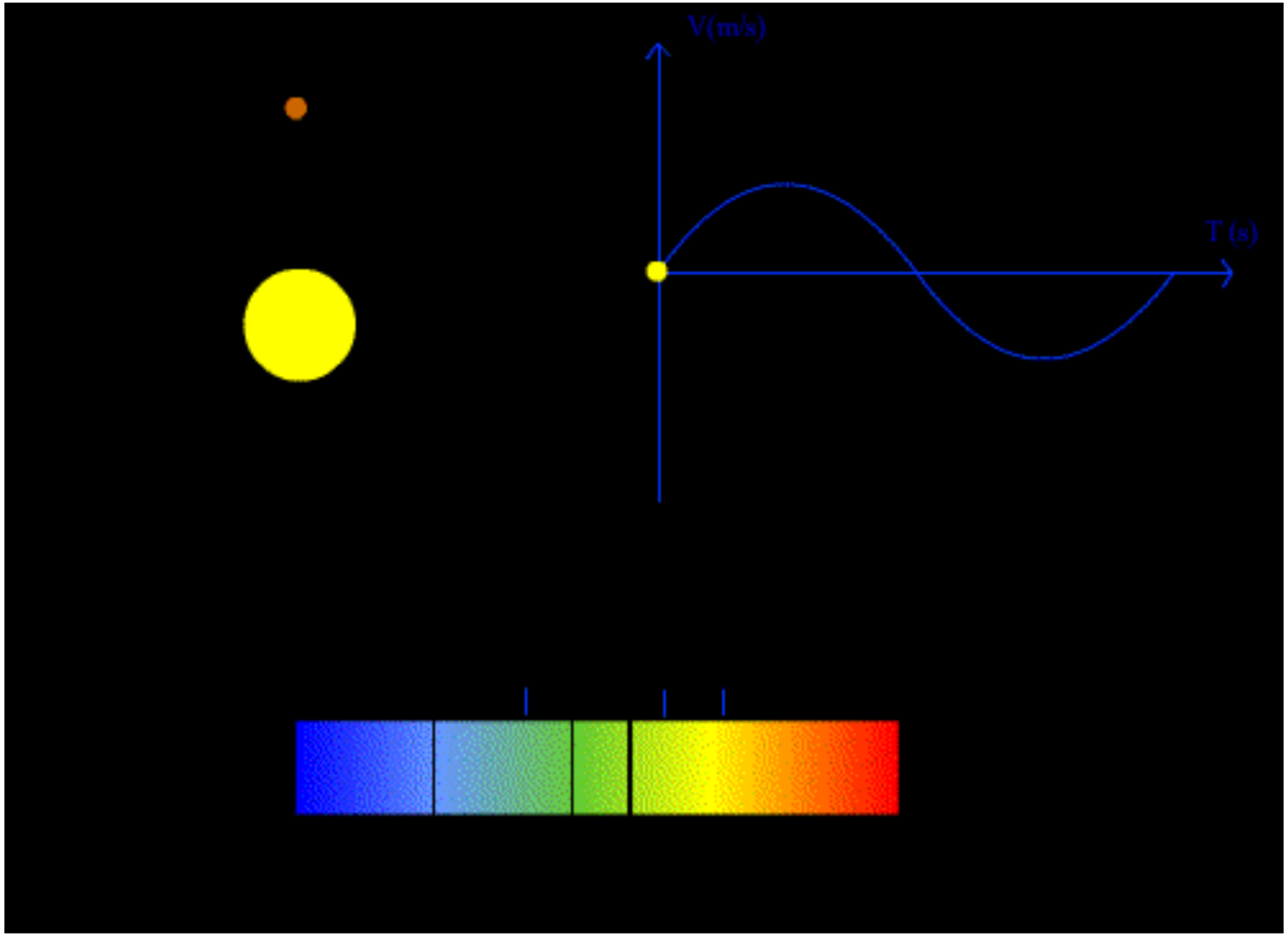
Exoplanet Demographics

- Solar System analogs are still difficult to find with current search techniques
  - Jupiter analogs have been found in the past 2 decades
  - It will take longer to find analogs of other solar system planets
- Exoplanet notation: The name of the host star, followed by
  - “b” (the first planet found around that star)
  - “c” (second planet found)
  - …
- 51 Peg b, HD 209458 b, Kepler 11 c, 51 Eri b, HR 8799 c…



The Radial Velocity Method

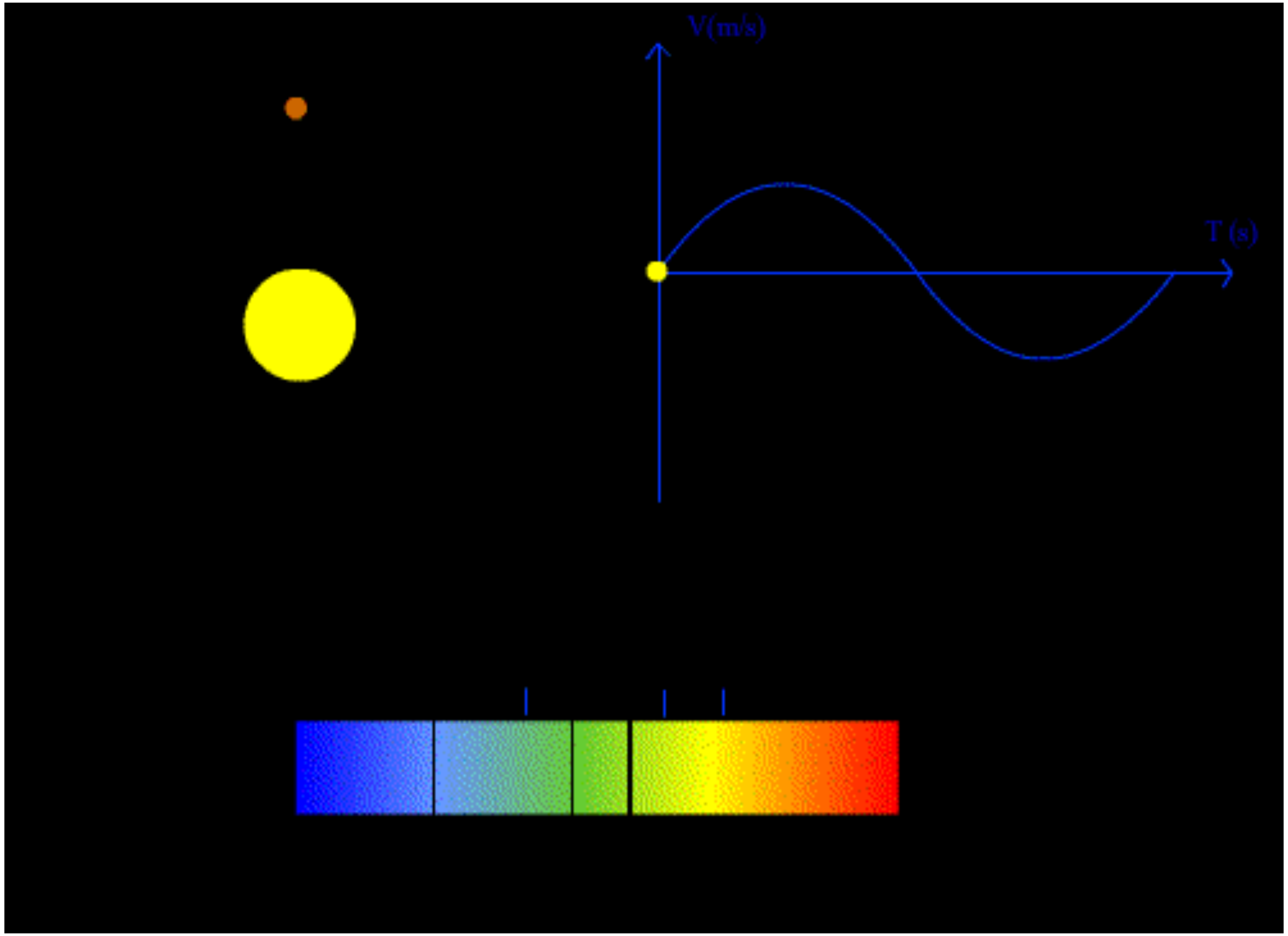
- A star with a planet will orbit the common center of mass of the star/ planet system
- Some component (most of the time) of the star's motion will be along the line-of-sight
- That radial velocity (RV) can be detected by a shift in the star's absorption lines over an orbit



OASA

Response Card Question

- Given the RV curve on this movie, where is the observer? (Hint: positive RVs are moving away from the observer)
  - (A) — At the top, looking down
  - (B) — At the left, looking right
  - (C) — At the bottom, looking up
  - (D) — At the right, looking left



