ASTR 620: Planetary Processes
Professor Eric Nielsen

Lecture 18: Atmospheres

Logistics

- Masks are encouraged

- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**

- Remember to bring you response card to class

Review of the last class

- If Earth didn't rotate, Hadley circulation would consist of:
  - (A) — One convection cell, rising at the summer pole and falling at the winter pole
  - (B) — Two convection cells, rising at the poles and falling at the equator
  - (C) — Two convection cells, rising at the equator and falling at the poles
  - (D) — Six convection cells, rising at the poles and at +/- 60 degrees lattiude
  - (E) — Six convection cells, rising at the equator and at +/- 30 degrees latitude

Review of the last class

- Since Earth does rotate, circulation on Earth consists of:
  - (A) — One convection cell, rising at the summer pole and falling at the winter pole
  - (B) — Two convection cells, rising at the poles and falling at the equator
  - (C) — Two convection cells, rising at the equator and falling at the poles
  - (D) — Six convection cells, rising at the poles and at +/- 30 degrees lattiude
  - (E) — Six convection cells, rising at the equator and at +/- 60 degrees latitude

Review of the last class

- If an acoustic wave is propagating in the x direction, then particles are oscillating in the:
  - (A) — x direction
  - (B) — y direction
  - (C) — z direction
  - (D) — y and z direction
  - (E) — Particles don’t move in an acoustic wave

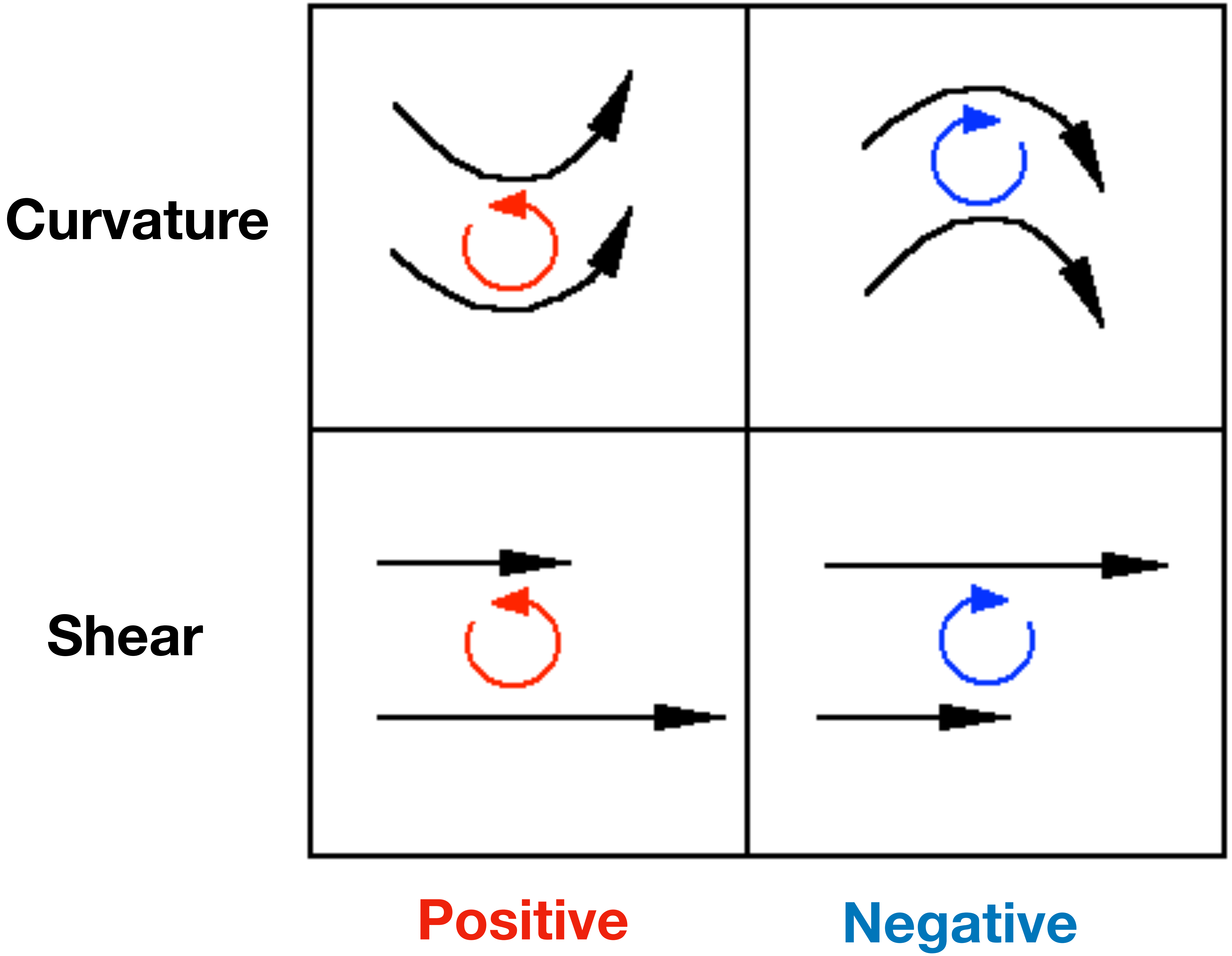
Review of the last class

- The restoring force in an acoustic wave is:
  - (A) — Gravity
  - (B) — Tension on magnetic field lines
  - (C) — Pressure
  - (D) — Bouyancy
  - (E) — Acoustic waves are a myth, like the aether

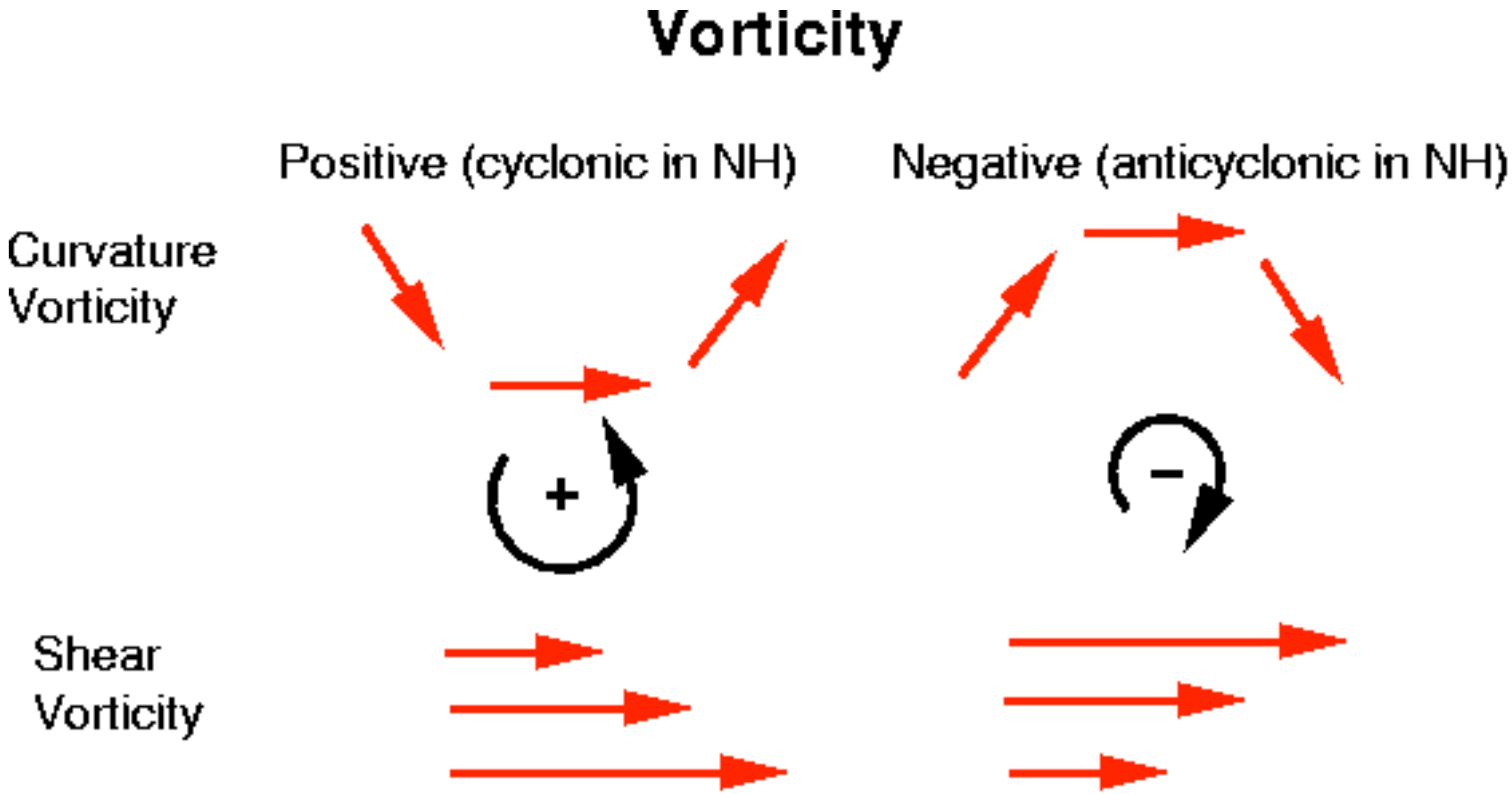
Review of the last class

- Wind is moving in a vortex in a clockwise direction. This vortex has:
  - (A) — positive vorticity
  - (B) — negative vorticity
  - (C) — There's no way to tell

Vorticity



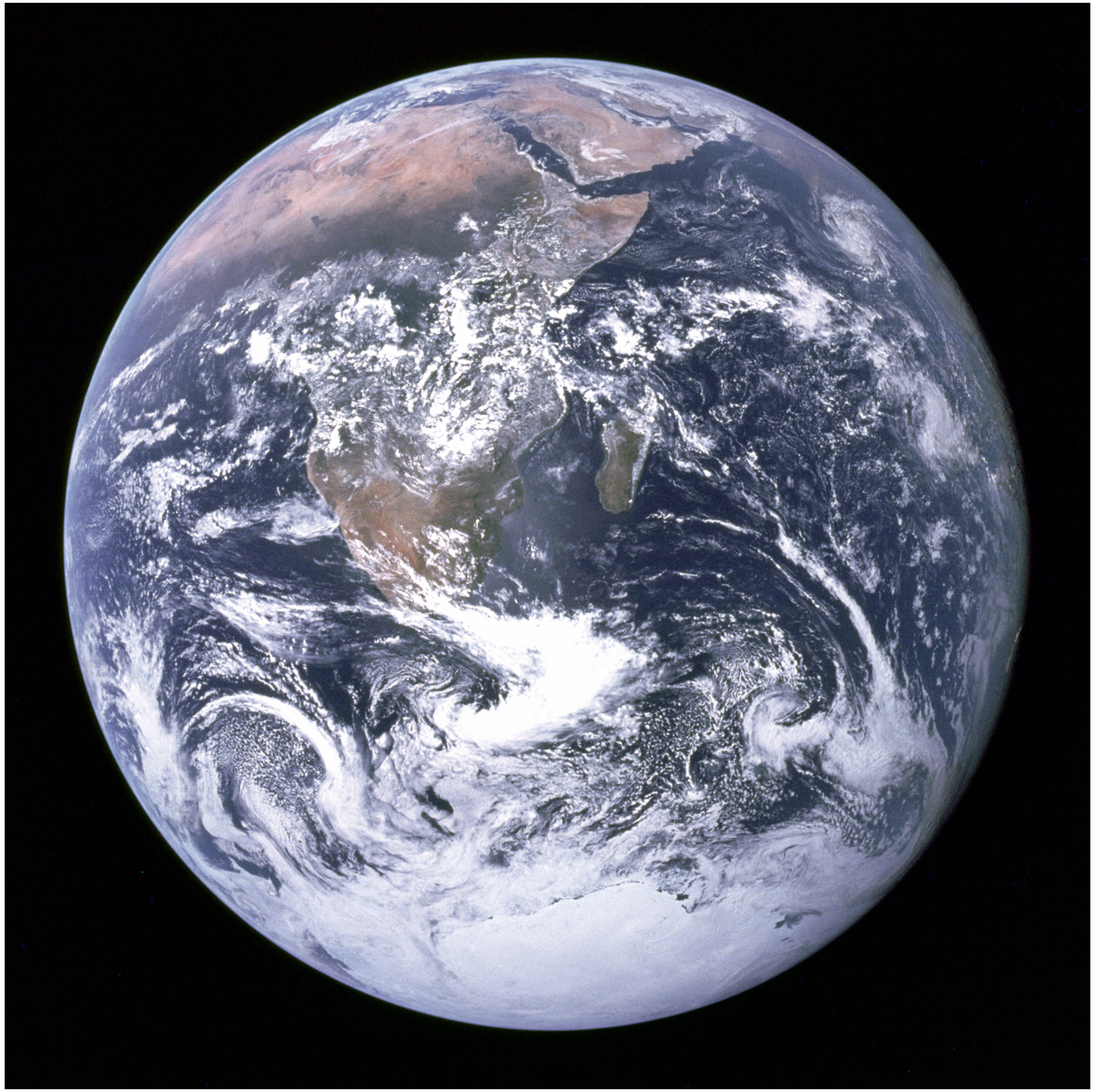
Vorticity


Vorticity
Positive (cyclonic in NH)
Negative (anticyclonic in NH)
Curvature Vorticity
+
−
Shear Vorticity

Vorticity

- Defined as the curl of the velocity vector
- Measures local rotation of fluid (s^{-1})
- $\overrightarrow{\omega} = \overrightarrow{\nabla} \times \overrightarrow{u} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$
    - For: $\overrightarrow{u} = (u, v, w)$
- For 2D flow in (x,y) plane, vorticity only has a z-component
- $\overrightarrow{\omega} = \omega(0,0,\xi)$

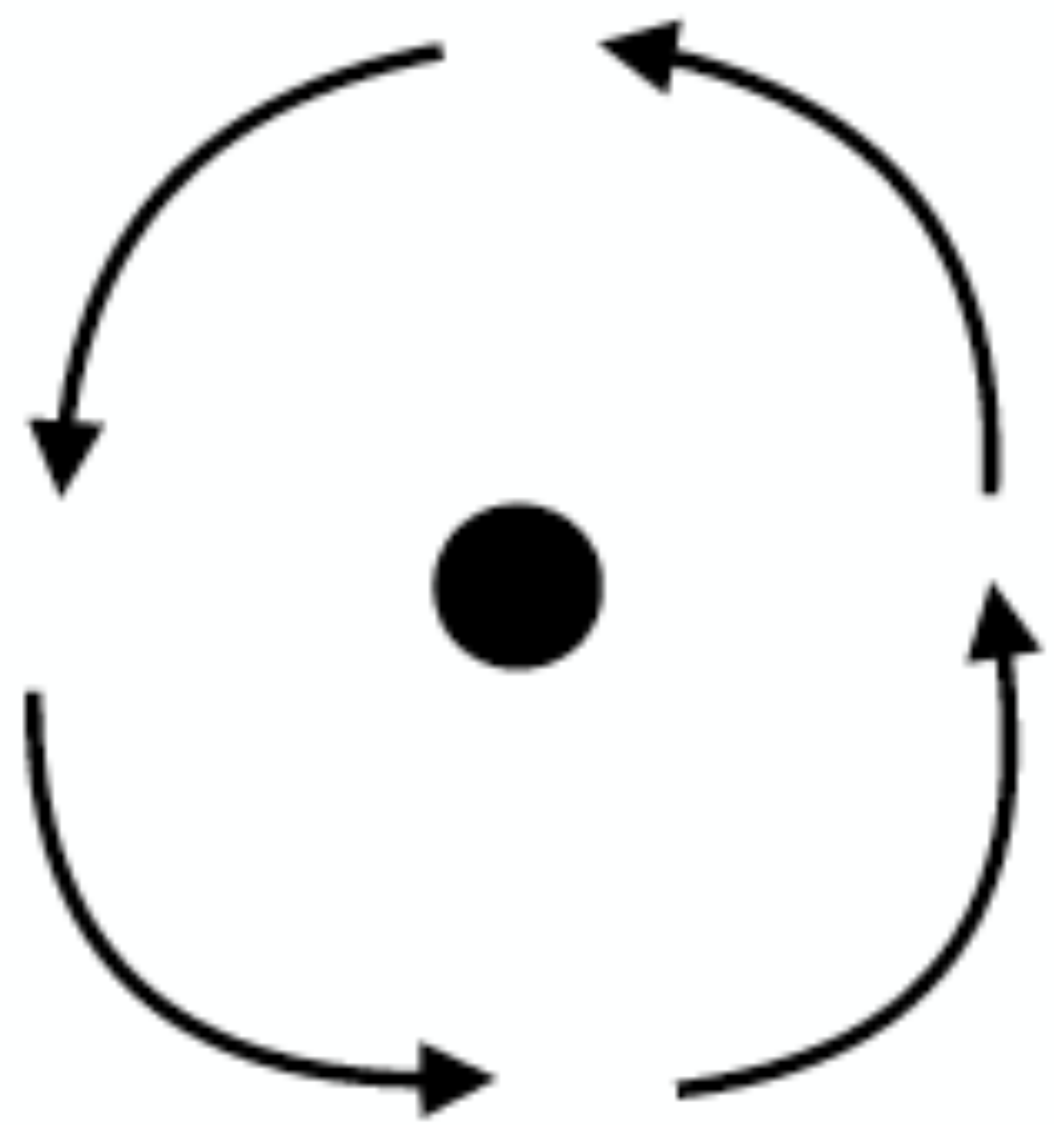
$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ This measures the “spin” of the flow



Vorticity

- Absolute vorticity, η, is the sum of the relative vorticity and planetary vorticity:
    - $\eta = \xi + f$ (generally conserved)
- ξ is highly correlated with synoptic scale weather disturbances.
    - A large, positive ξ means a cyclonic storm in the northern hemisphere (counterclockwise rotation)
    - $\frac{\partial v}{\partial x} > 0$ (northward wind increases as x increases)
    - $\frac{\partial u}{\partial y} < 0$ (eastward wind decreases as y increases)

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} > 0$$



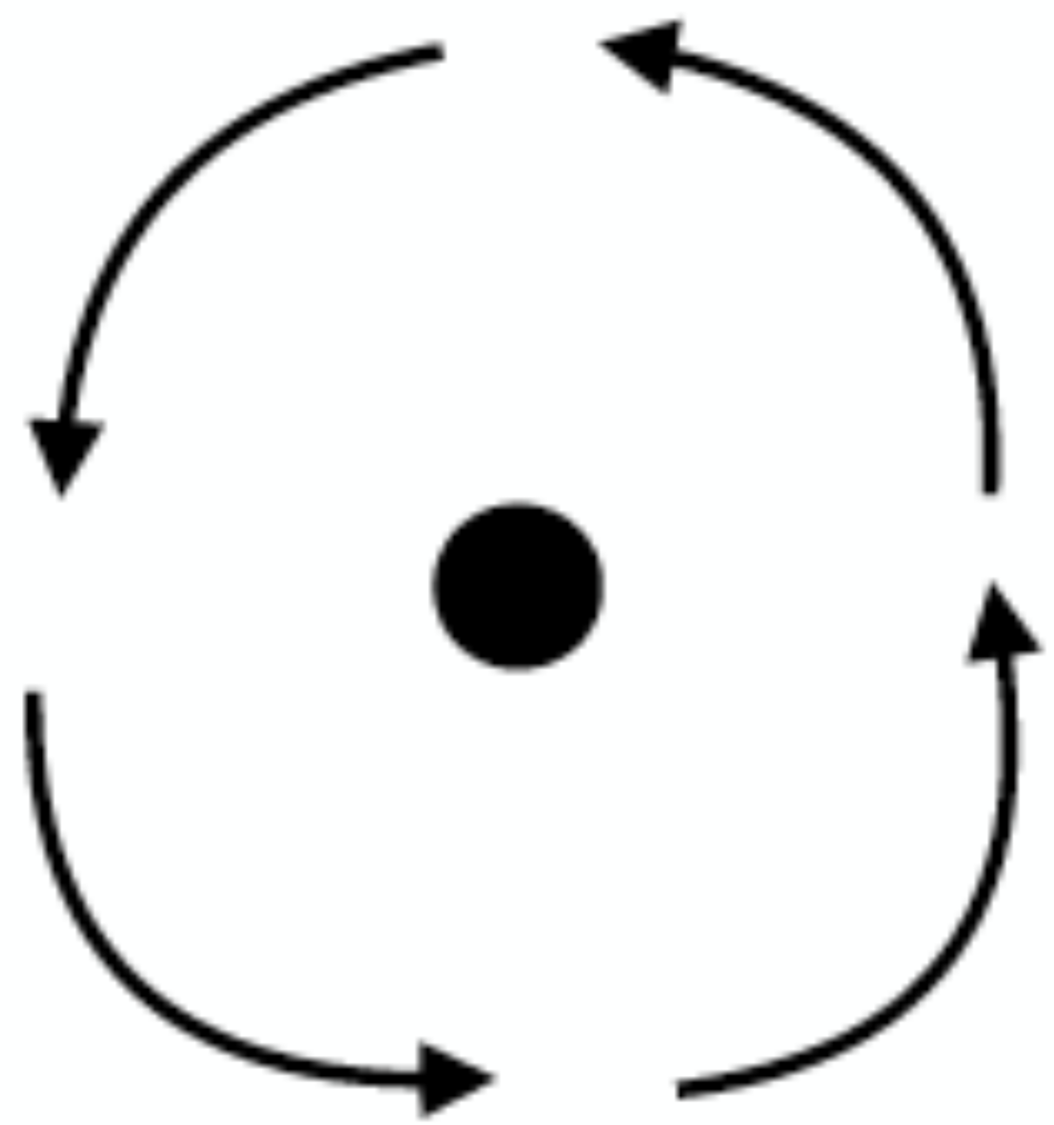
Potential Vorticity

- The "potential temperature" of a parcel of fluid at pressure P is defined as the temperature that parcel would have if moved adiabatically from its current pressure to P=1 bar

$$\theta = T\left(\frac{P_0}{P}\right)^{\frac{R}{C_P}}$$

- If we take an air parcel and compress/stretch it in the z-direction, the absolute vorticity η will change

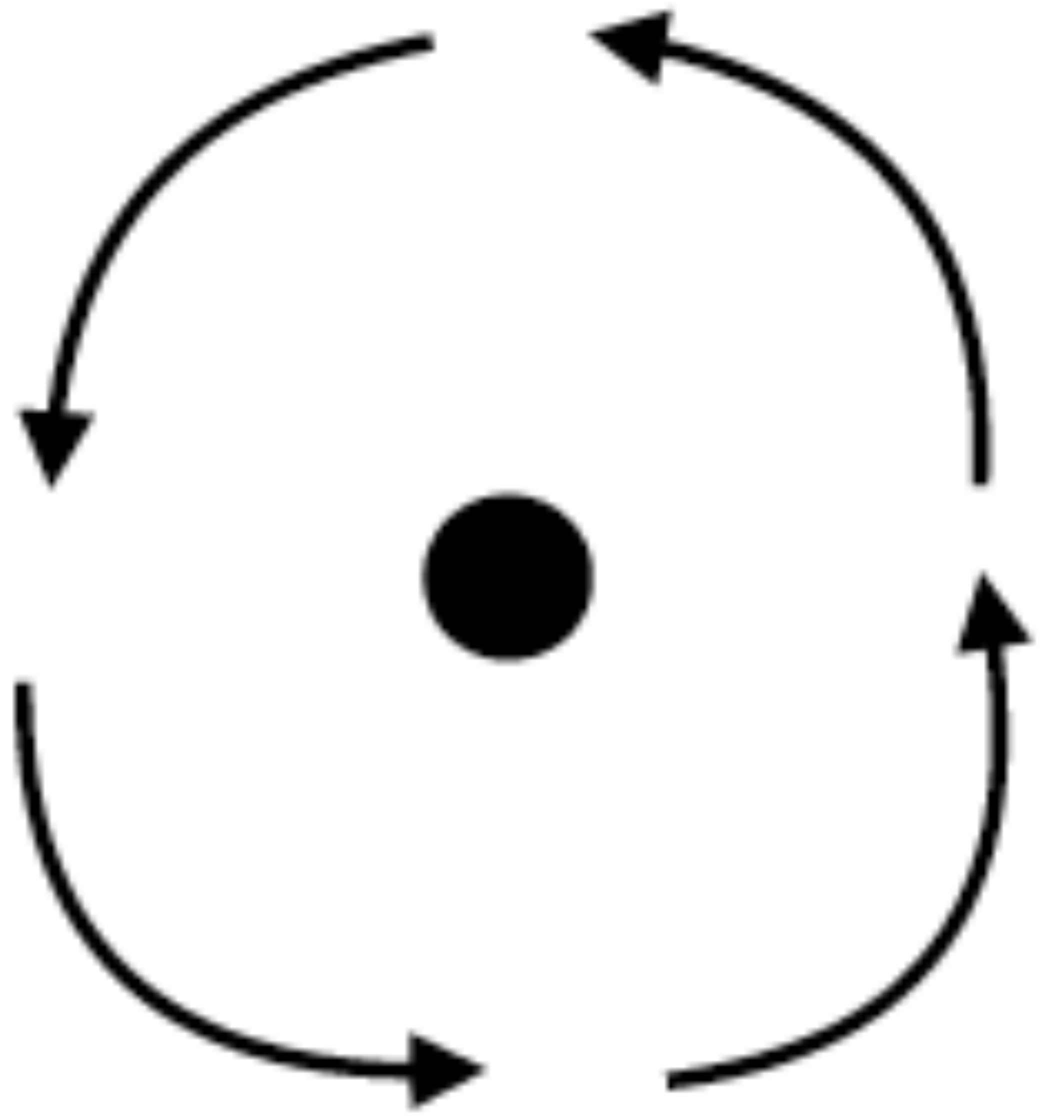
- But, if we divide η by the vertical spacing of levels of constant potential temperature, get conserved adiabatic flow