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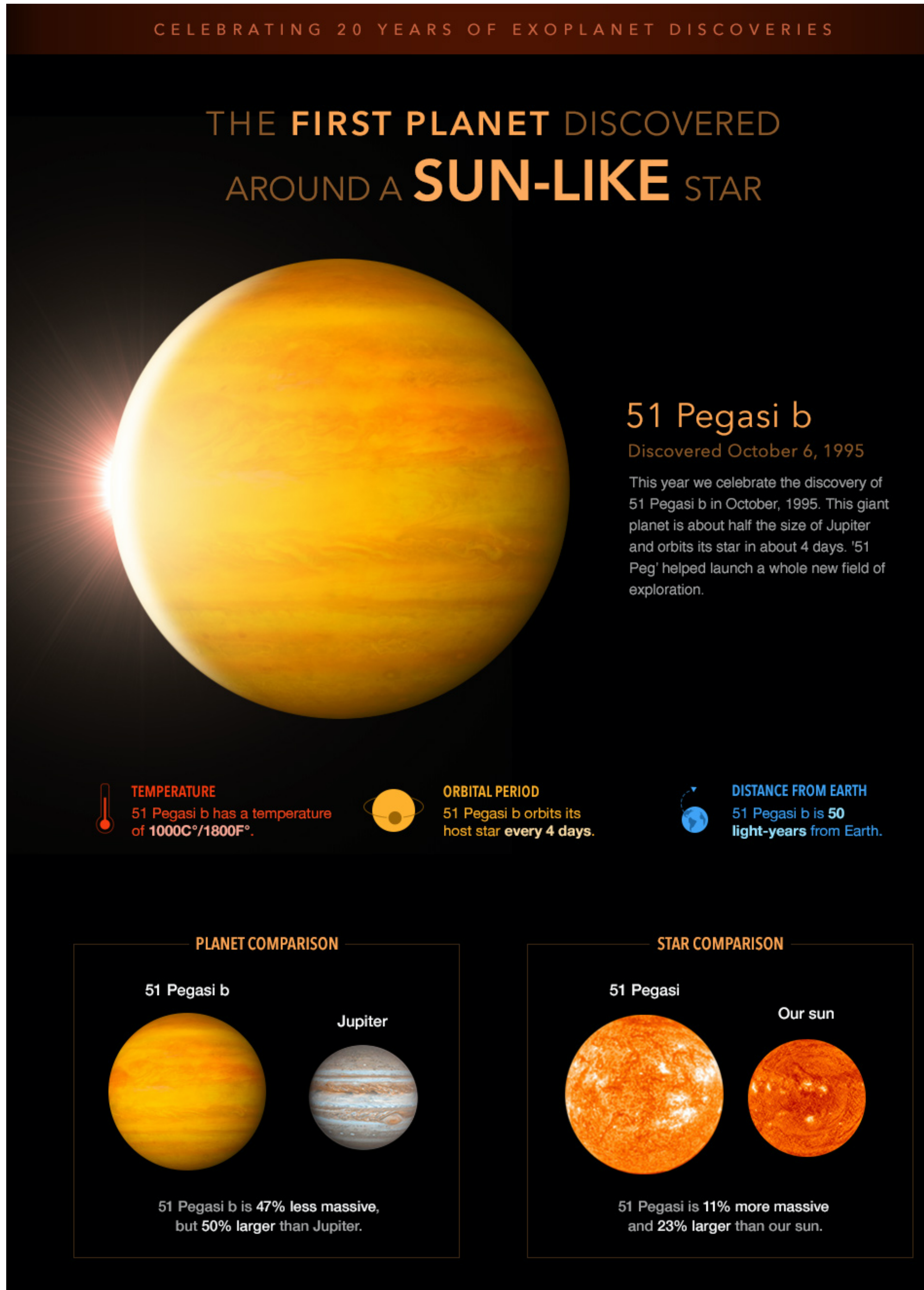
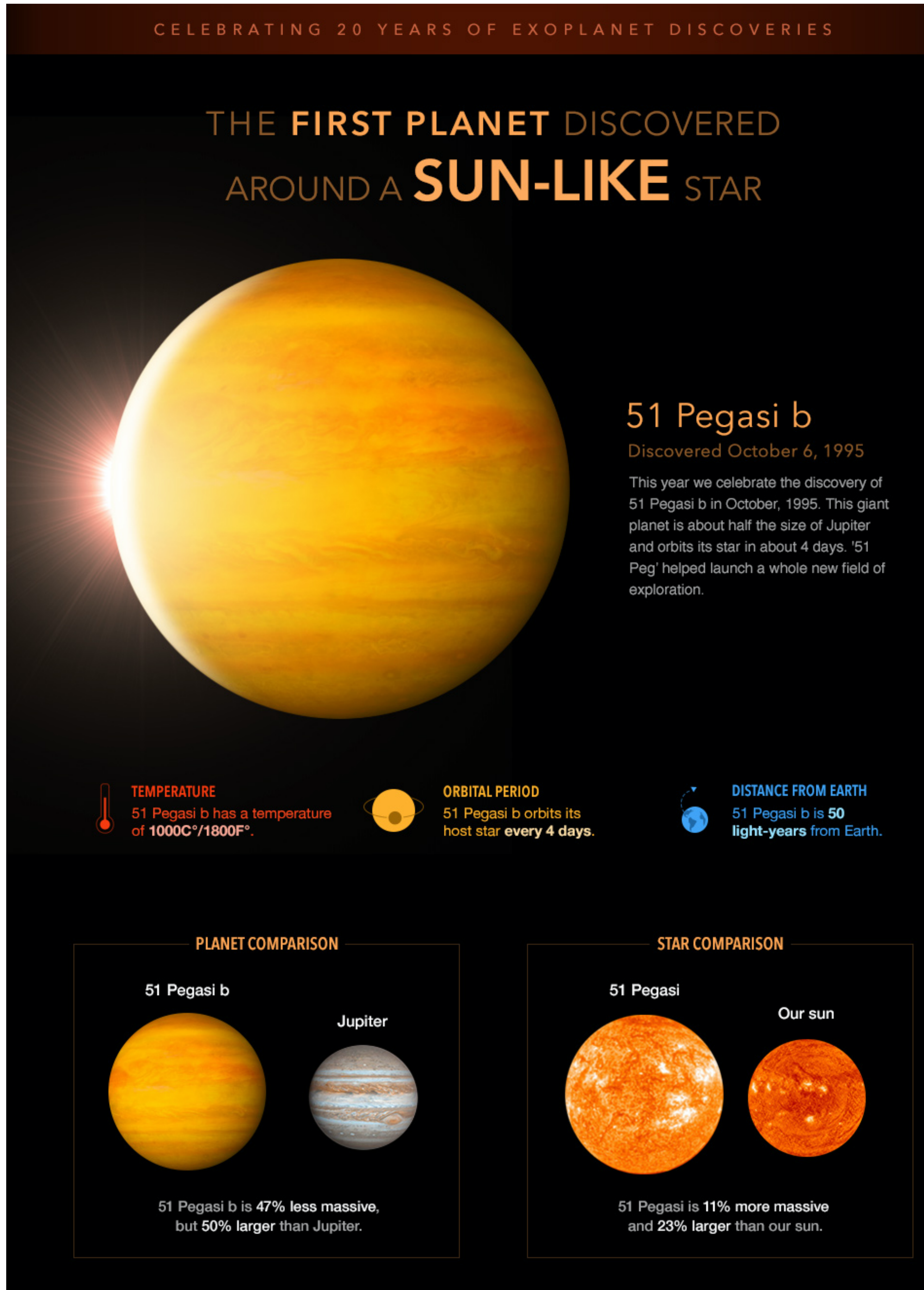
The Radial Velocity Method



The Radial Velocity Method

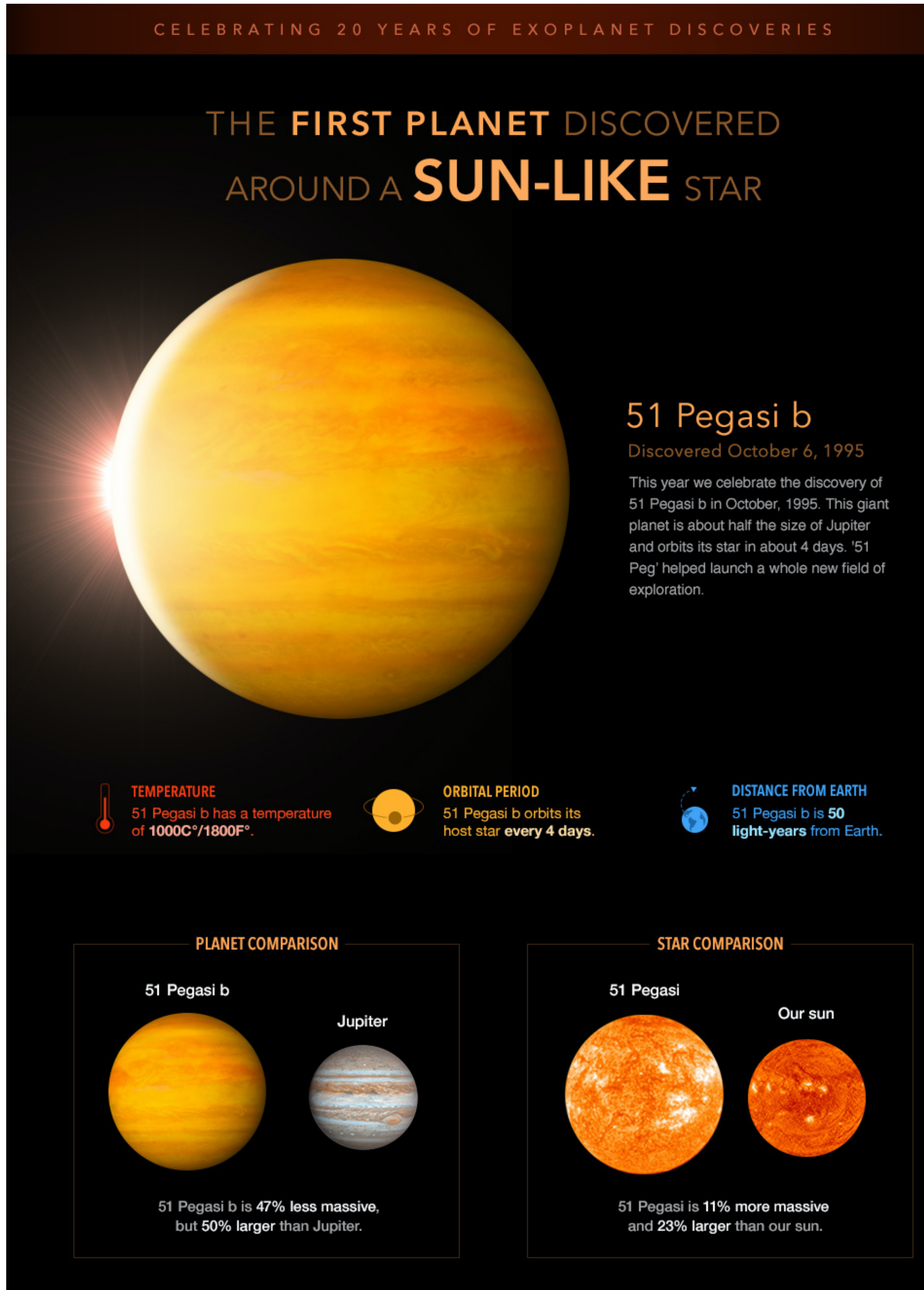
Springer



The Radial Velocity Method

Springer

Keystone/Martial Trezzini



The Radial Velocity Method

- What's the radial velocity amplitude a planet induces on its star?
- Start with Kepler's third law:

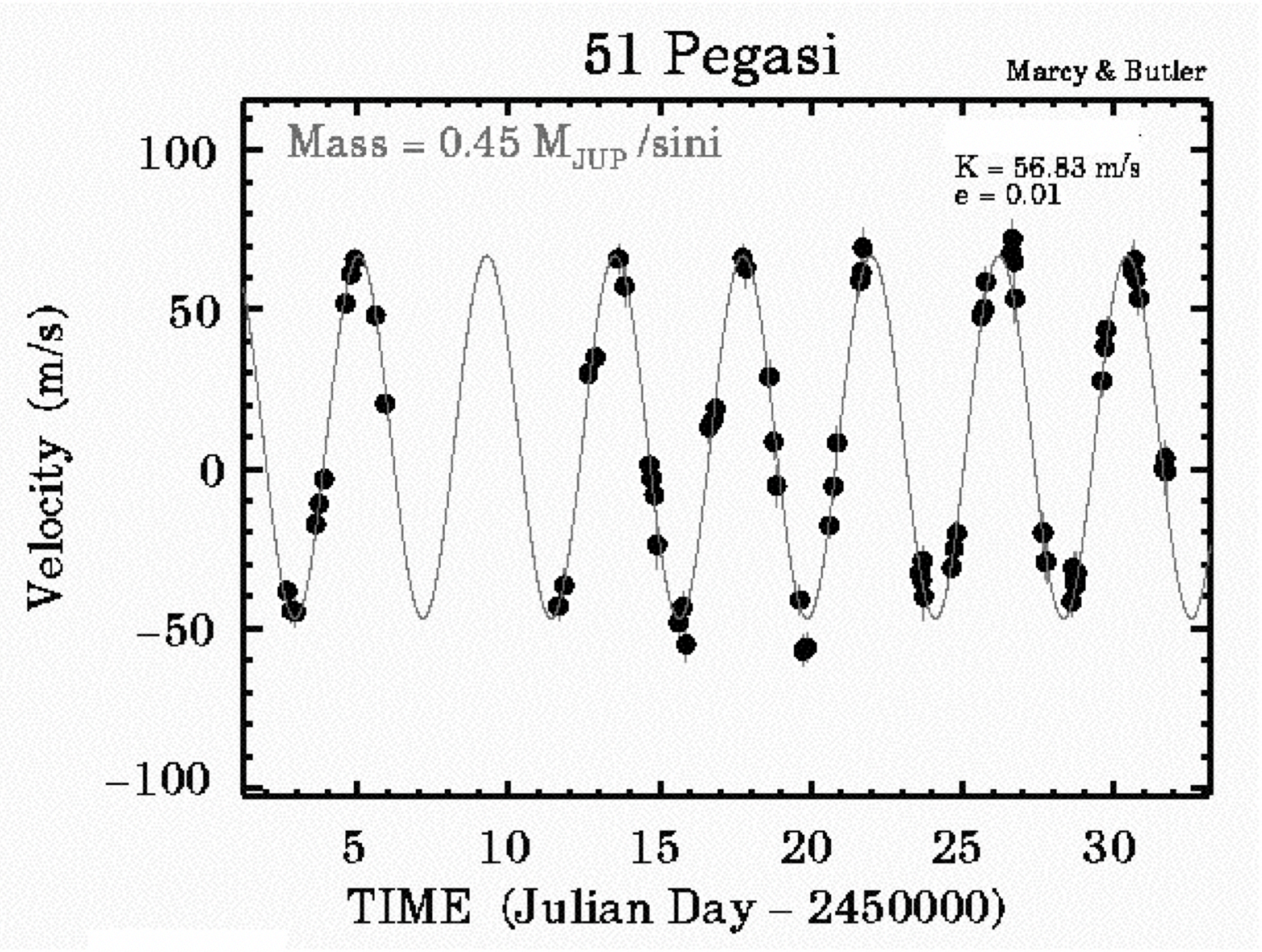
$$P^2 = \frac{4\pi^2(a_1 + a_2)^3}{G(M_1 + M_2)}$$

- Let's also use the center of mass equation:

$$a_1 M_1 = a_2 M_2$$

- And solve for a_2:

$$a_2 = \frac{M_1}{M_2} a_1$$



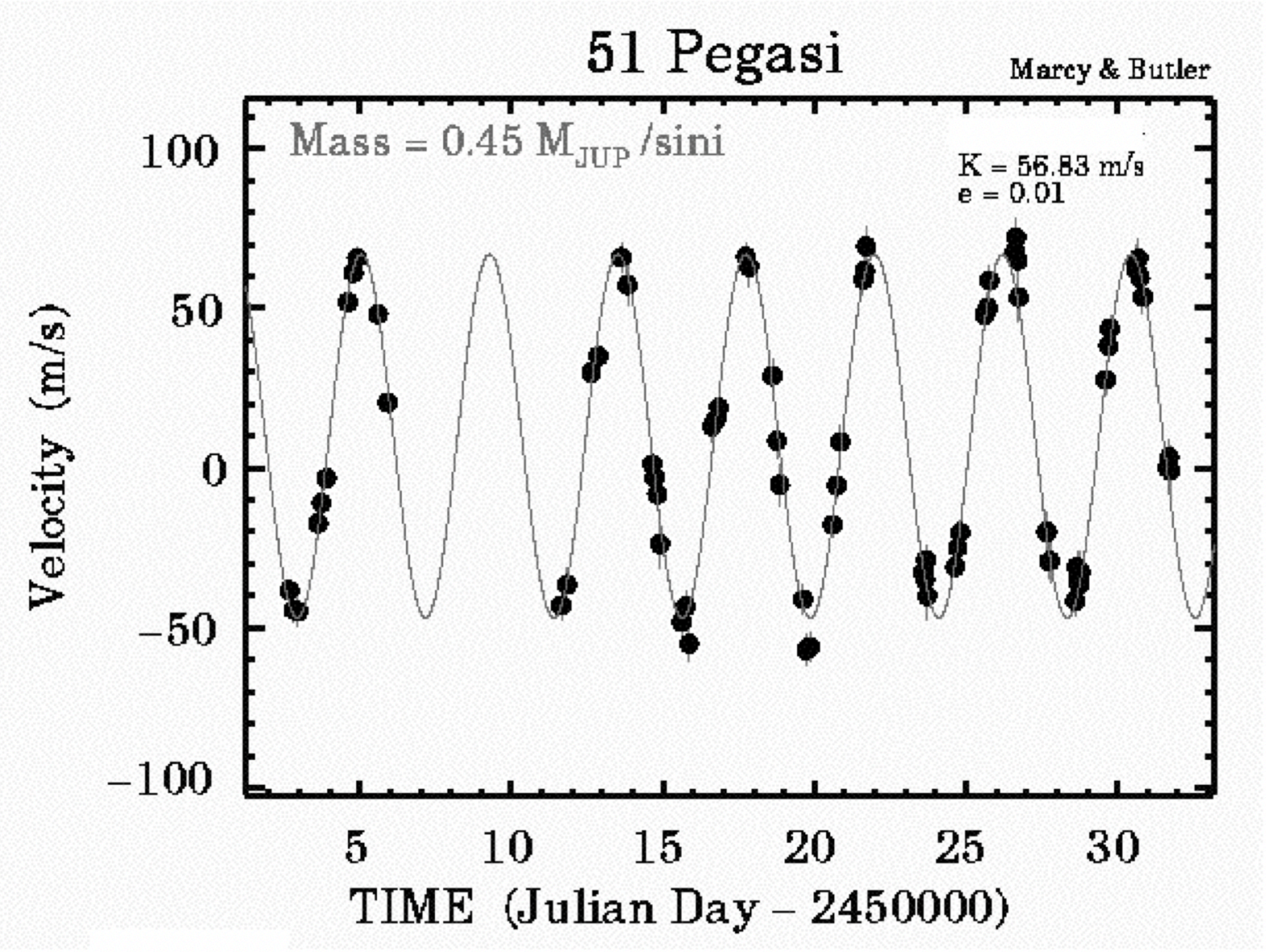
The Radial Velocity Method

- Then rewrite Kepler's third law to get rid of a_2:

$$P^2 = \frac{4\pi^2(a_1 + a_2)^3}{G(M_1 + M_2)} = \frac{4\pi^2(a_1 + \frac{M_1}{M_2}a_1)^3}{G(M_1 + M_2)}$$

$$P^2 = \frac{4\pi^2 a_1^3(1 + \frac{M_1}{M_2})^3}{G(M_1 + M_2)} = \frac{4\pi^2 a_1^3(\frac{M_2 + M_1}{M_2})^3}{G(M_1 + M_2)}$$

$$P^2 = \frac{4\pi^2 a_1^3(M_1 + M_2)^2}{G(M_2)^3}$$



The Radial Velocity Method

- $$P^2 = \frac{4\pi^2 a_1^3 (M_1 + M_2)^2}{G(M_2)^3}$$
- Then let's solve for a_1:

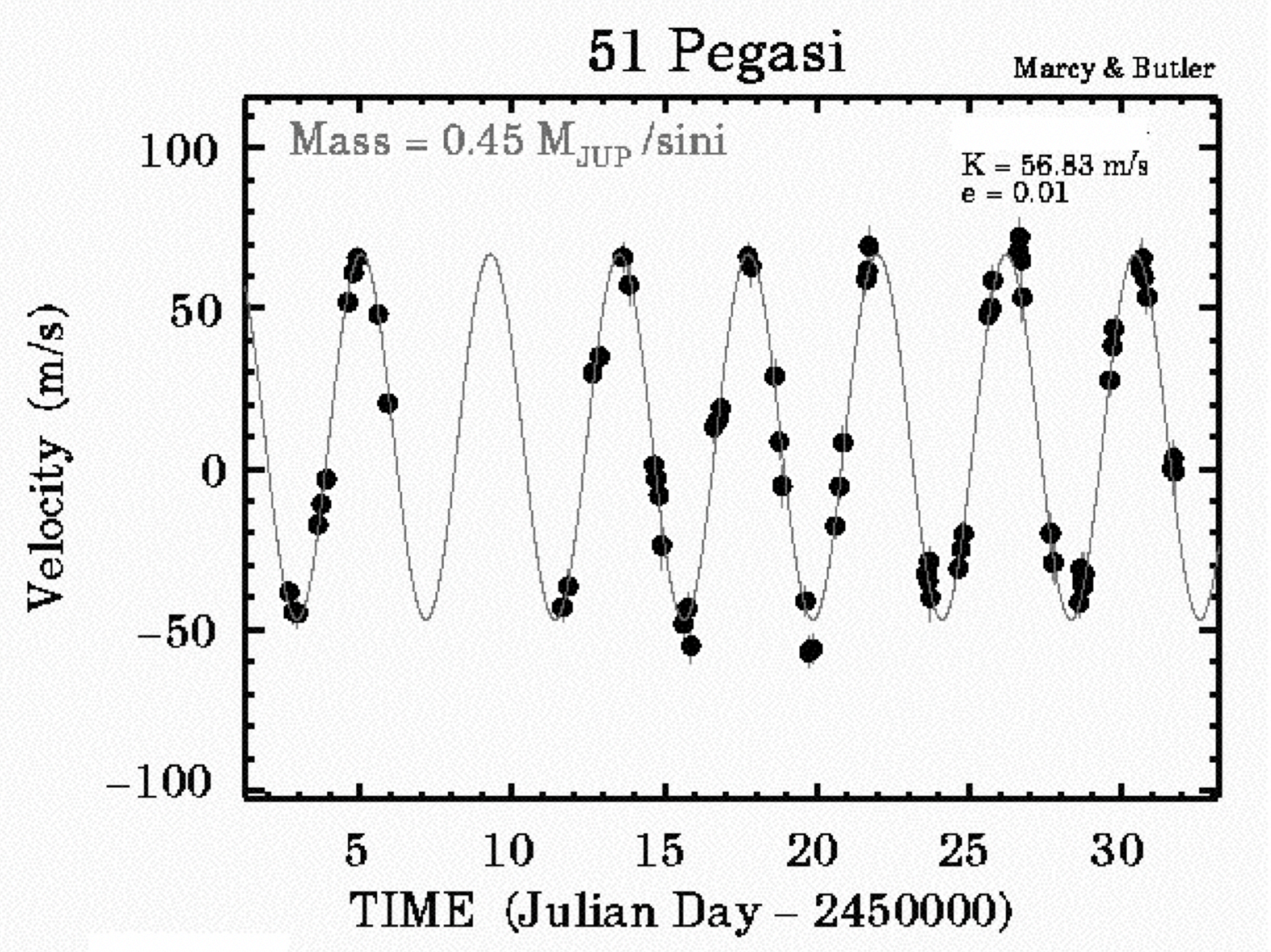
$$a_1 = \left(\frac{G}{4\pi^2}\right)^{1/3} P^{2/3} \frac{M_2}{(M_1 + M_2)^{2/3}}$$

- If the orbit is a circle (e=0), the velocity of the planet in each point in the planet's orbit is:

$$v_P = \frac{2\pi a_2}{P}$$

- And for the star:

$$v_S = \frac{2\pi a_1}{P}$$



The Radial Velocity Method

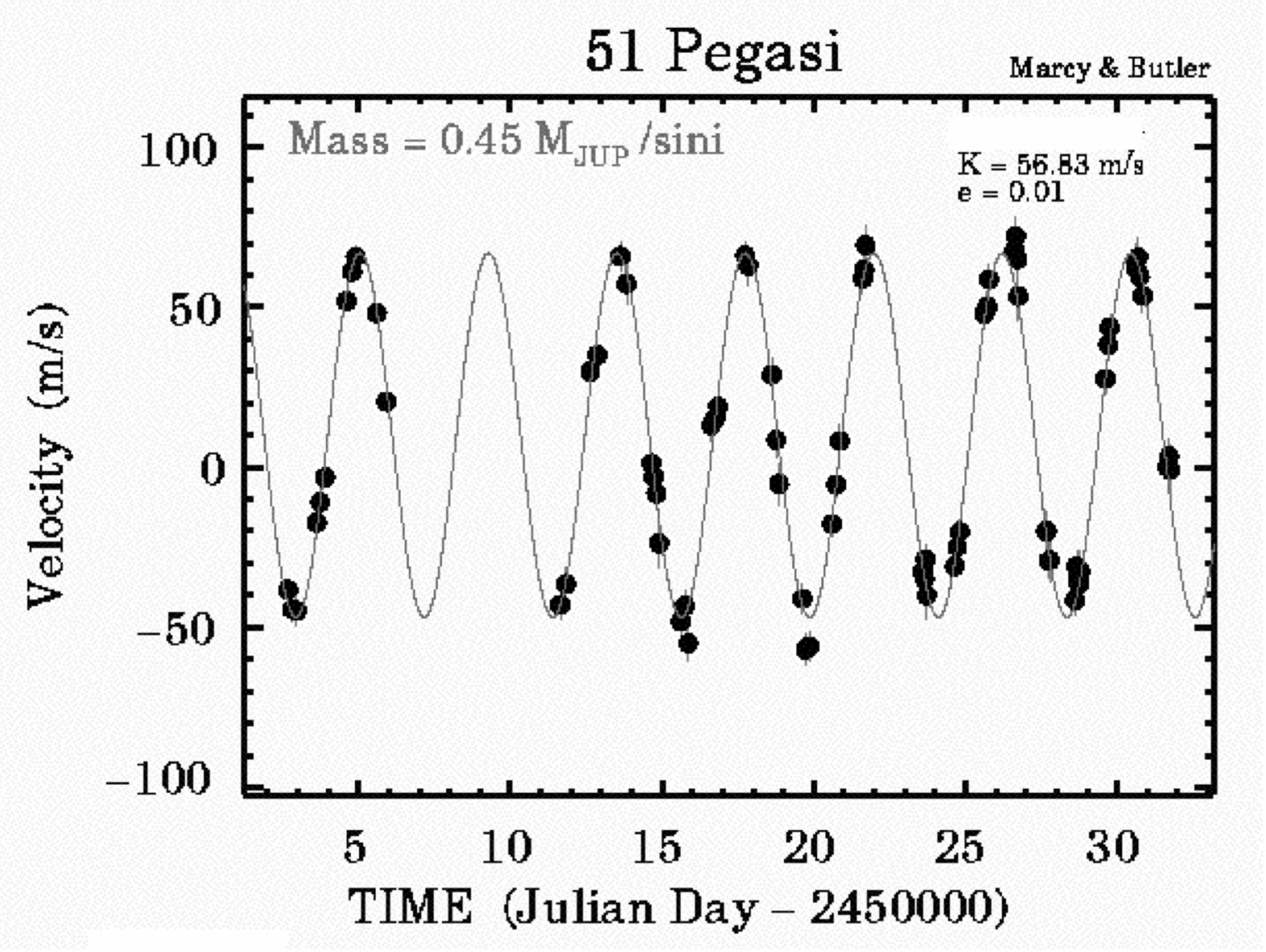
- Then combine equations:

$$v_S = \frac{2\pi a_1}{P} = \frac{2\pi}{P}\left(\frac{G}{4\pi^2}\right)^{1/3} P^{2/3}\frac{M_2}{(M_1+M_2)^{2/3}}$$

- And simplifying:

$$v_S = \left(\frac{2\pi G}{P}\right)^{1/3}\frac{M_2}{(M_1+M_2)^{2/3}}$$

- This is the maximum velocity (for a circular orbit) that's edge-on to the observer
    - When the star is coming directly toward (or moving directly away from) the observer