Potential Vorticity

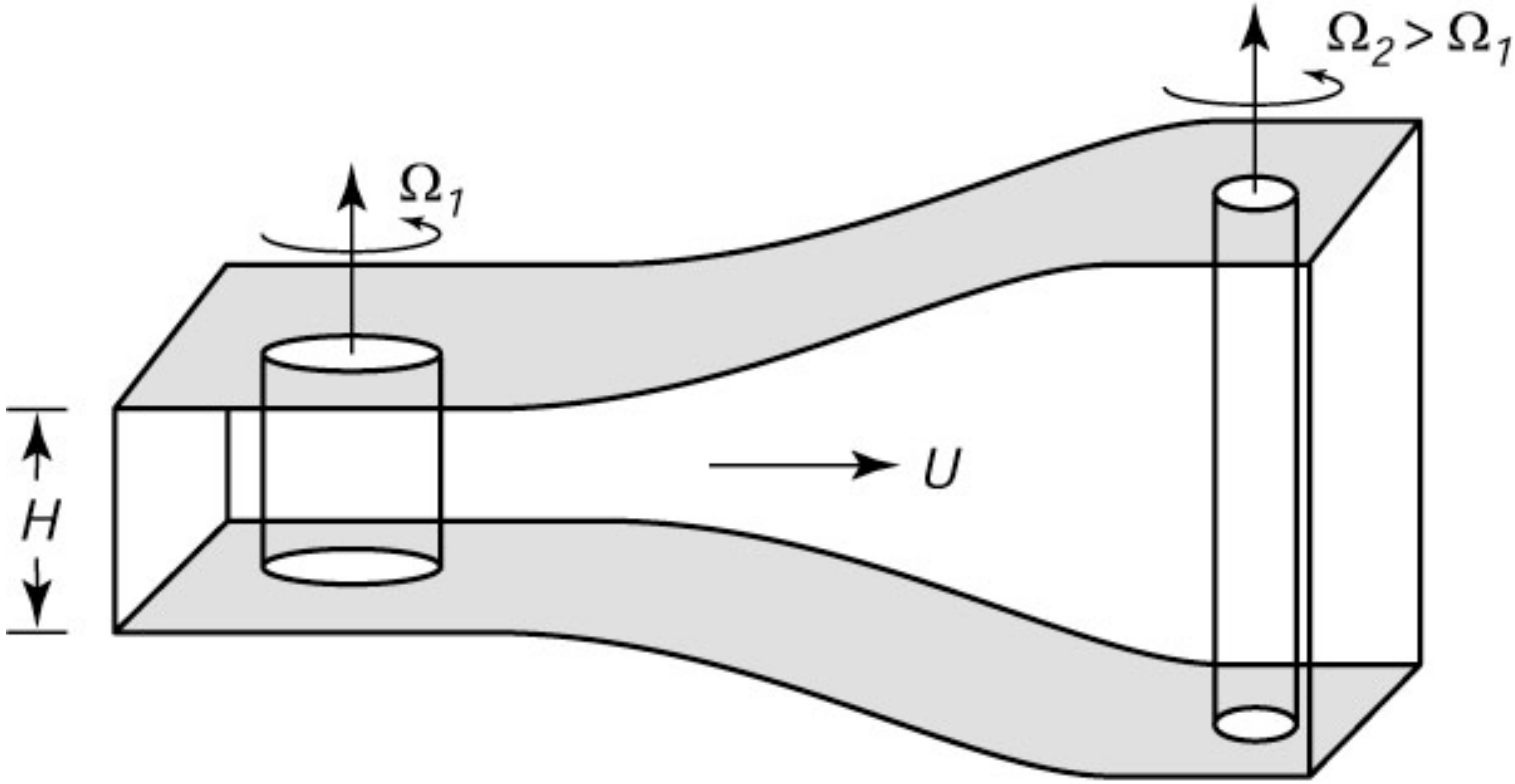
- $$PV = \frac{\eta}{\rho} \nabla \theta = (\xi + f) g \left(\frac{\partial \theta}{\partial P} \right)$$

- Potential vorticity represents a measure of the ratio of absolute vorticity to effective depth of the vortex



Conservation of PV

- Changes in the depth H of the flow cause changes in the relative vorticity
- As the vertical fluid column moves left to right, vertical stretching reduces the moment of inertia of the column, causing it to spin faster



Conservation of PV

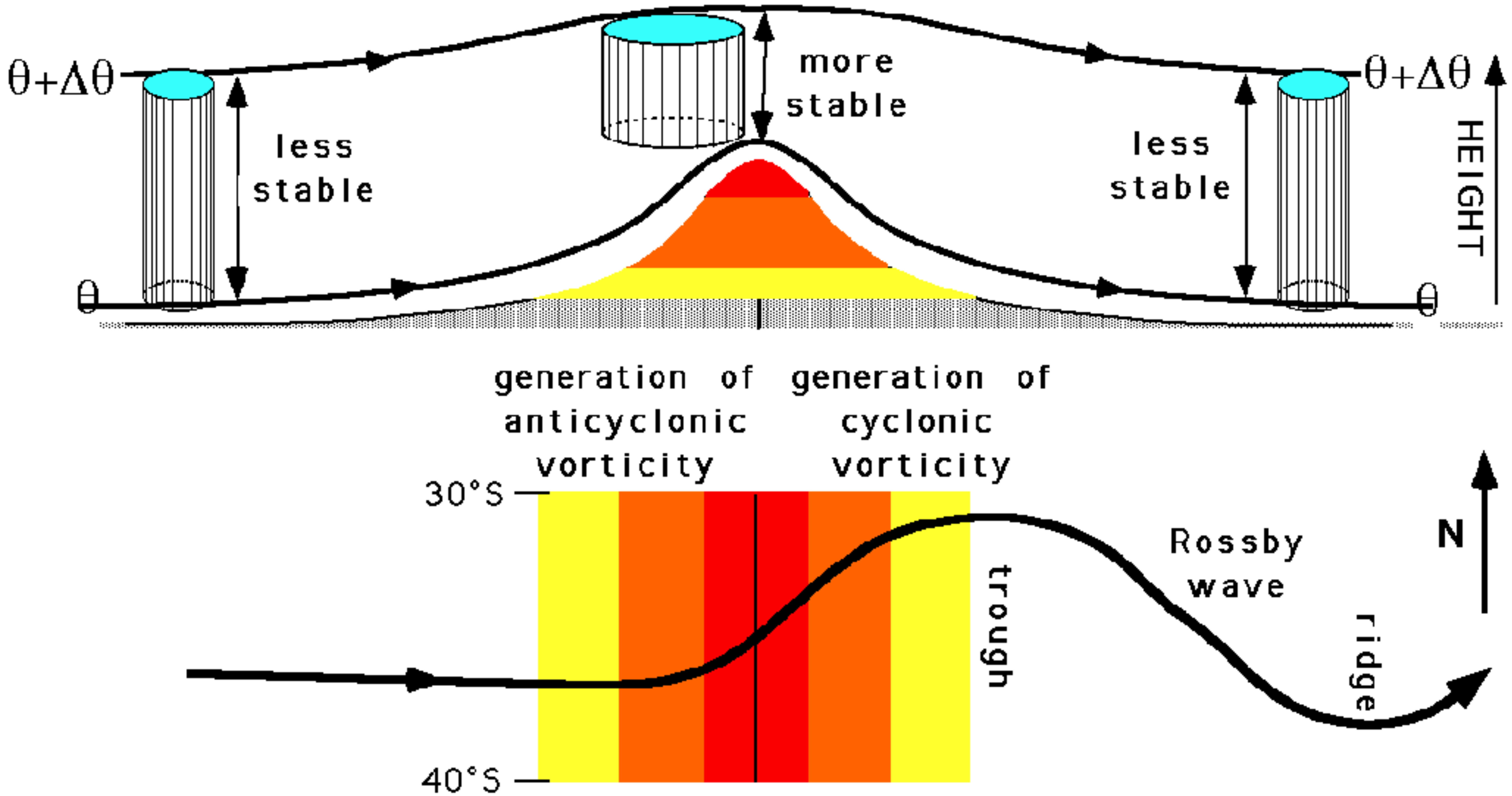
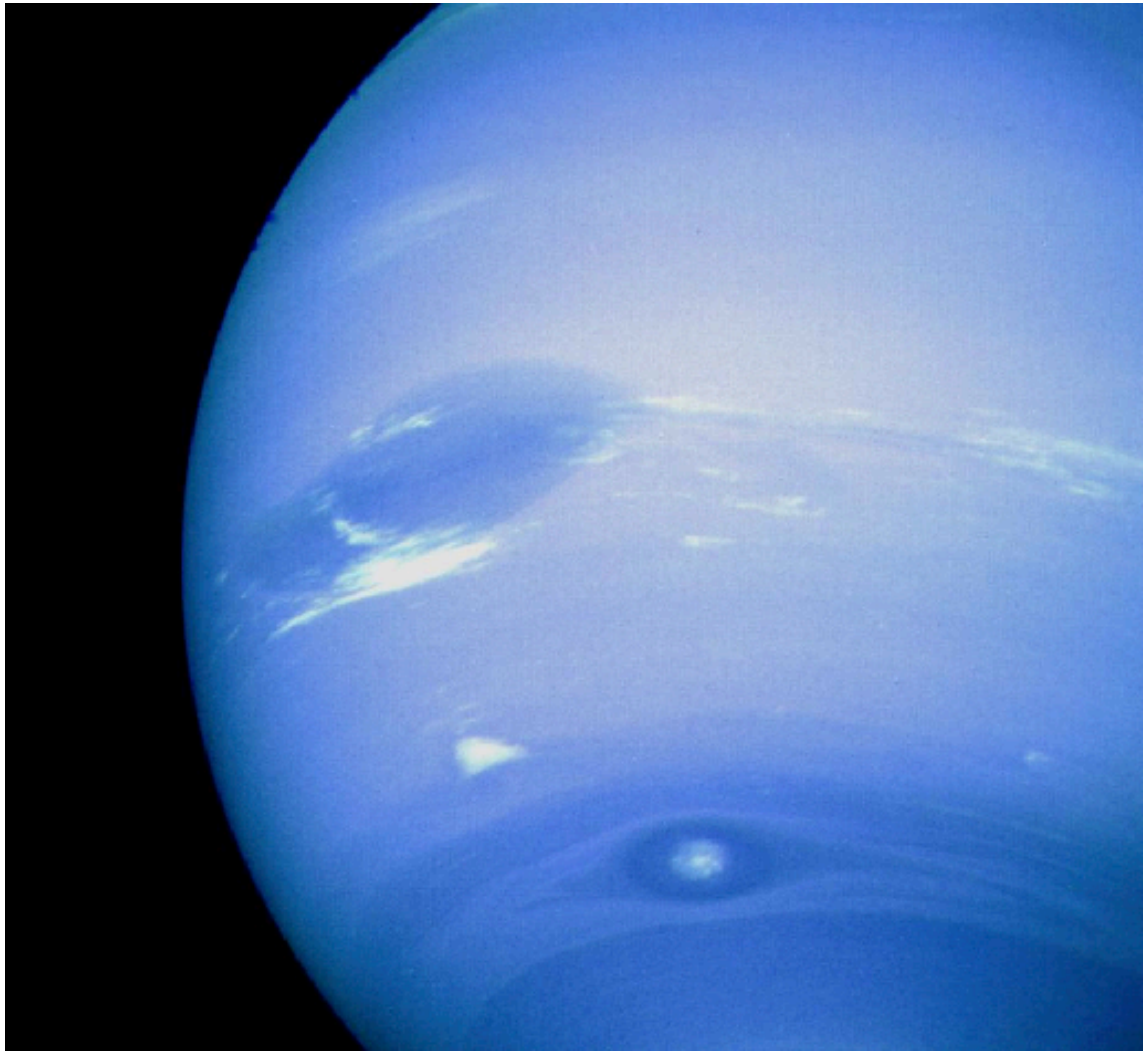


Fig 12.K.4 The formation of a trough in the lee of a mountain range in the southern hemisphere. The top figure is a vertical cross section, and the bottom one is a plan view. The mountain ridge is at the same location in both figures.

Neptune’s Great Dark Spot

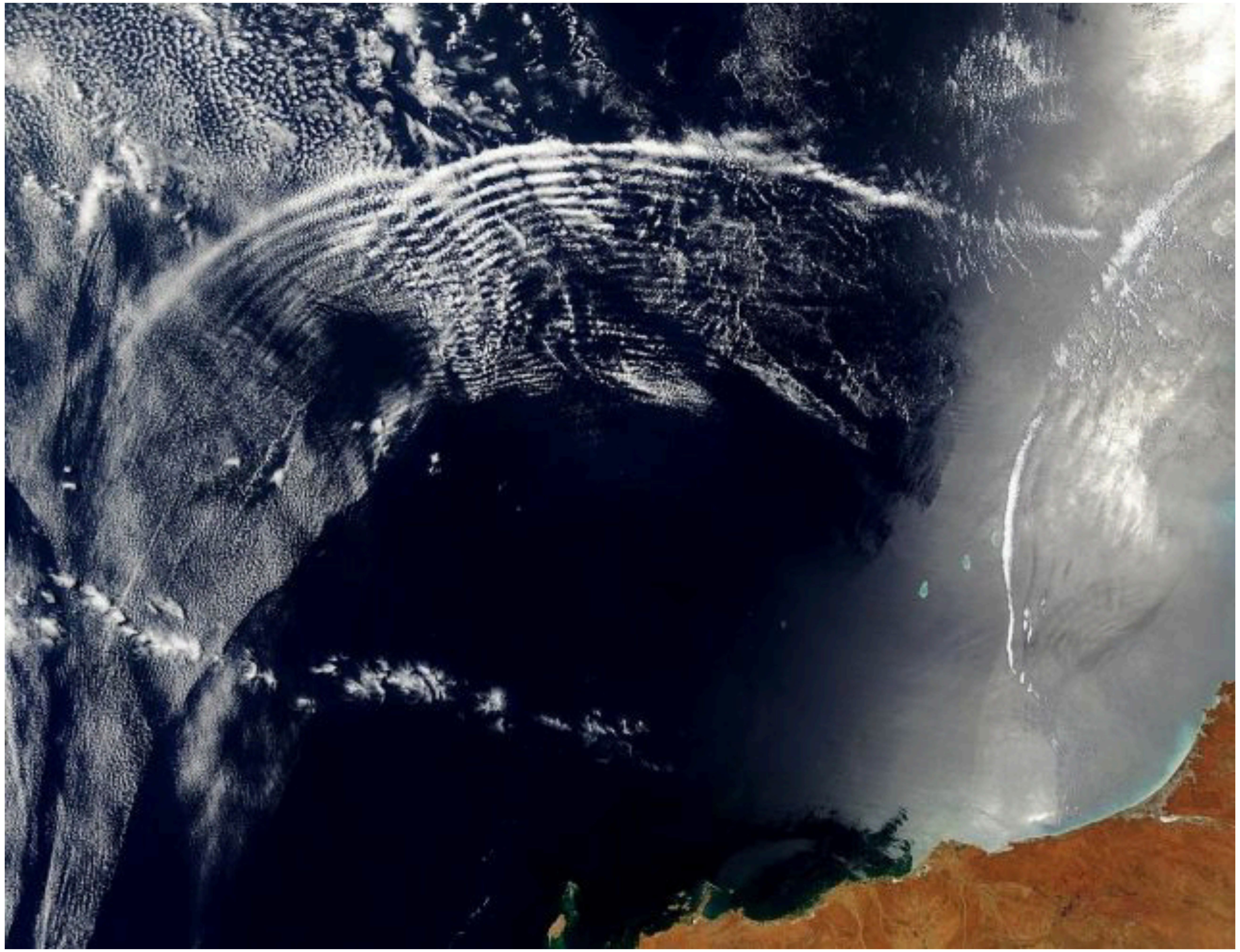
- Seen in Voyager images
- Observed to migrate slowly toward the equator
- $PV = \frac{\eta}{\rho} \nabla \theta = (\xi + f) g \left(\frac{\partial \theta}{\partial P} \right)$
- If latitude is changing, either vertical extent or relative vorticity must change