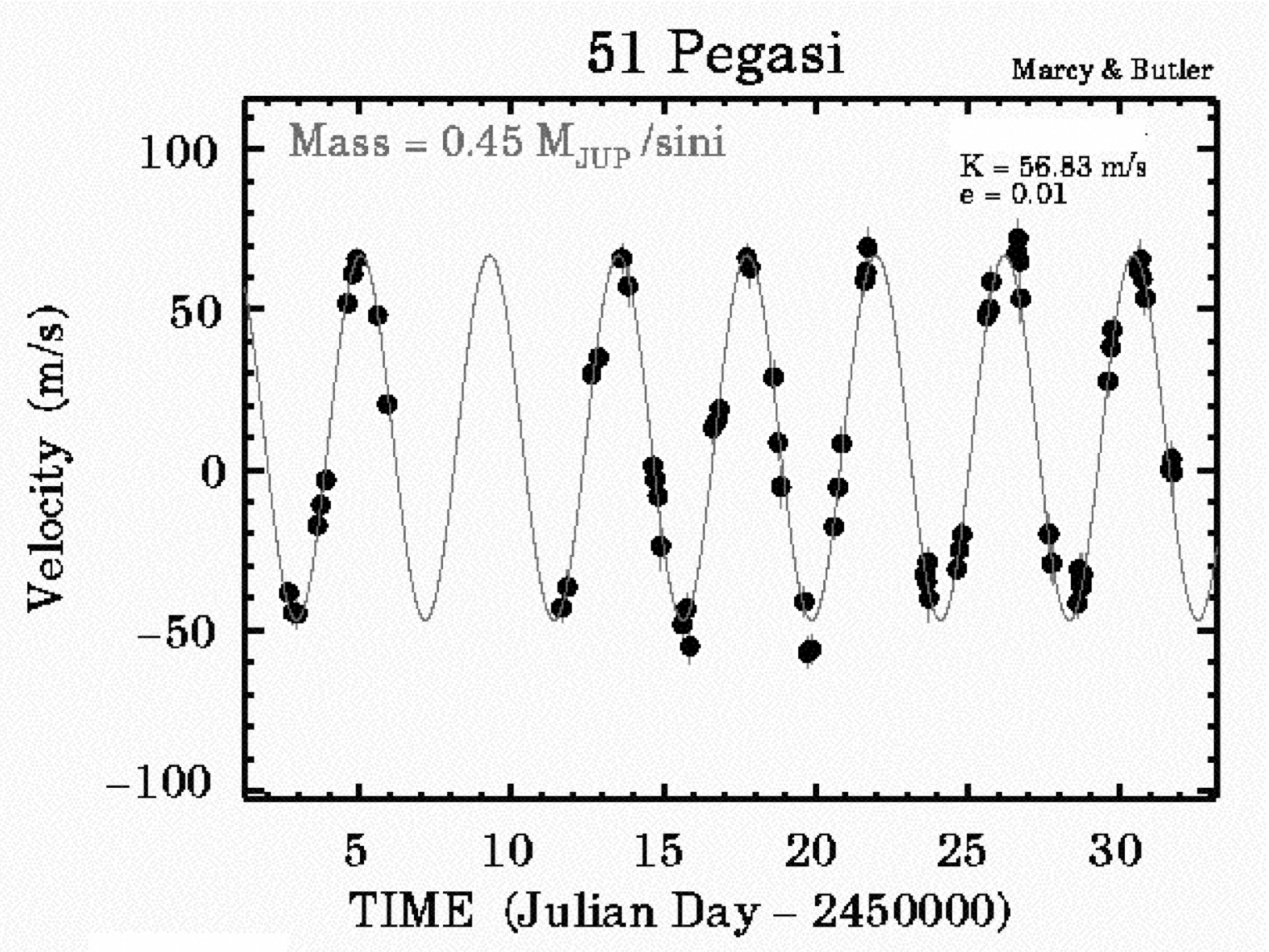


The Radial Velocity Method

- If we allow for non-circular orbits ($0 < e < 1$), and orbits that aren't exactly edge-on ($i \neq 90°$), we pick up extra terms:

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

- K is the semi-amplitude of the star: half the peak-to-peak velocity change
  - $RV_{max} - RV_{min} = 2K$



The Radial Velocity Method

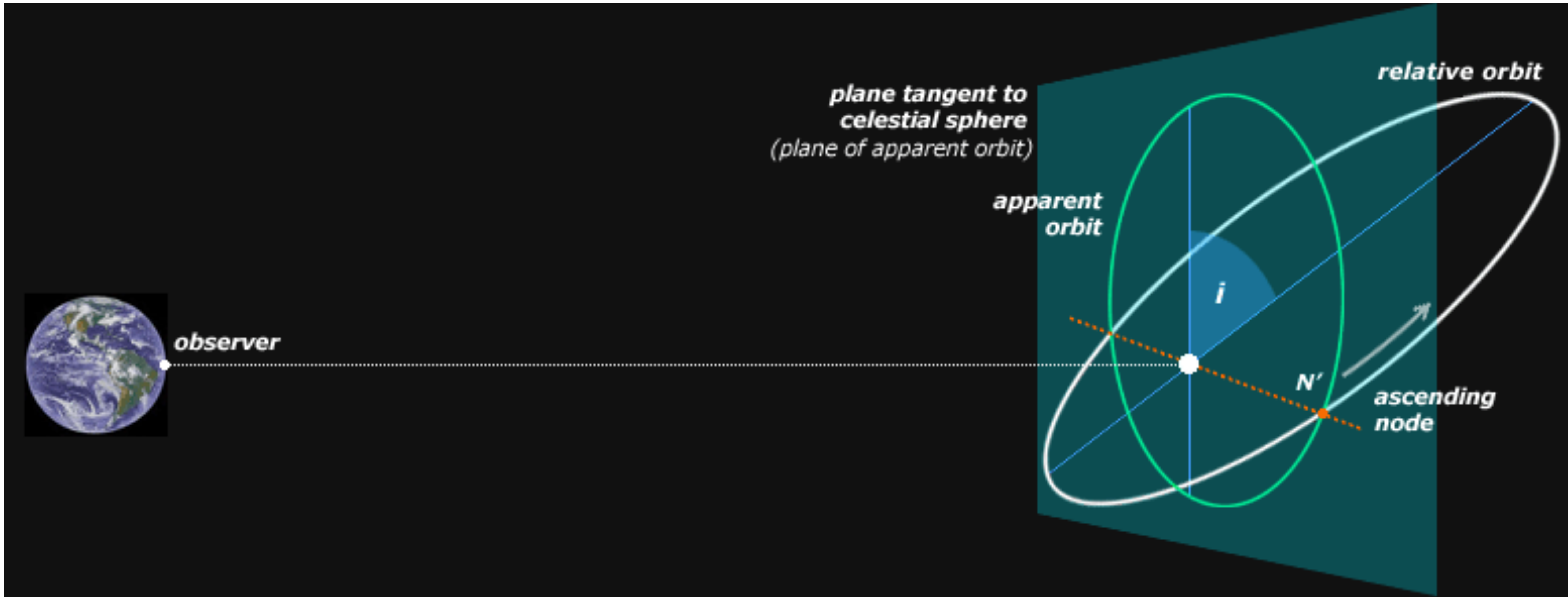
- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

- Inclination angle (i) is the angle between the angular momentum vector of the orbit and the observer

  - 0 degrees: face-on, counterclockwise orbit

  - 90 degrees: edge-on orbit

  - 180 degrees: face-on, clockwise orbit



Response Card Question

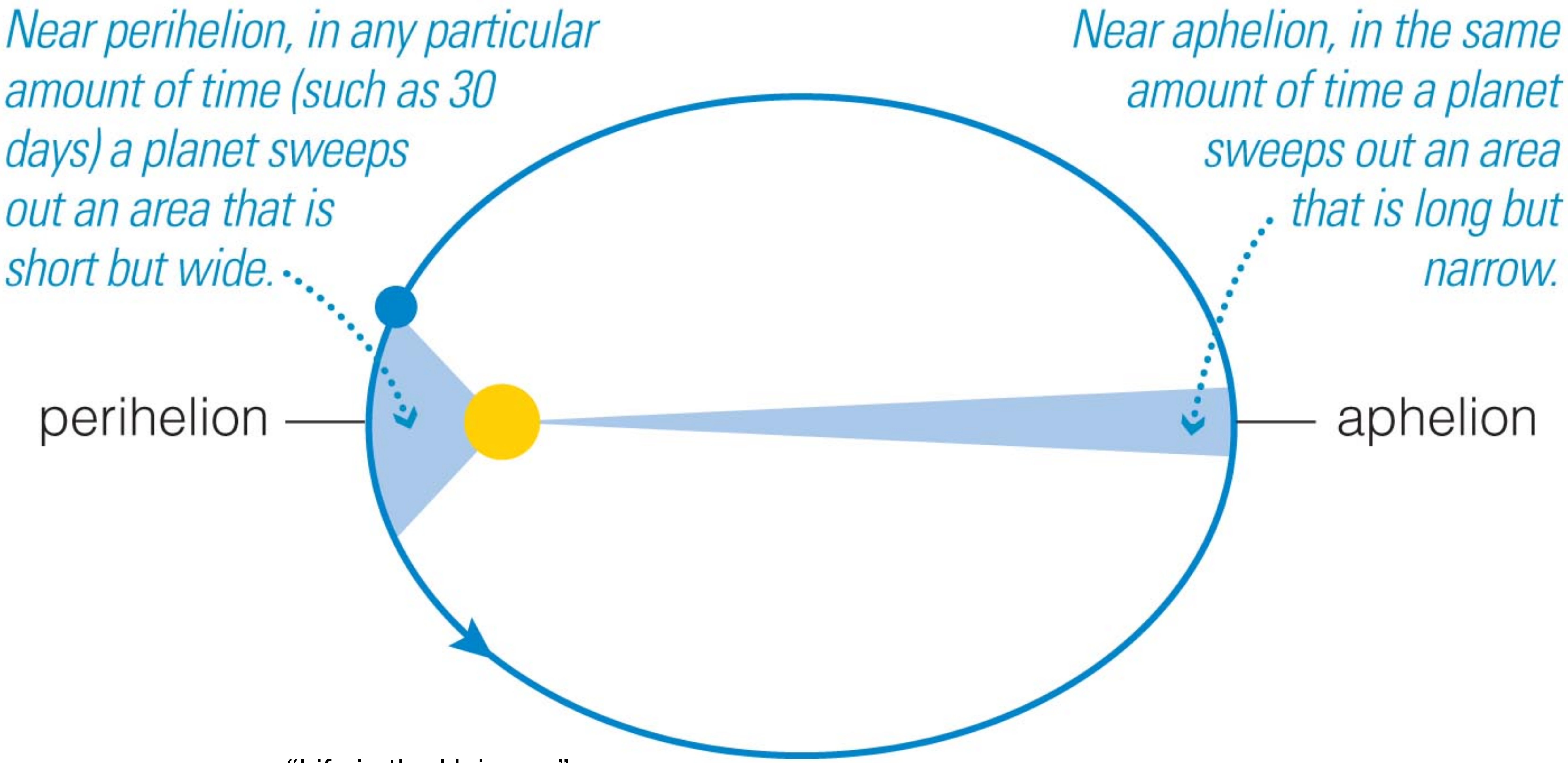
- Planet 1 and Planet 2 have identical orbital periods, identical masses, and orbit identical stars. But Planet 1 is on a circular orbit and Planet 2 is on a highly eccentric orbit. Which planet has a larger velocity semi-amplitude?

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

  - (A) — Planet 1
  - (B) — Planet 2
  - (C) — They have identical semi-amplitudes
  - (D) — There's no way to tell

Kepler's Second Law

- As a planet moves around its orbit, it sweeps out equal areas in equal times.
- This means that a planet travels faster when it is nearer to the Sun and slower when it is farther from the Sun.
- Angular momentum (m * v * r) is conserved.



"Life in the Universe"

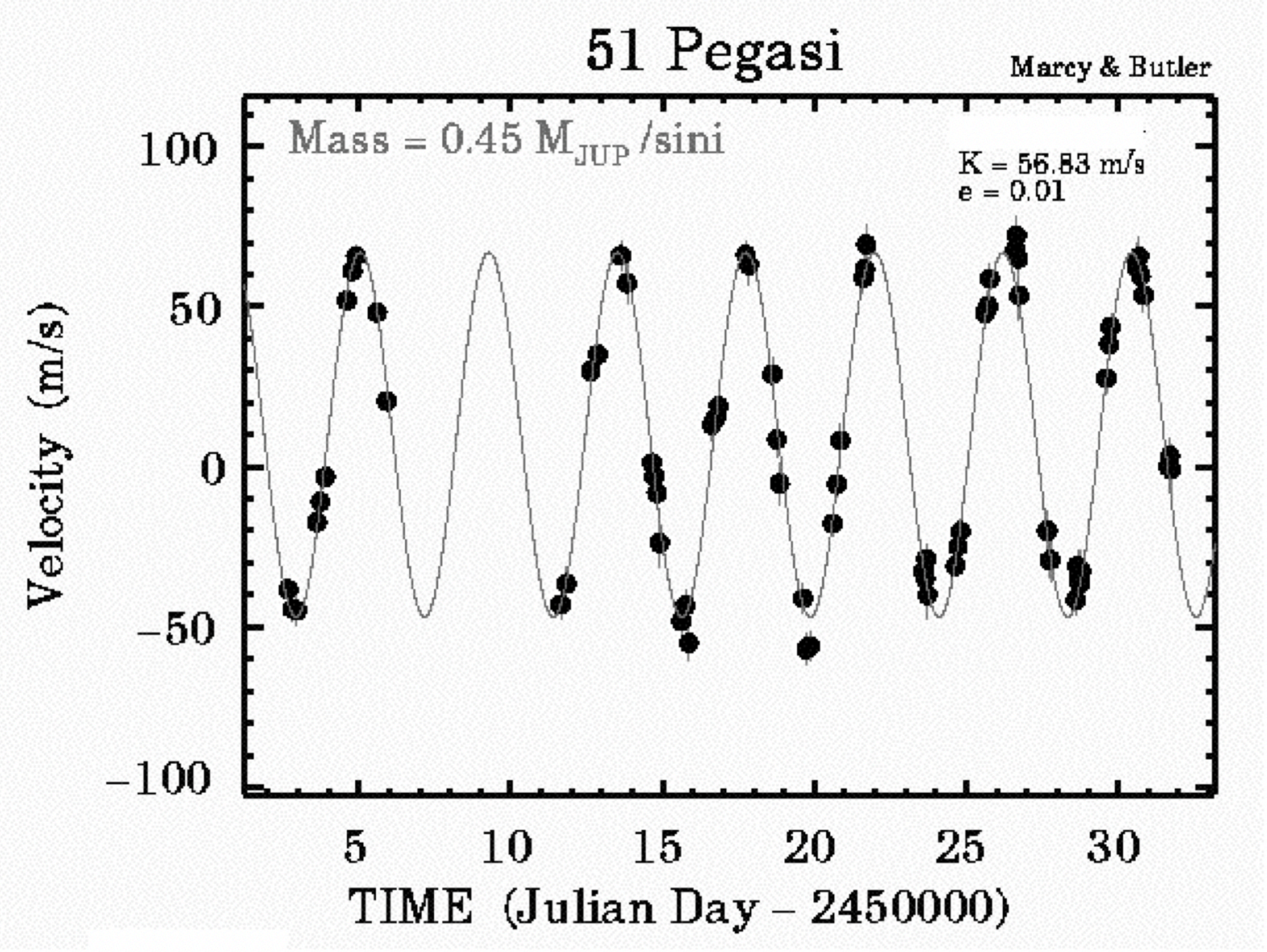
The Radial Velocity Method

- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1-e^2}}$$
- Using Kepler's third law to replace period with semi-major axis:

$$P^2 = \frac{4\pi^2 (a_1 + a_2)^3}{G(M_1 + M_2)}$$

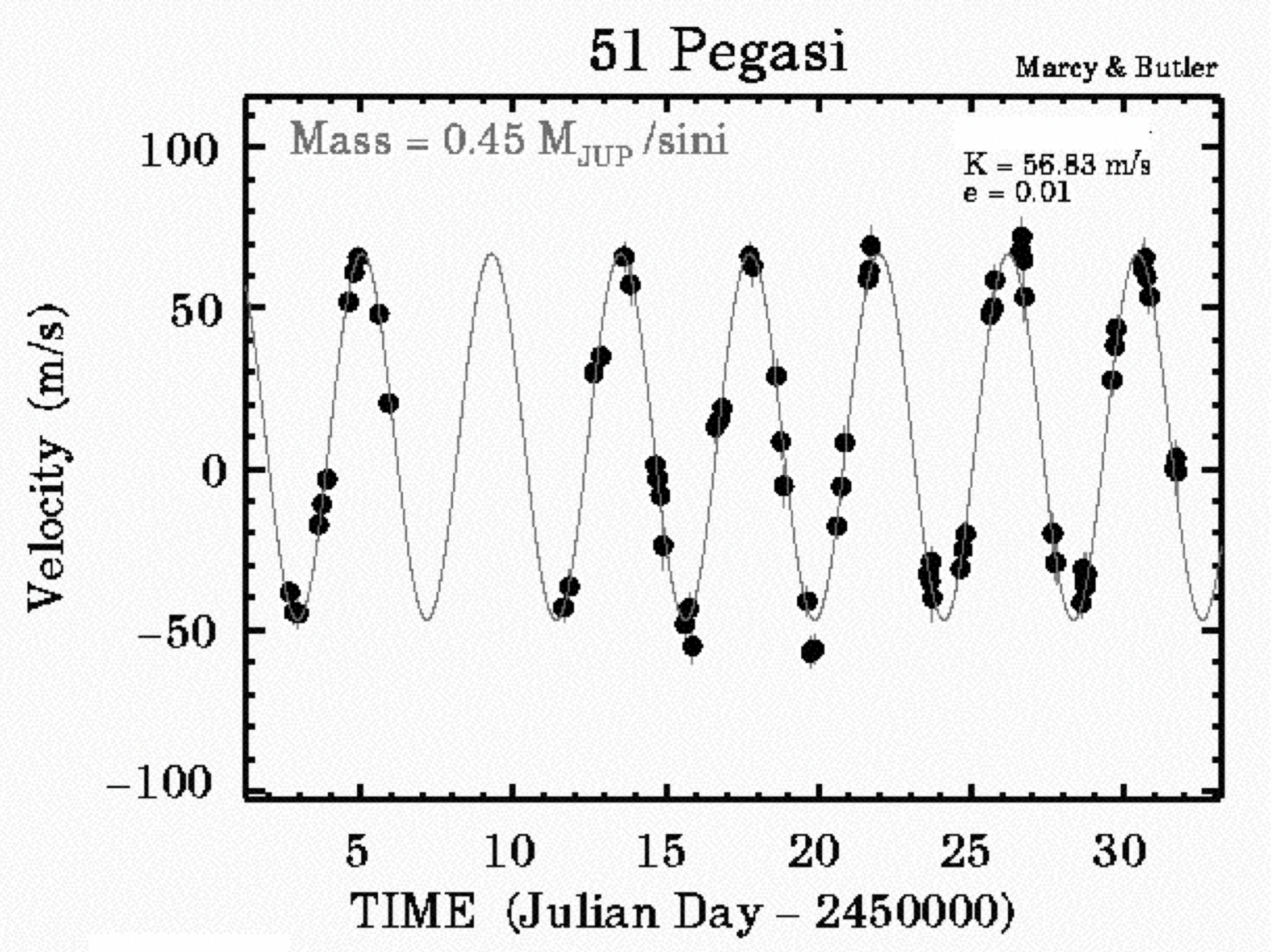
- $$K = \left(\frac{2\pi G G^{1/2} (M_1 + M_2)^{1/2}}{2\pi (a_{tot}^{3/2})}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

$$K = \sqrt{\frac{G}{a_{tot}}} \frac{M_2 \sin i}{\sqrt{M_1 + M_2}} \frac{1}{\sqrt{1-e^2}}$$