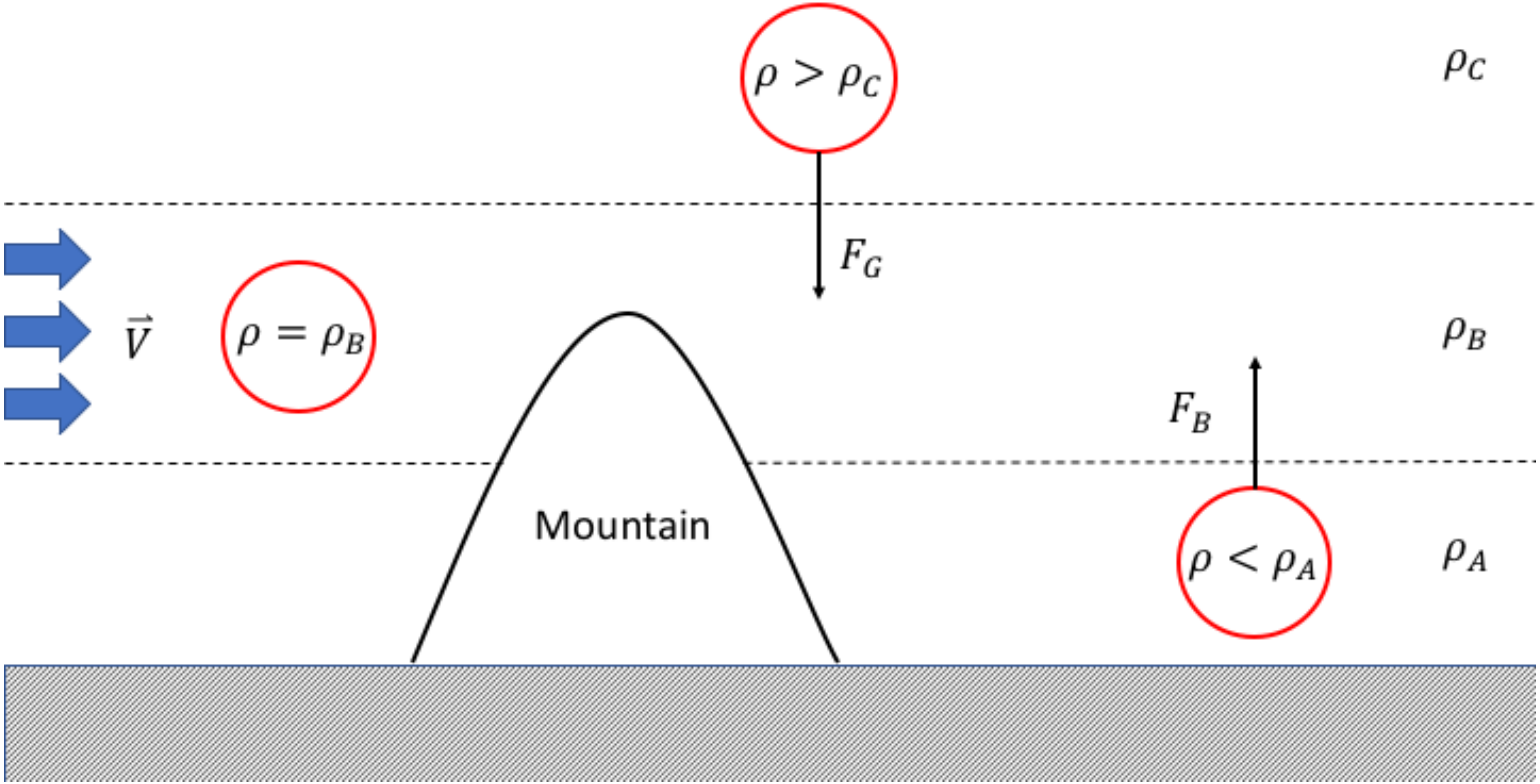
Break

Gravity Waves

- Air parcel (in a stable atmosphere) that is displaced vertically from its equilibrium position will undergo oscillations
  - balance between buoyancy and gravity
- Propagate vertically and horizontally, transporting momentum

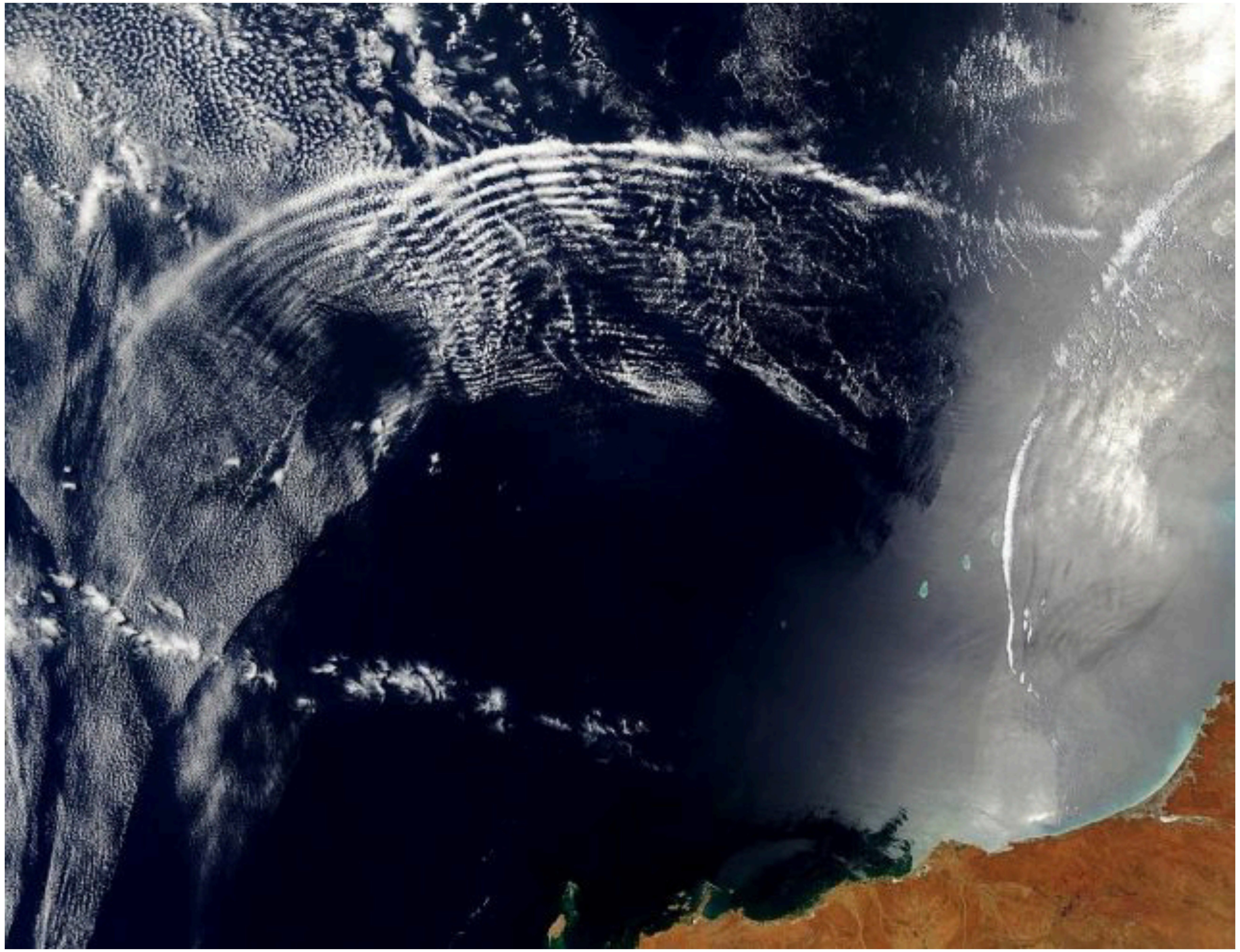


Gravity Waves



Gravity Waves

- Can be derived with the following assumptions:
    - Consider motions in the (x,z) plane
    - Neglect rotation (Coriolis) effects and friction
    - Unperturbed atmosphere is at rest
    - Density is constant except when coupled with gravity in buoyancy term of vertical motion equation
        - “Boussineq approximation”
    - Atmosphere is adiabatic



Gravity Waves

- Momentum Equation

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial x} = 0$$

$$\frac{dw}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial z} = 0$$

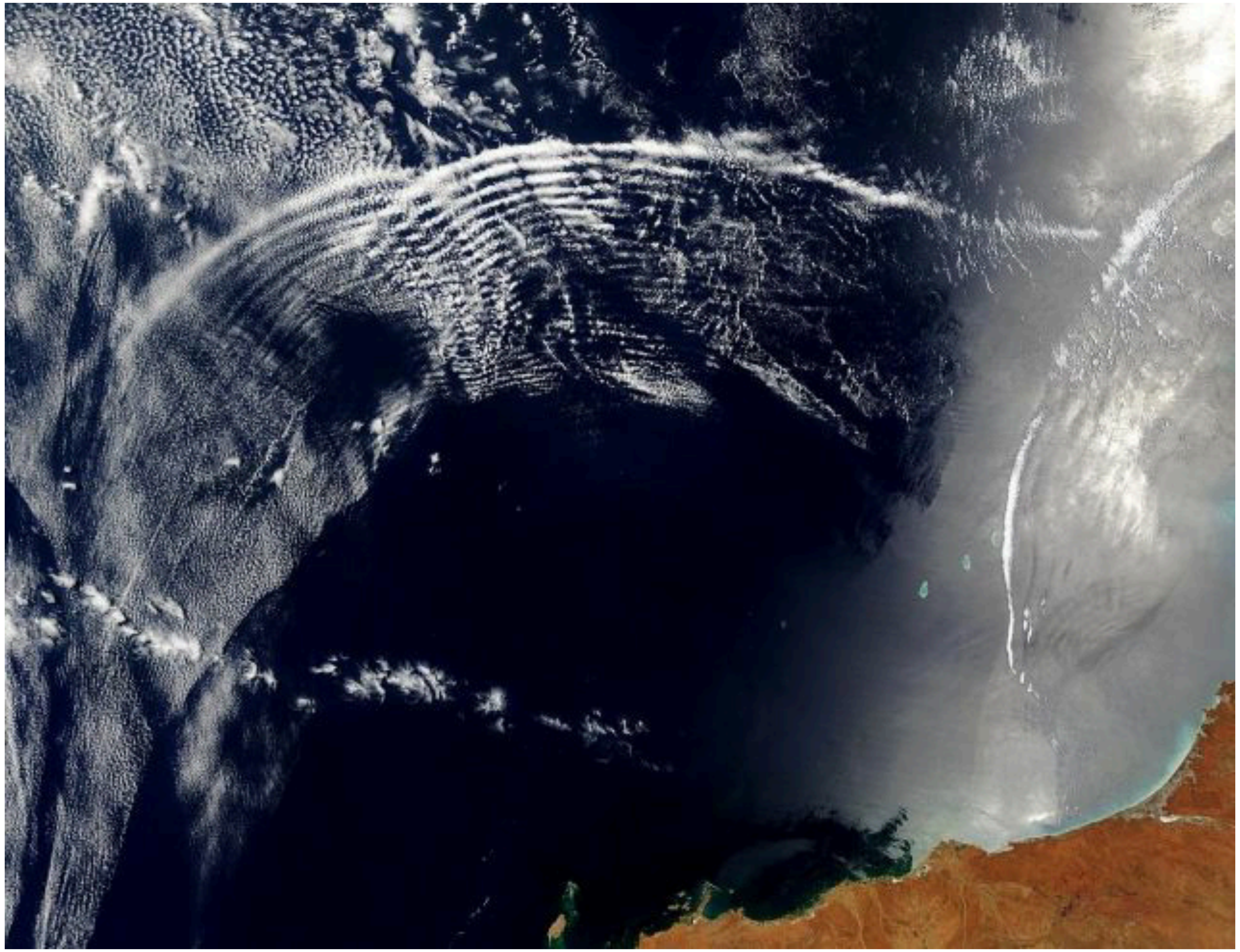
- Continuity Equation

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

- Thermodynamics:

$$P\rho^{-\gamma} = constant$$

$$\frac{d\theta}{dt} = 0$$



Gravity Waves

- Use perturbation theory to expand variables and linearize equations:

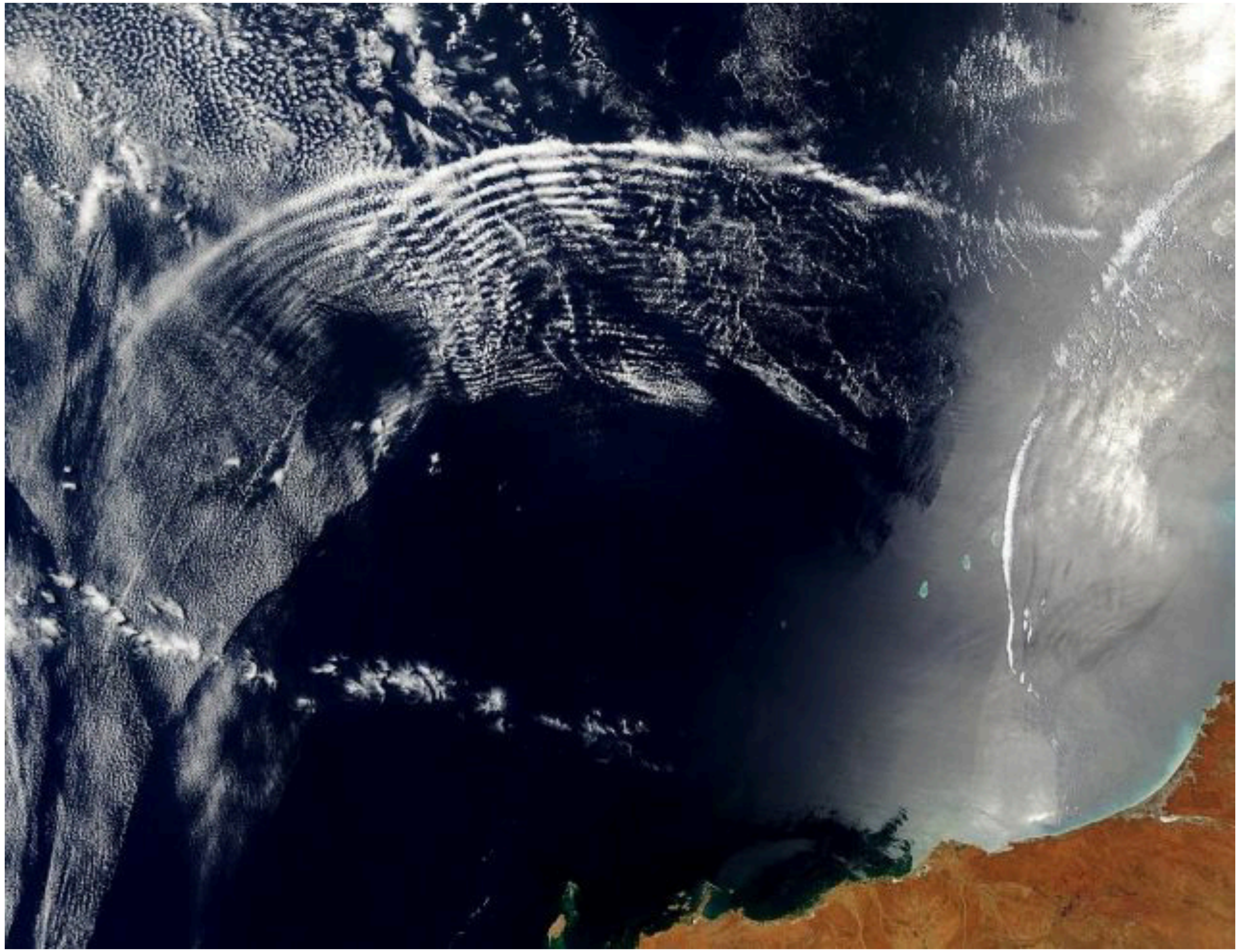
$$u(x,t) = \bar{u} + u'$$

$$P(x,t) = \bar{P}(z) + P'$$

$$\rho = \bar{\rho} + \rho'(x,t)$$

$$\theta = \bar{\theta}(z) + \theta'$$

$$w = w'$$



Gravity Waves

- After manipulating, can get wave equation:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 \left(\frac{\partial w'^2}{\partial x^2} + \frac{\partial w'^2}{\partial z^2}\right) + N_B^2 \frac{\partial w'^2}{\partial x^2} = 0$$

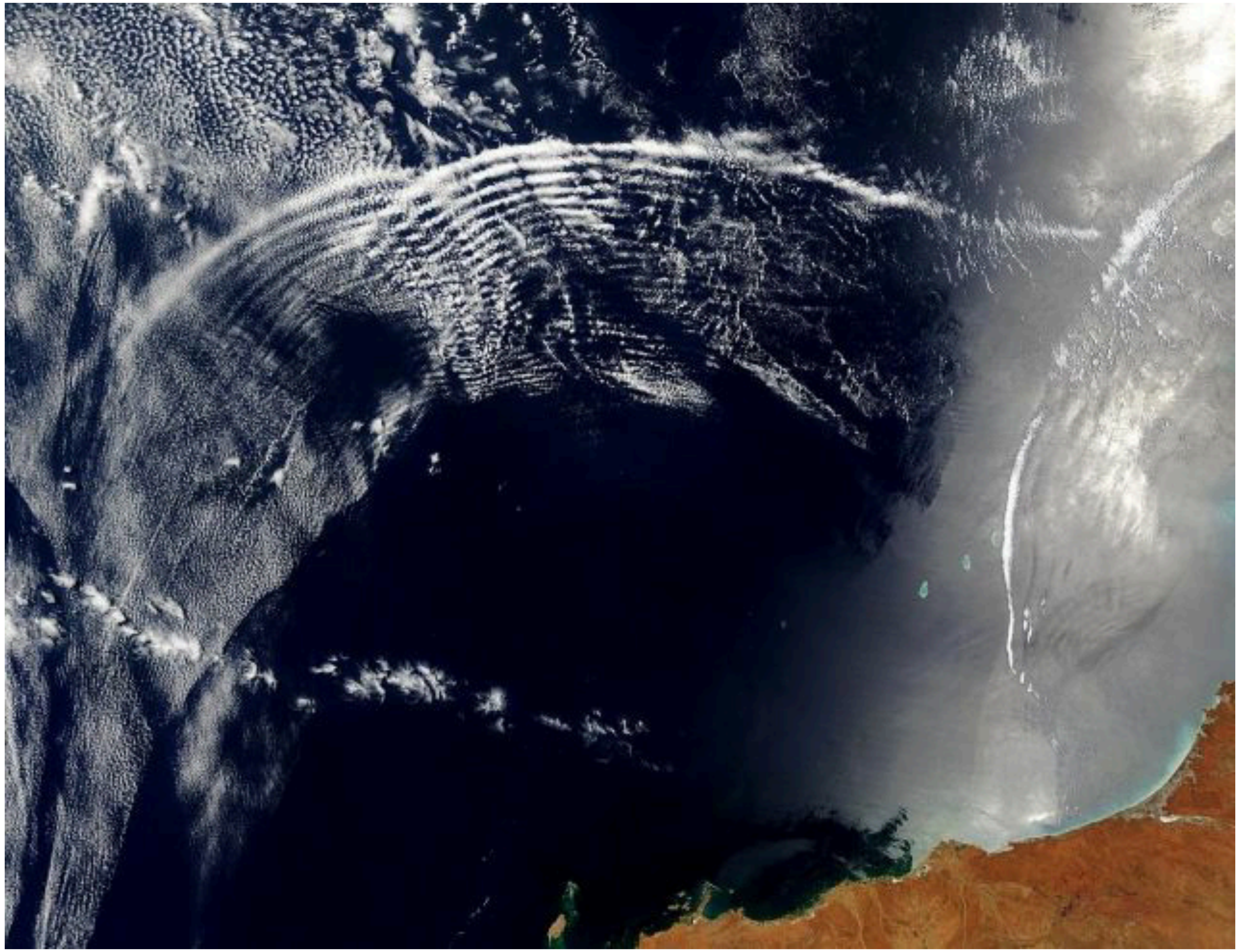
- N_B: the "Brunt-Vaisala frequency," assumed to be constant with altitude

- This equation has the solution:

$$w' = Real\{w_0 e^{i(kx+mz-\omega t)}\}$$

$$w_0 = w_{real} + iw_{imaginary}$$

- These oscillations are called "inertial gravity waves," they occur in the (x,z) plane with a phase ($kx + mz - \omega t$) that depends on (x,z,t)



Gravity Waves

- $w' = Real\{w_0 e^{i(kx+mz-\omega t)}\}$
- The horizontal wavenumber is real:

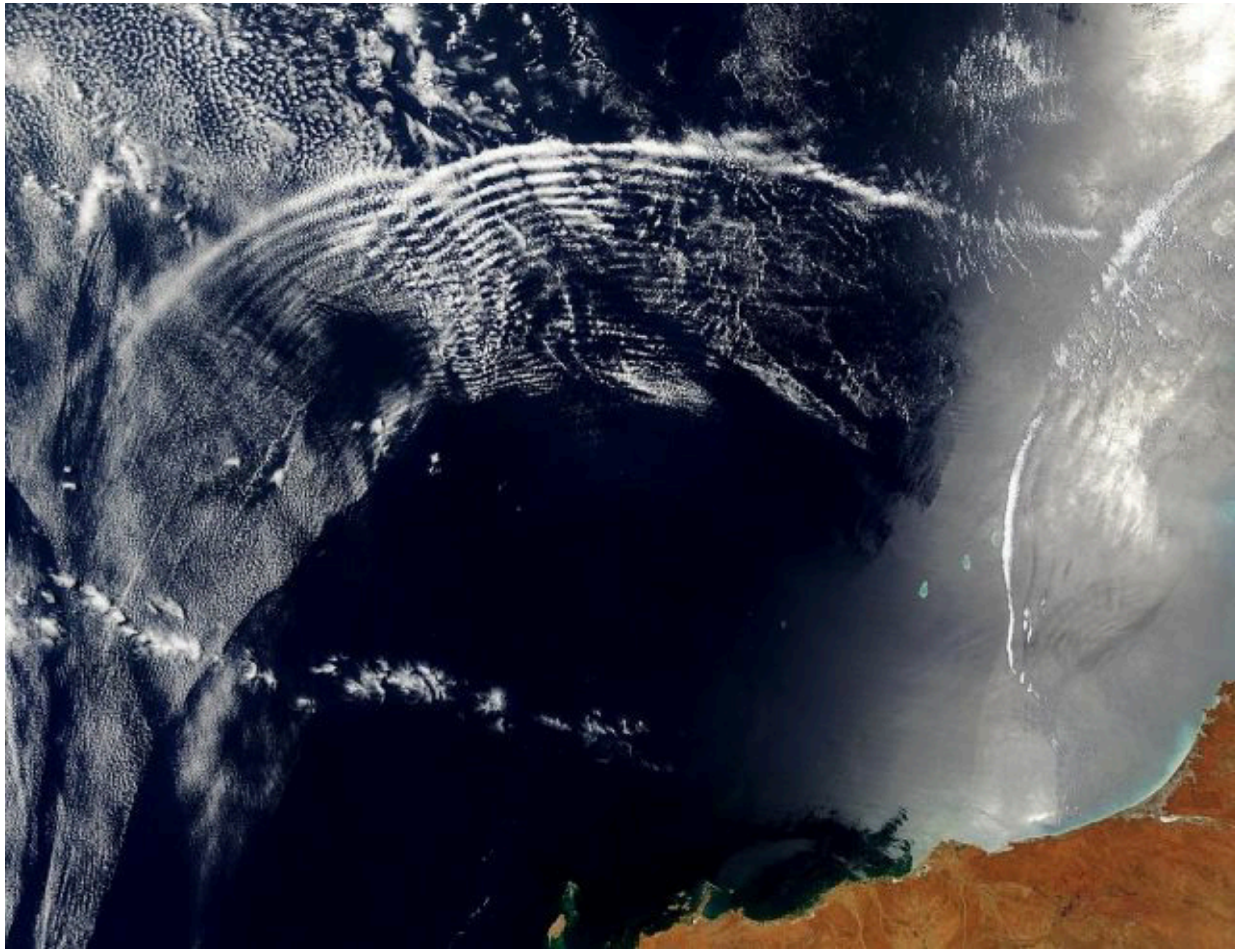
$k = \frac{2\pi}{\lambda_x}$ Solution is sinusoidal in x

- The vertical wavenumber is complex:

$m = m_{real} + i m_{imaginary}$

$m_{real} = \frac{2\pi}{\lambda_z}$: inverse of vertical oscillation distance

$m_{imaginary}$: decay/growth of wave in the z-direction

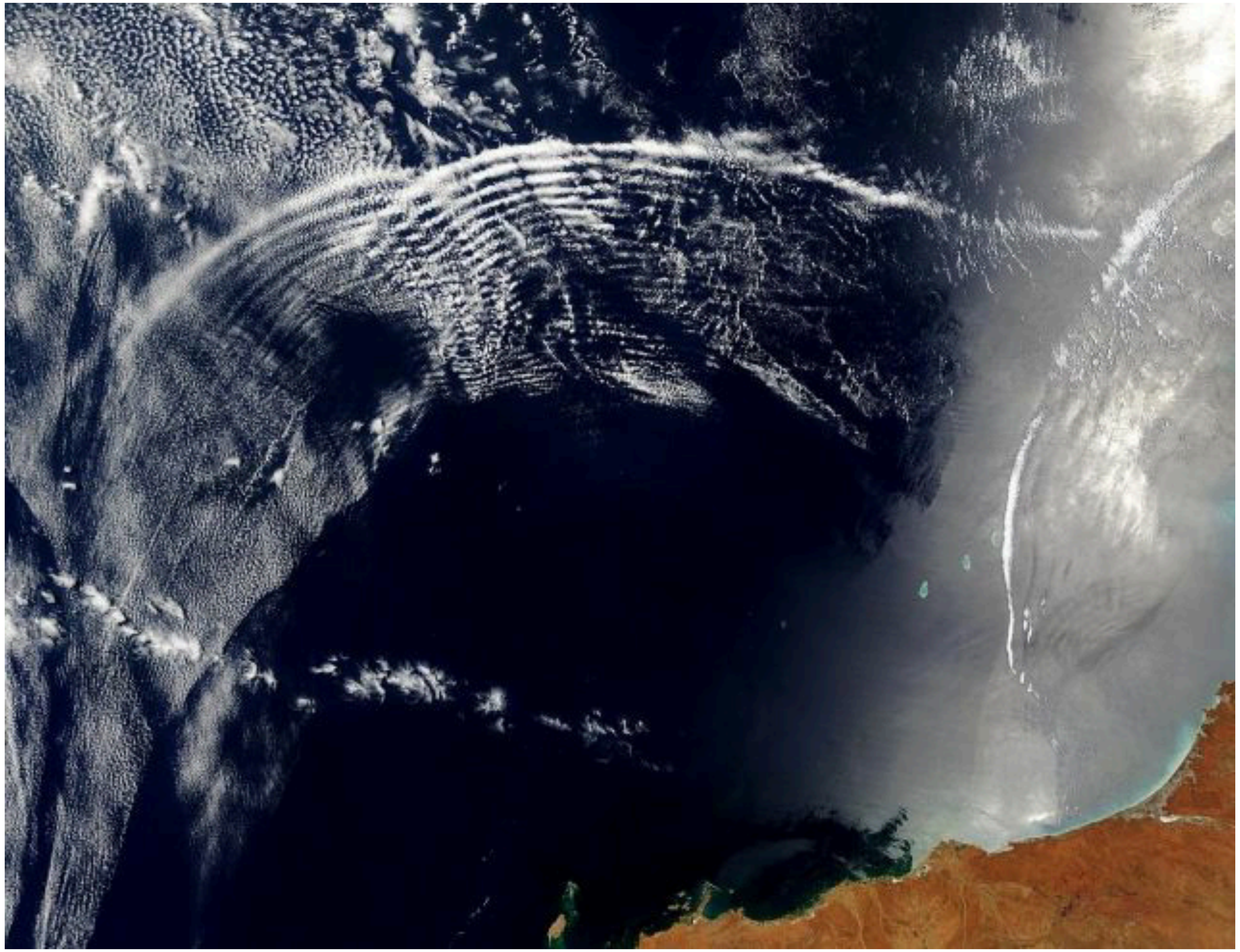


Gravity Waves

- Get dispersion relation by plugging solution back into wave equation

$$\omega - \bar{u}k = \pm \frac{N_B k}{\sqrt{k^2 + m^2}}$$

$$N_B^2 = \frac{(\gamma - 1)g}{\gamma H}$$



Gravity Waves

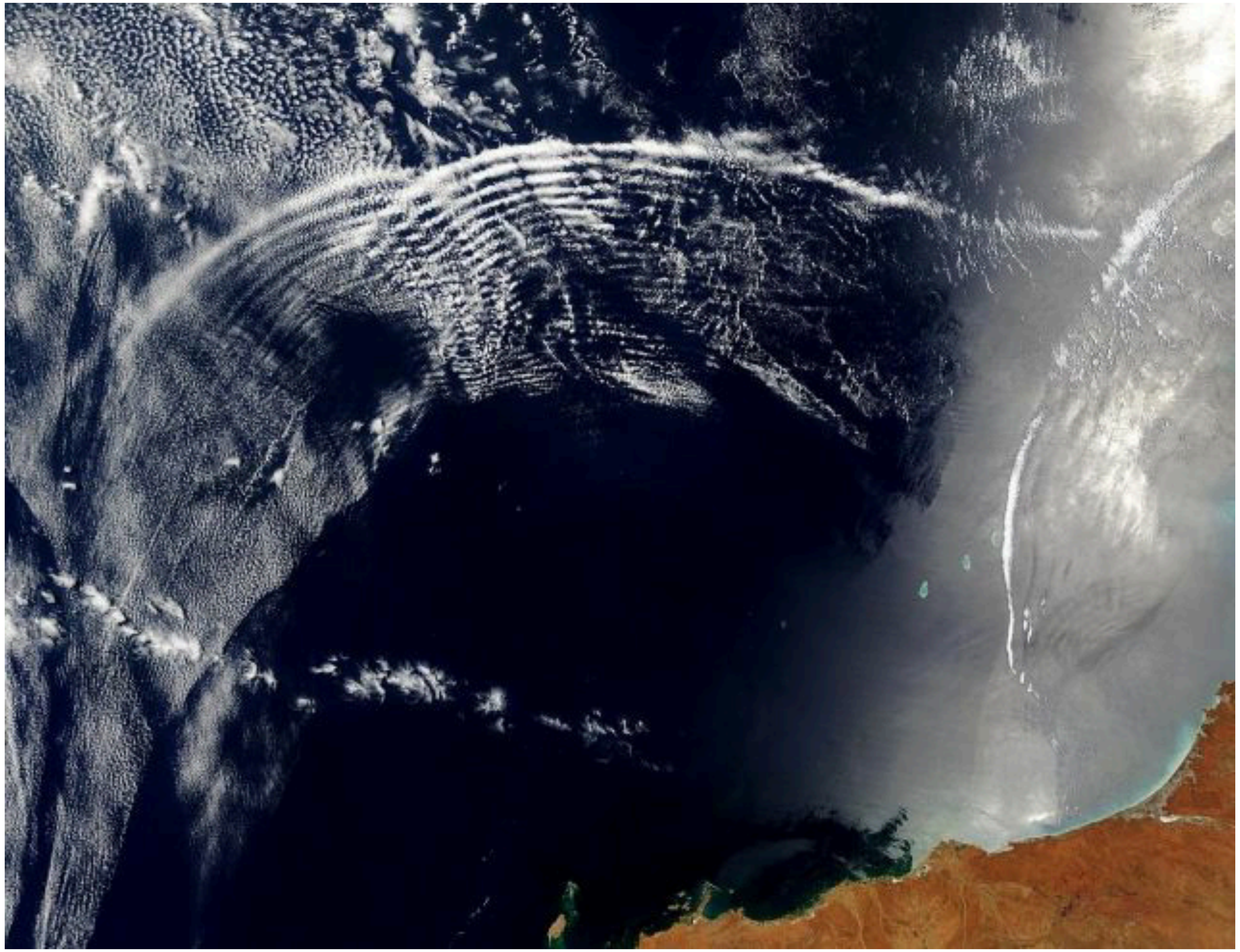
- We can get “topographical stationary waves,” where air flowing over topographical features (like mountains) can trigger a gravity wave if the atmosphere is statically stable

- These waves are stationary relative to the surface and have w’ dependent on (x,z) but not t:

- $$\left(\frac{\partial w'^2}{\partial x^2} + \frac{\partial w'^2}{\partial z^2}\right) + \frac{N_B^2}{\bar{u}^2} w' = 0$$

$$m^2 = \frac{N_B^2}{\bar{u}^2} - k^2$$

- When $m^2 > 0$ (real solution) the wave propagates in the vertical direction



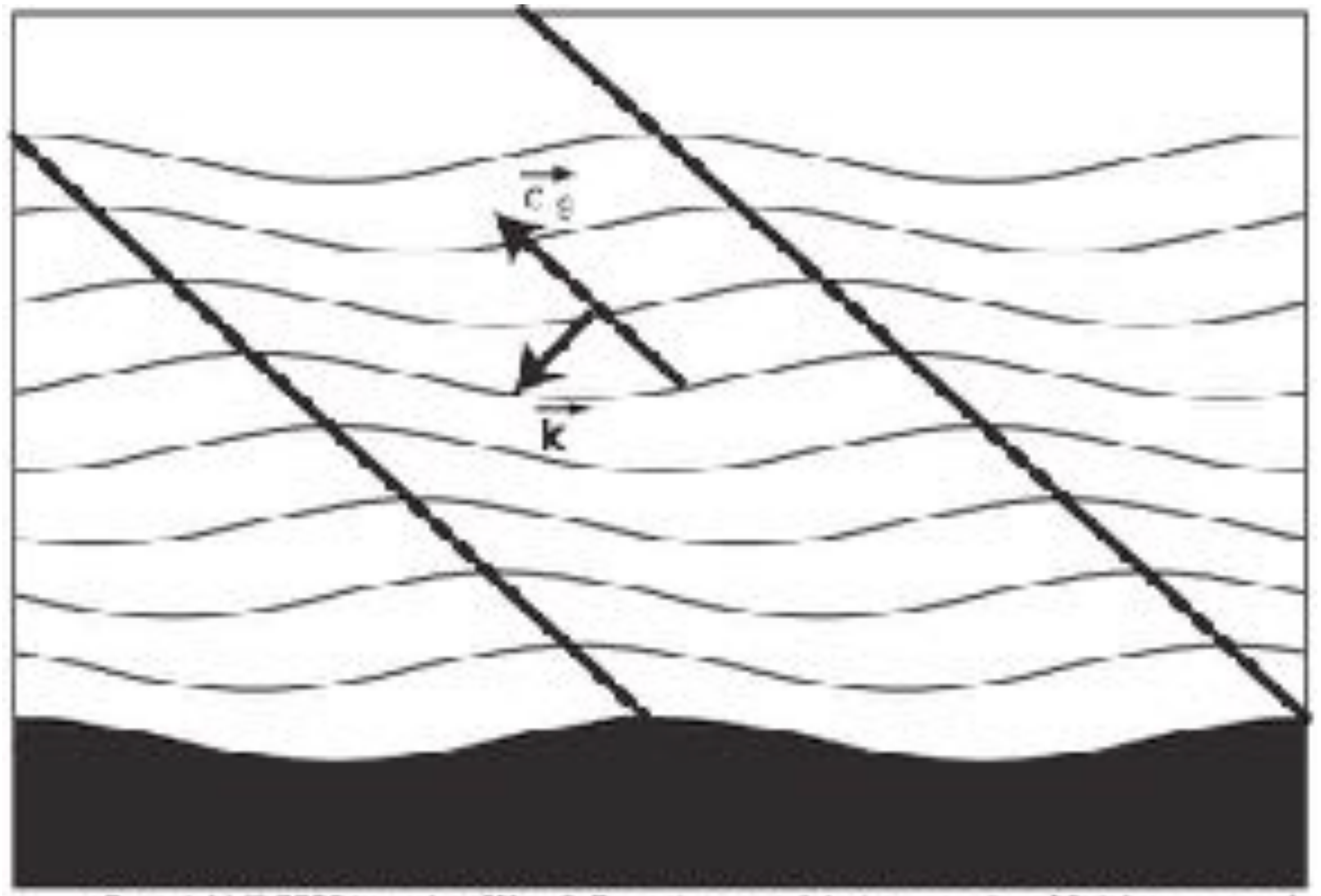
Order of Magnitude: Gravity Waves

- $m^2 = \frac{N_B^2}{\bar{u}^2} - k^2$ $\qquad k = \frac{2\pi}{\lambda_x}$

- When $m^2 > 0$ (real solution) the wave propagates in the vertical direction

- Stationary (topographic) gravity waves form on Mars when air flows with a speed of $\bar{u} = 15m/s$ over mountain ridges. If $N_B = 5x10^{-2}s^{-1}$, what is the limiting separation between the ridges (in km) for vertical propagation of the gravity waves?

- Do we get vertical propagation if the separation is larger or smaller than this value?