The Radial Velocity Method

- $$K = \sqrt{\frac{G}{a}} \frac{M_2 \sin i}{\sqrt{M_1 + M_2}} \frac{1}{\sqrt{1 - e^2}}$$
- Radial velocity of the host star increases as:
  - The semi-major axis of the orbit becomes smaller ($\propto \frac{1}{\sqrt{a}}$)
  - The mass of the planet becomes larger ($\propto M_2$)
  - The eccentricity becomes larger
  - The orbit becomes more edge-on (i closer to 90)



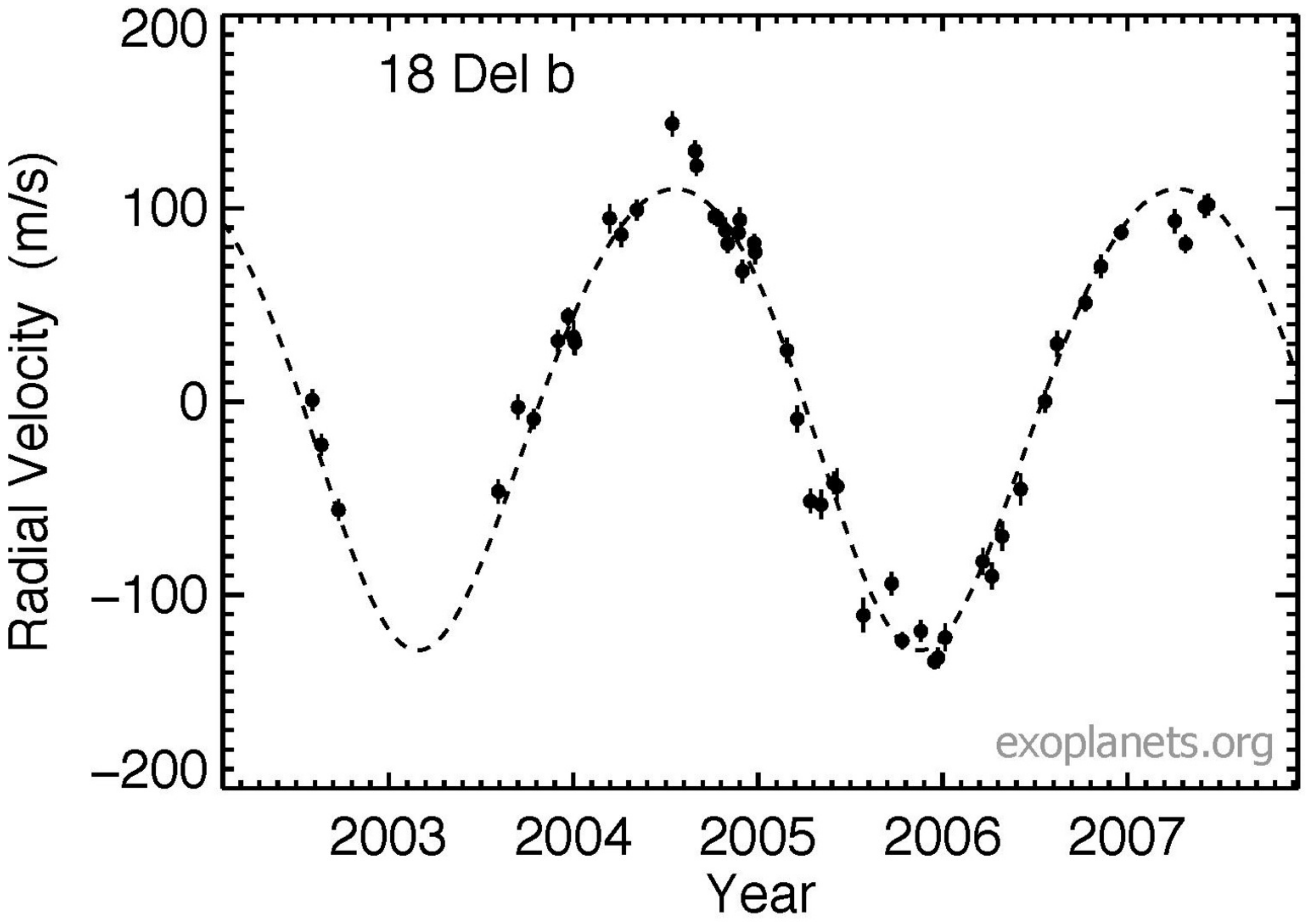
The Radial Velocity Method

- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

- We can get period by measuring the amount of time for the RV curve to repeat (peak to peak, or valley to valley)

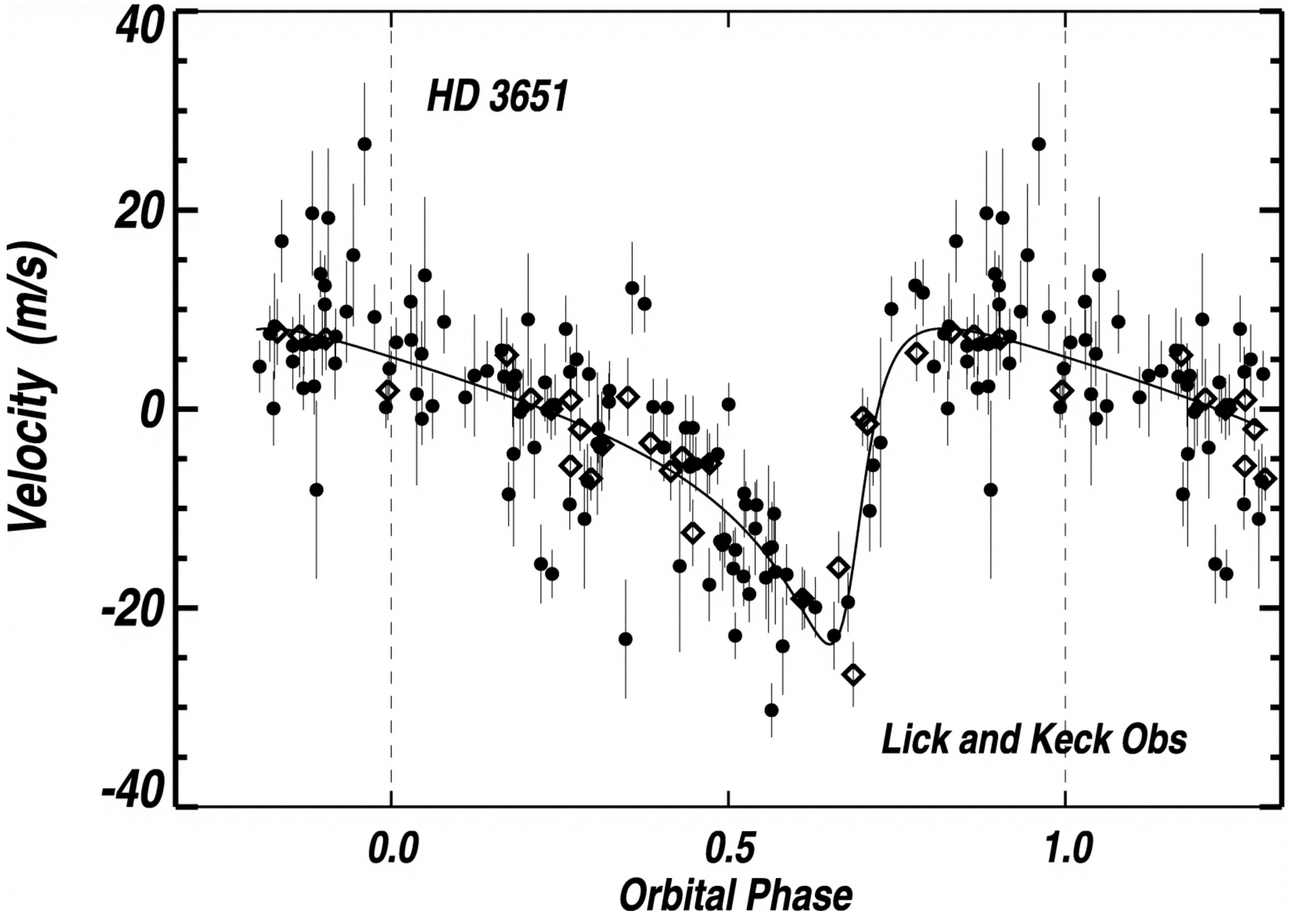
- The 1D component of circular motion is a sine curve, so a circular (e=0) orbit will look like a sine curve