Order of Magnitude: Gravity Waves

- $m^2 = \frac{N_B^2}{\bar{u}^2} - k^2$ $\qquad k = \frac{2\pi}{\lambda_x}$
- When $m^2 > 0$ (real solution) the wave propagates in the vertical direction
- Stationary (topographic) gravity waves form on Mars when air flows with a speed of $\bar{u} = 15m/s$ over mountain ridges. If $N_B = 5x10^{-2}s^{-1}$, what is the limiting separation between the ridges (in km) for vertical propagation of the gravity waves?
- We need m^2 to be positive to have a real solution so:
- $\frac{N_B^2}{\bar{u}^2} > k^2$
- If we take the square root of both sides (and make sure to take the absolute value of all terms), the limiting value of k happens at: $k = \frac{N_B}{\bar{u}}$

Order of Magnitude: Gravity Waves

- $\frac{N_B^2}{\bar{u}^2} > k^2$ $\qquad$ $k = \frac{N_B}{\bar{u}}$
- But we don't want k (the horizontal wavenumber), we want the horizontal wavelength:

$k = \frac{2\pi}{\lambda_x}$ To have vertical propagation, k must be smaller than the limiting value, so wavelength must be larger

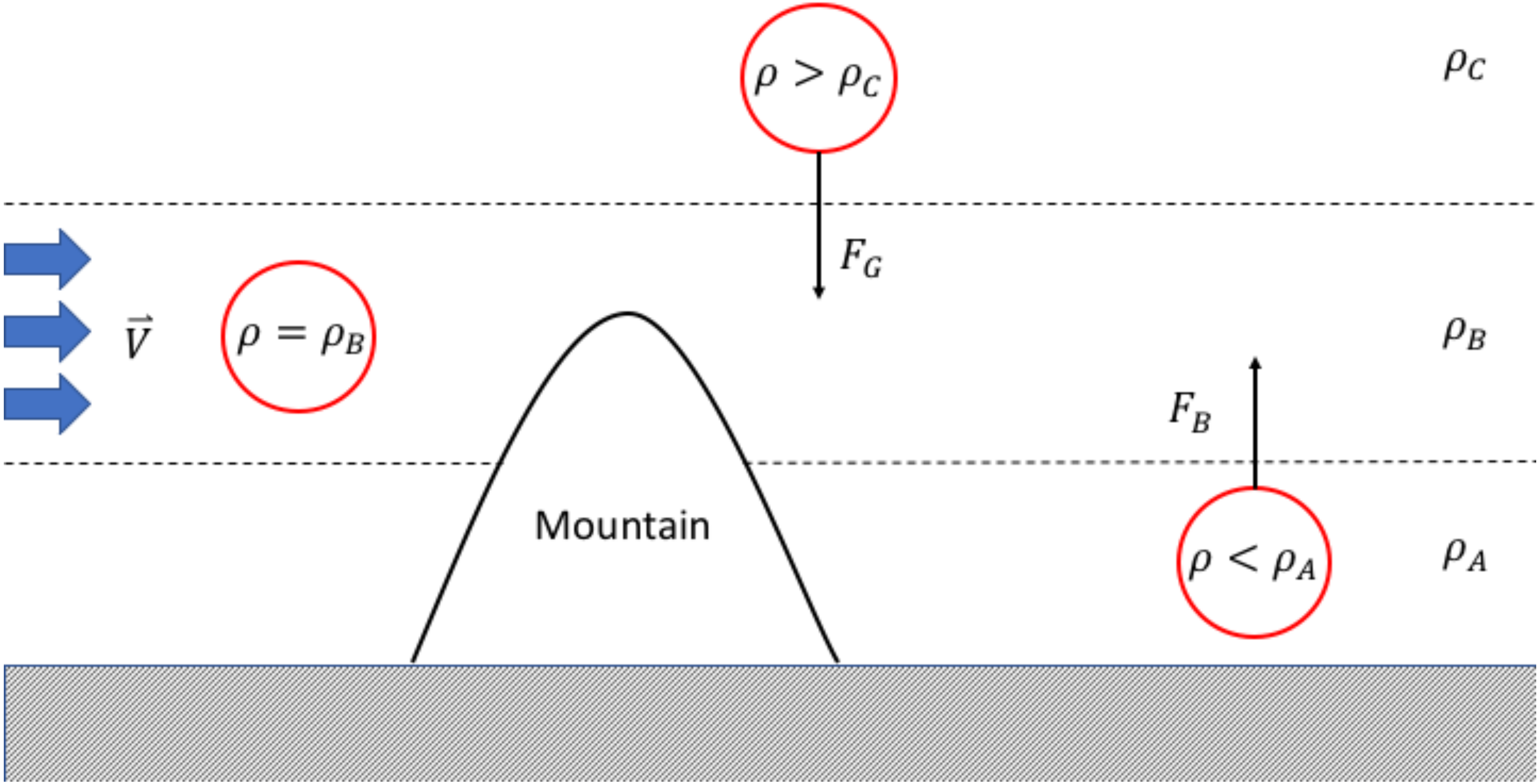
- Combining equations:

$$k = \frac{N_B}{\bar{u}} = \frac{2\pi}{\lambda_x}$$

- And solving for wavelength, which will be the ridge spacing:

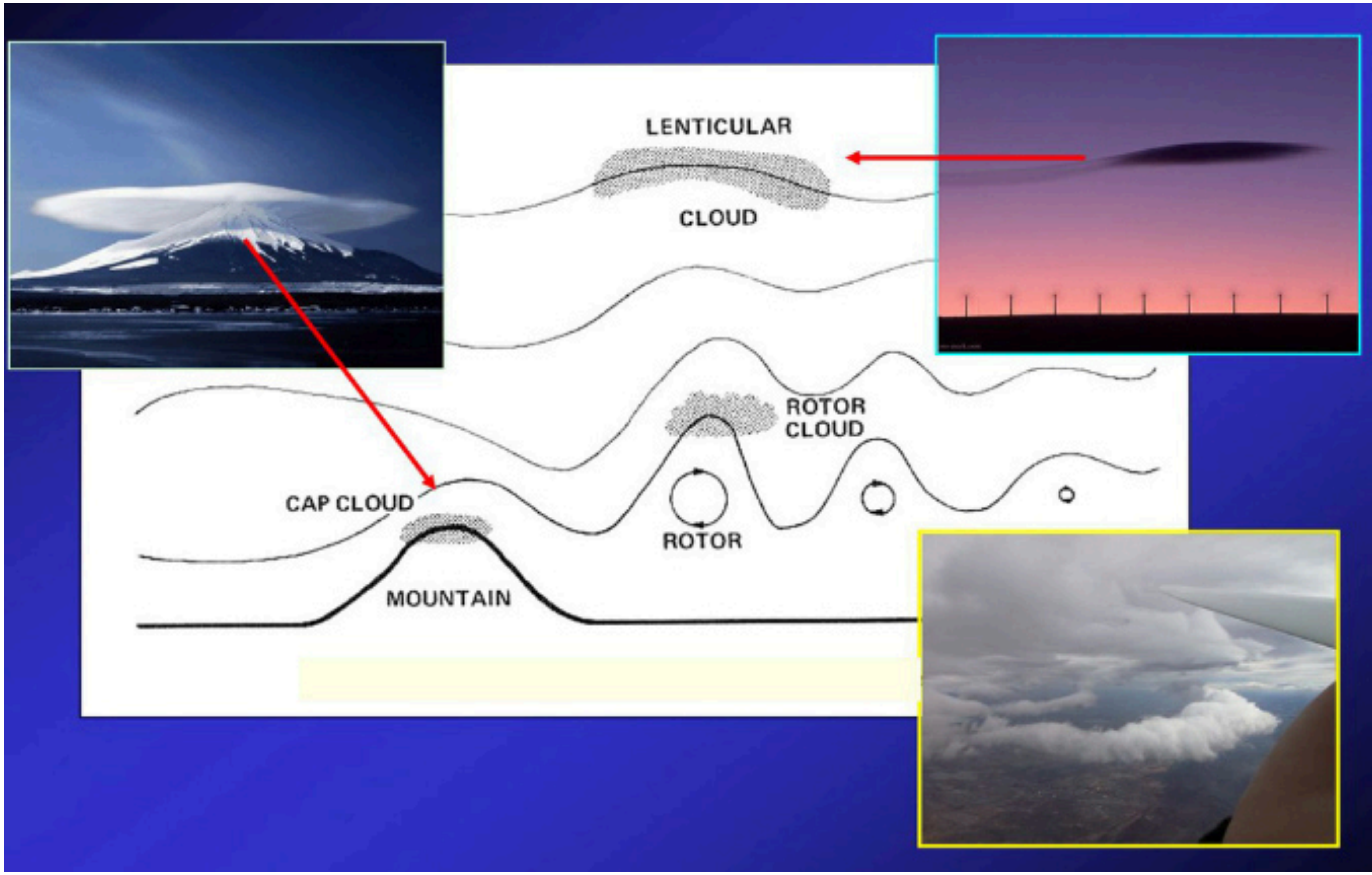
$\lambda_x = \frac{2\pi\bar{u}}{N_B} = \frac{2x3x15m/s}{5x10^{-2}s^{-1}} = 18x10^2m = 1800m = 1.8km$ The ridge spacing must be LARGER than 1.8 km

Gravity Waves



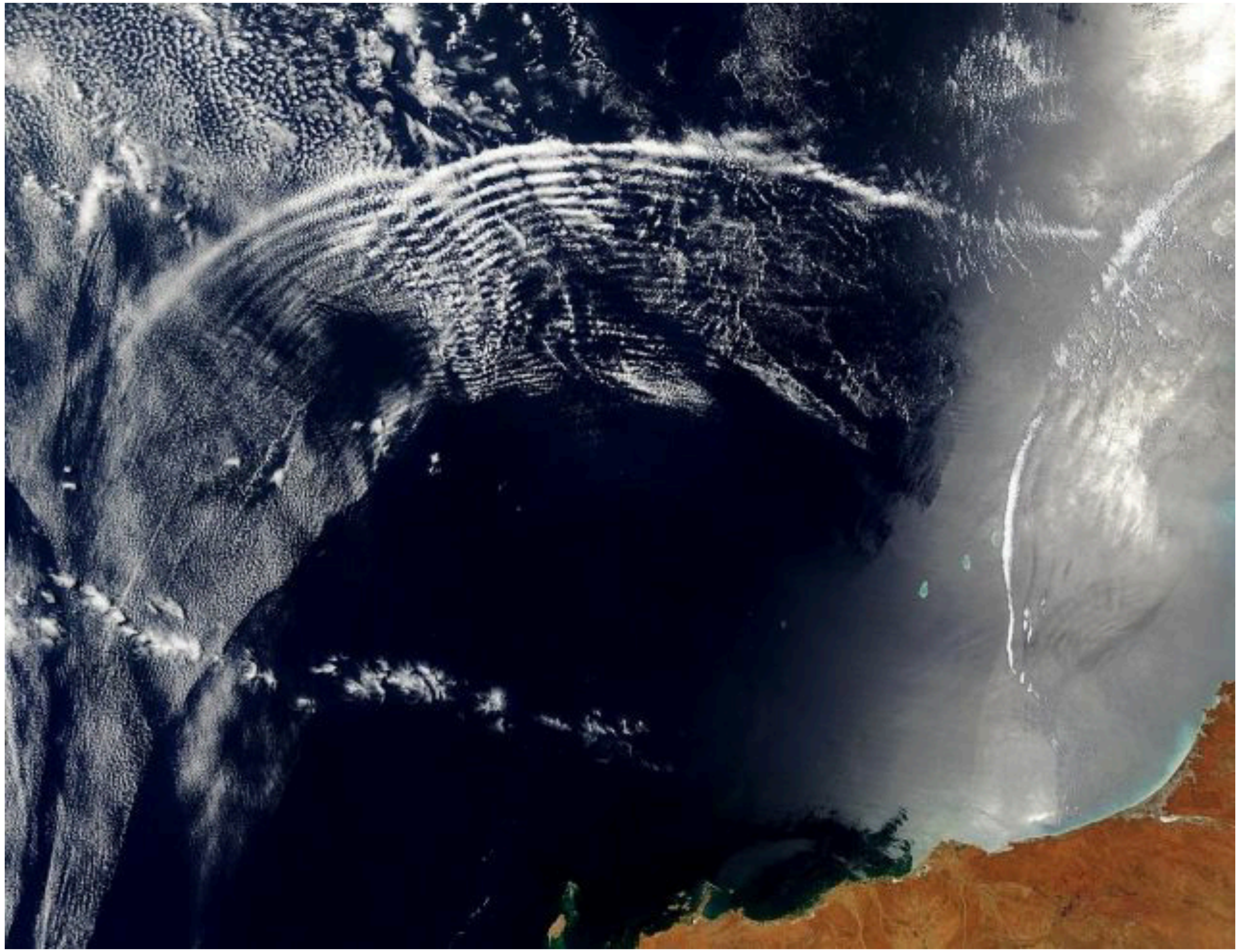
Gravity Waves

- Formation and structure of mountain (lee) waves formed by obstacles and associated clouds
- At the wave crests, moisture in the air parcel condenses to form clouds