    - We're only seeing the component of velocity directly toward or away from us



The Radial Velocity Method

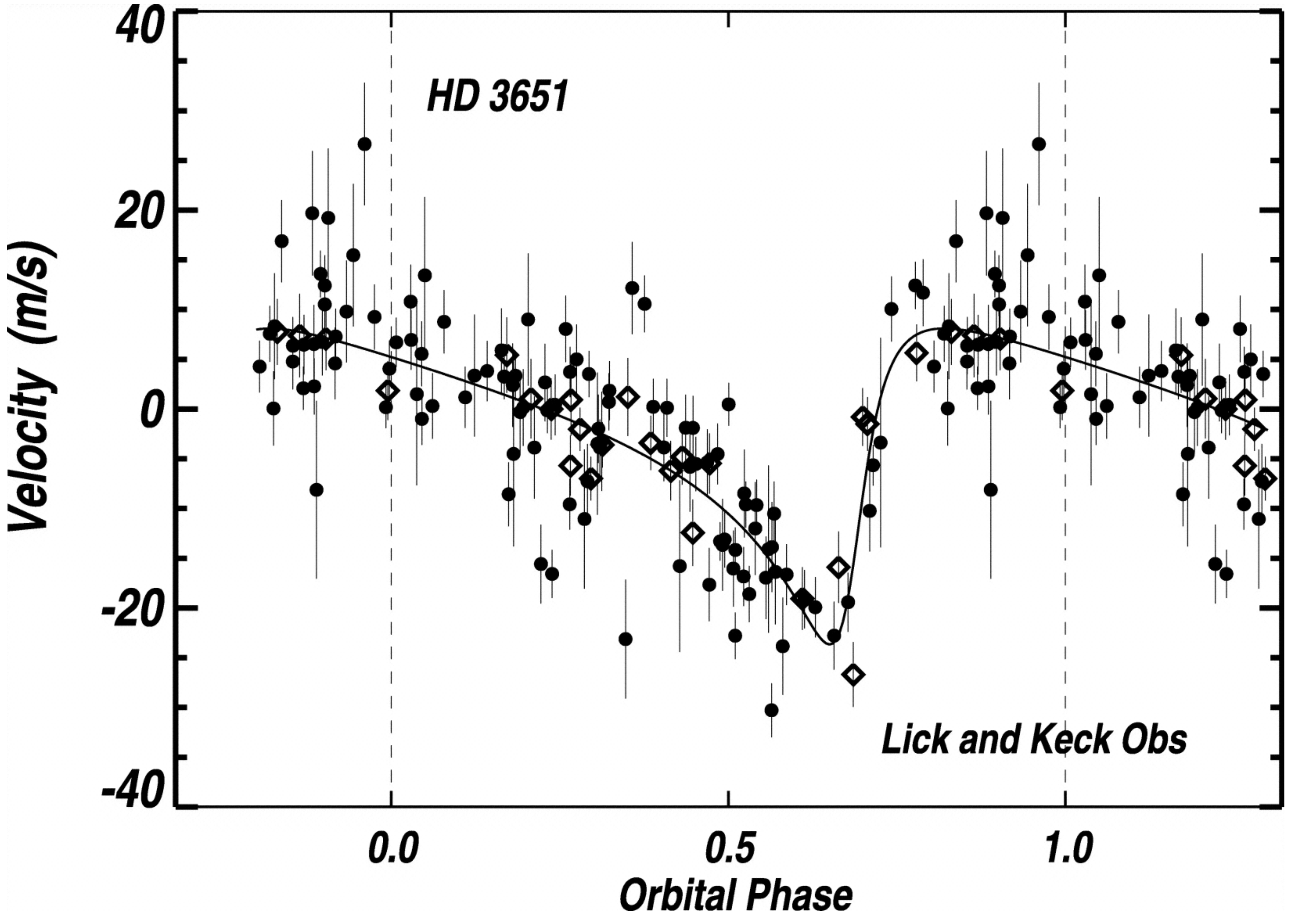
- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1-e^2}}$$
- In an eccentric orbit (by Kepler's second law), the star's velocity changes over the orbit
    - an eccentric orbit will have an asymmetric RV curve
- Planets are usually much less massive than the star, so $M_1 + M_2 \approx M_1$
    - Can get stellar mass from spectrum, luminosity, and stellar models



Fischer et al. 2003

The Radial Velocity Method

- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$
- These observations (and stellar models) let us relate K (something we measure) to $M_2 \sin i$
- We can't measure inclination angle from radial velocity data alone. We'd need additional information like:
    - If the planet passes in front of its star (transits)
    - directly imaging the planet
    - measuring the planet's gravitational perturbation on other planets in the system
    - measuring the star's motion in the plane of the sky (astrometry)
- $M_2 \sin i$ represents the minimum mass a planet could have



Fischer et al. 2003

Break

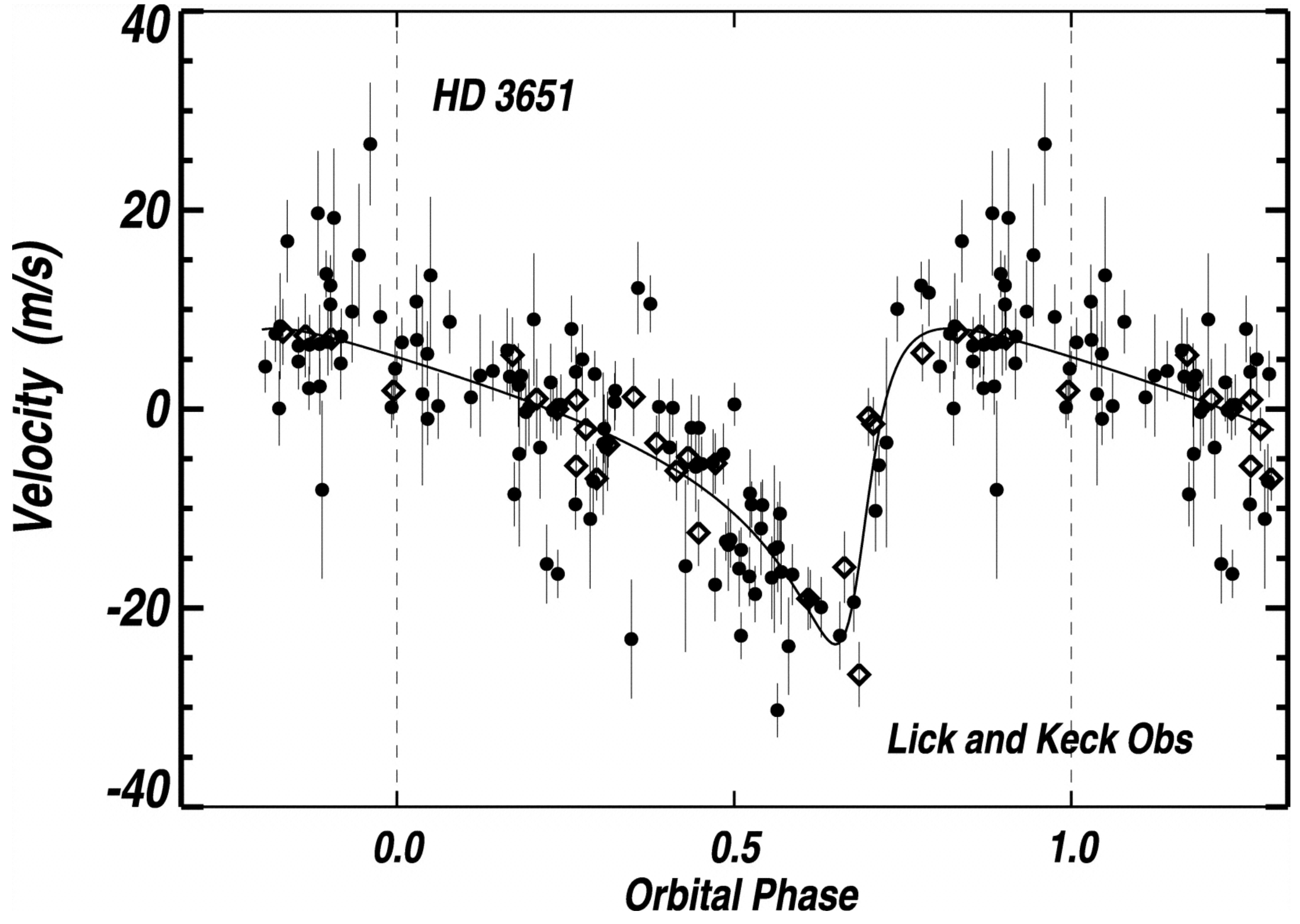
Order of magnitude: Radial Velocity

- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1-e^2}}$$

- Alien astronomers are observing the Sun with a high-precision spectrograph for many years, and are lucky enough to be oriented such that the inclination angle of our solar system is 90 degrees.

- (1) What is the radial velocity, in m/s, of the Sun's motion due to Jupiter?

- (2) What is the radial velocity, in m/s, of the Sun's motion due to the Earth?



Fischer et al. 2003

Order of magnitude: Radial Velocity

- $$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}}$$

- Alien astronomers are observing the Sun with a high-precision spectrograph for many years, and are lucky enough to be oriented such that the inclination angle of our solar system is 90 degrees.

- (1) What is the radial velocity, in m/s, of the Sun's motion due to Jupiter?

- We can make some simplifying assumptions here, both Jupiter and Earth have orbits that are very close to circular, so that last term involving eccentricity will reduce to 1. Also, both planets are much less massive than the Sun, so $M_1 + M_2 \approx M_1$. Jupiter has a 12 year period, and is 1000 times less massive than the Sun. And there are $\pi x 10^7$ seconds in a year.

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_2 \sin i}{(M_1 + M_2)^{2/3}} \frac{1}{\sqrt{1 - e^2}} = \left(\frac{2\pi(7x10^{-8})}{(12x3x10^7)}\right)^{1/3} \frac{(10^{-3}x2x10^{33})\sin 90}{(2x10^{33})^{2/3}}$$

$$K = \left(\frac{2x3x7x10^{-8}}{12x3x10^7}\right)^{1/3} \frac{2x10^{30}}{(2x10^{33})^{2/3}} cm/s = (10^{-15})^{1/3} \frac{2x10^{30}}{2x10^{22}} = 10^{-5}10^{8} = 10^{3} cm/s = 10 m/s$$

Order of magnitude: Radial Velocity

- (2) we could do that all again, but we're only changing two quantities: we're making period 12 times smaller, and mass 300 times smaller, so let's try scaling our answer:

$$K_E = K_J \frac{M_E}{M_J} \left(\frac{P_J}{P_E} \right)^{1/3} = 10m/s \frac{1}{300} \left(\frac{12}{1} \right)^{1/3} = 10m/sx3x10^{-3}x2 = 6x10^{-2}m/s = 6cm/s$$

For next time

- Reading: Planetary Science, 12.2.3-12.2.5
- Homework 2 due Monday, November 6, at 11:59pm
- Let me know via email if you have conflicts on:
  - Monday, November 14
  - Wednesday, November 16