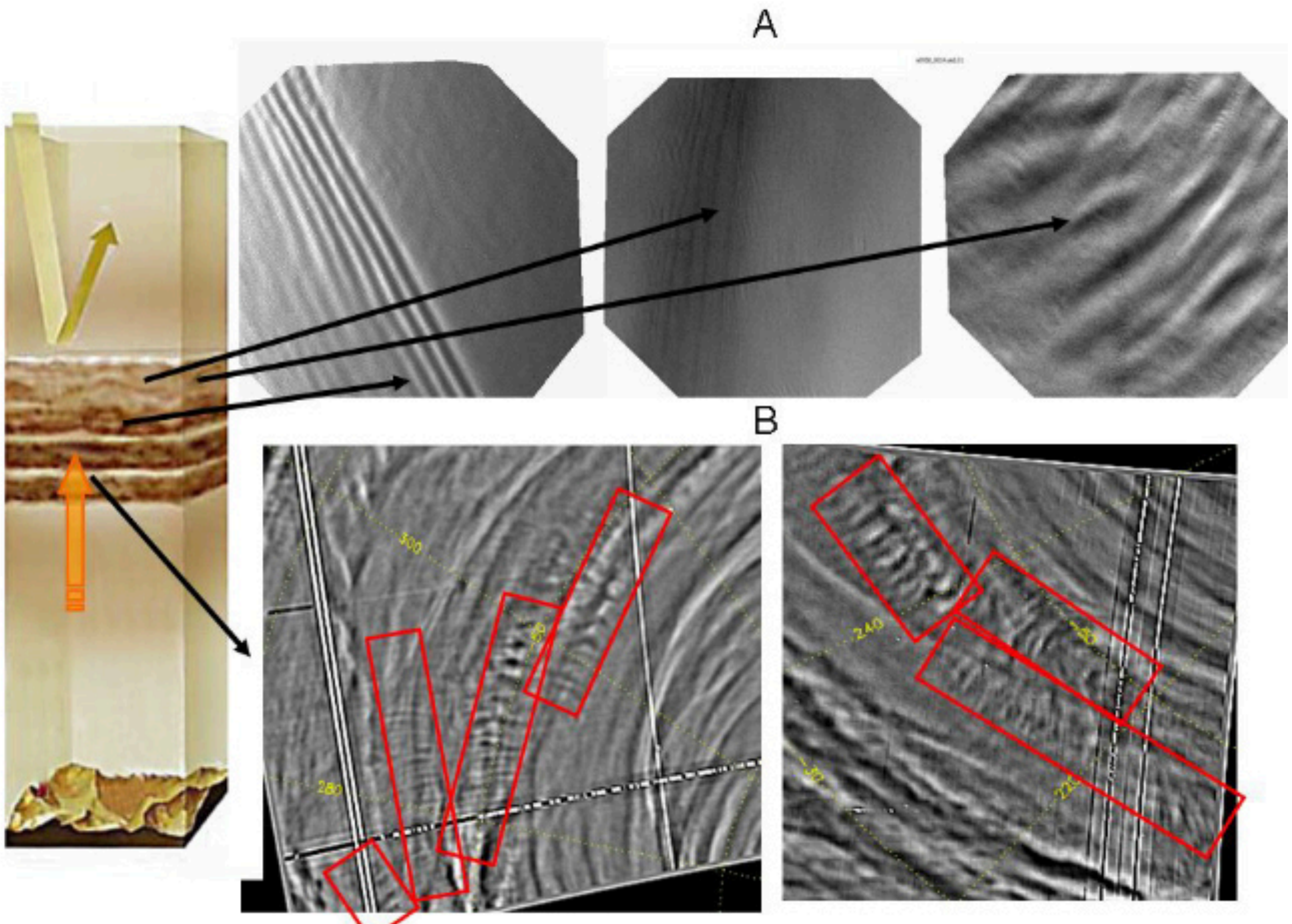


Gravity Waves

- Australia, observed by MODIS instrument on Terra satellite
- Packet of arc-shaped gravity waves is made visible from the condensation at the wave crests

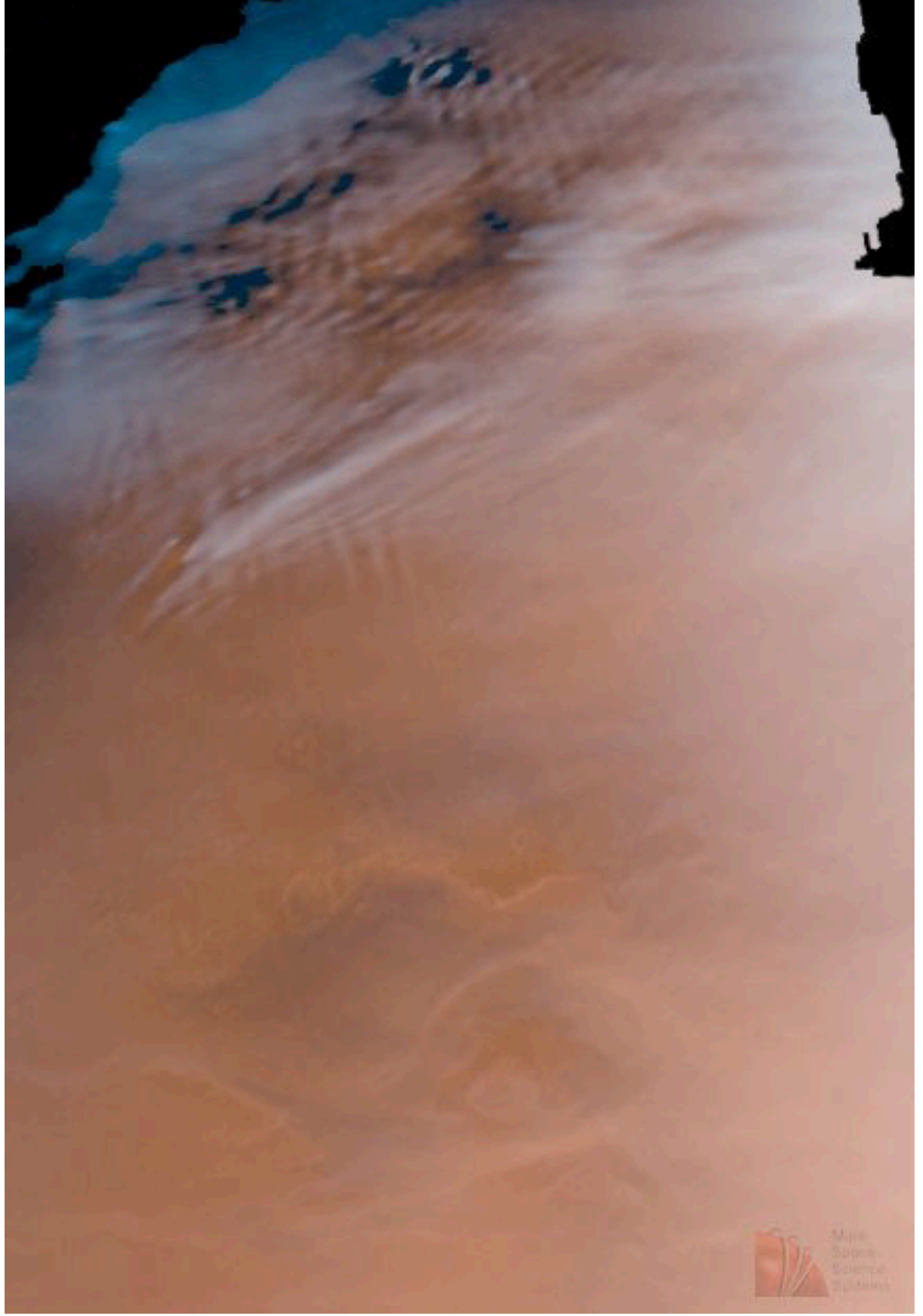


Venus (from Venus Express)

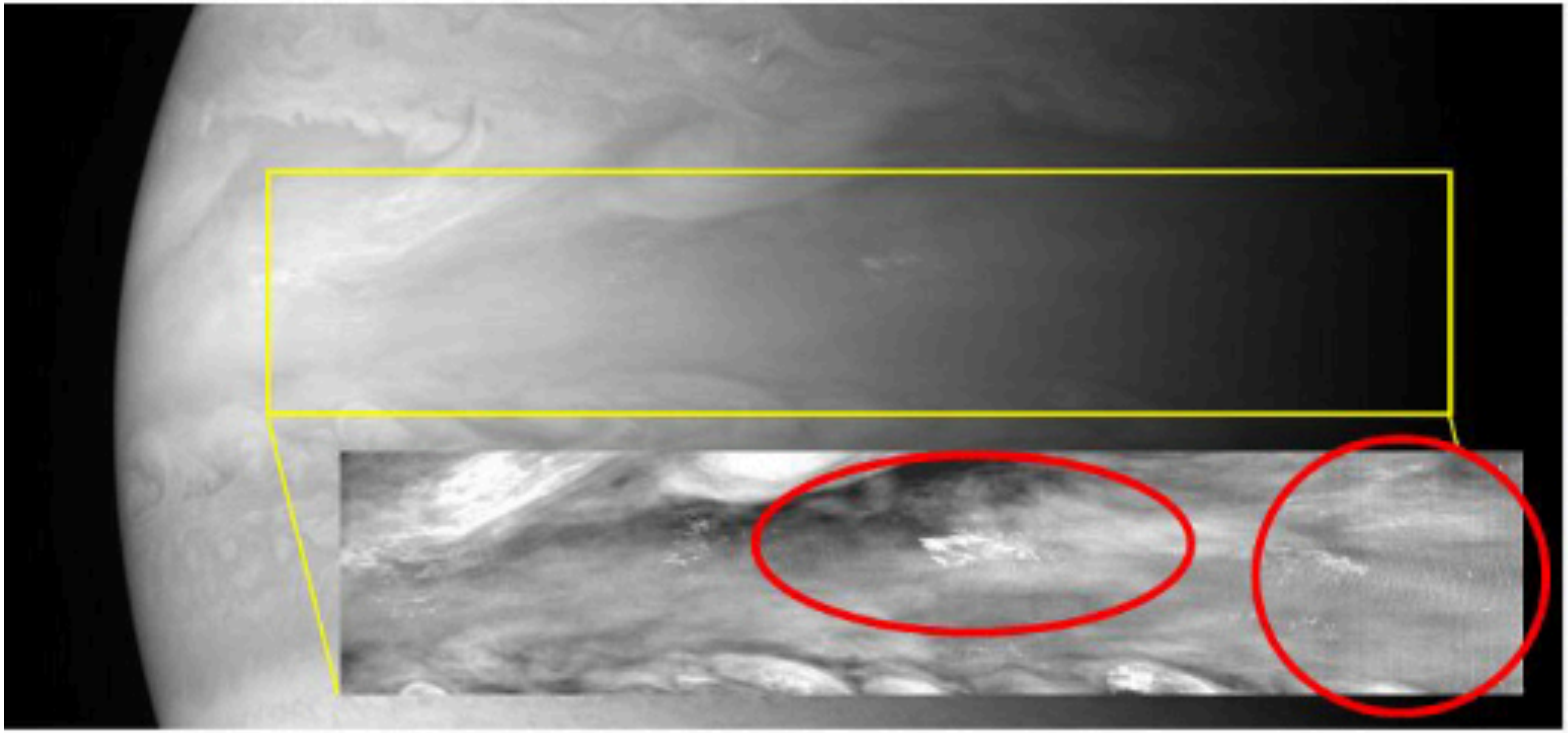


Mars (from Mars Global Surveyor)

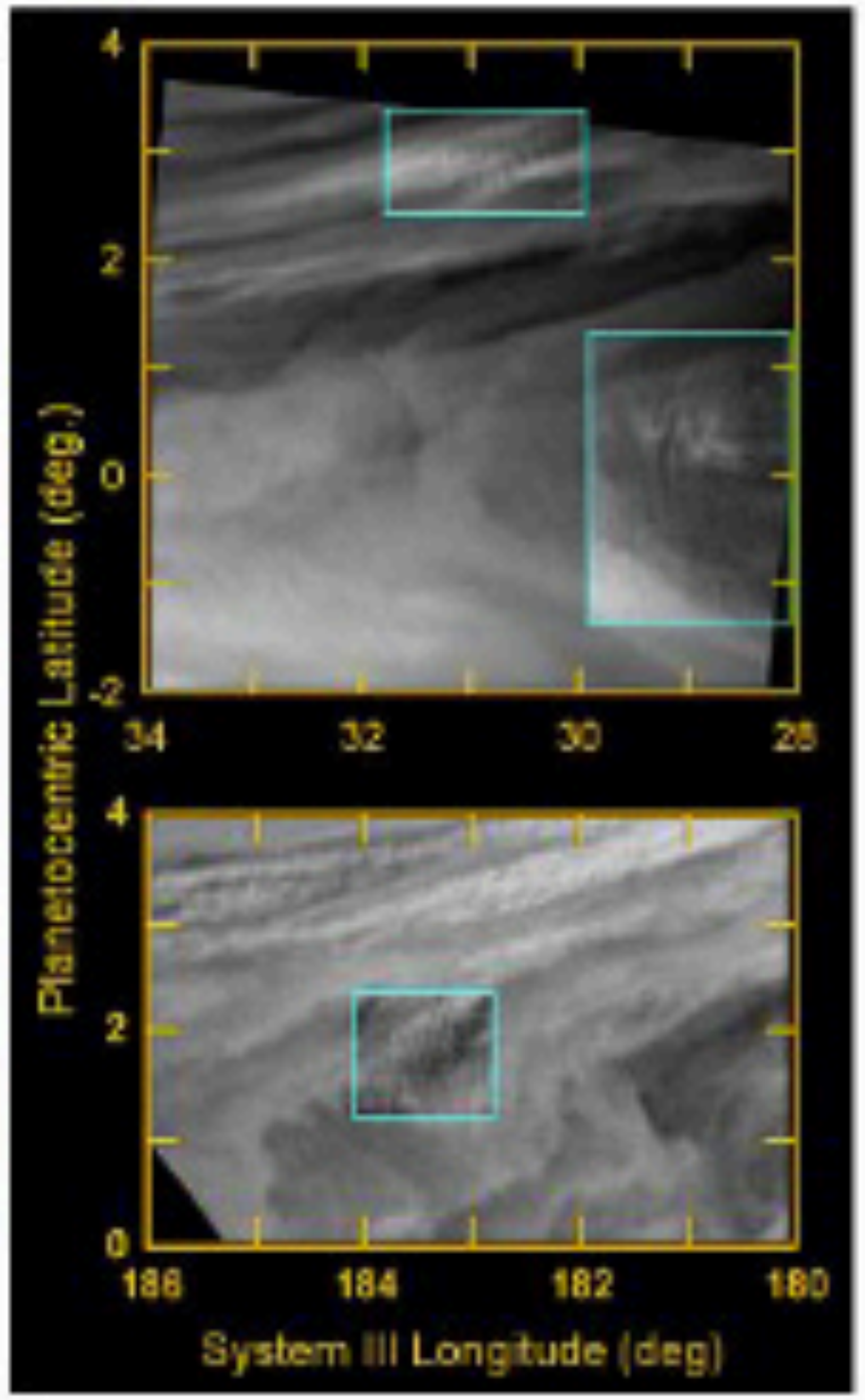
- Gravity waves traced by CO_2 ice condensation in the wave crests



Jupiter



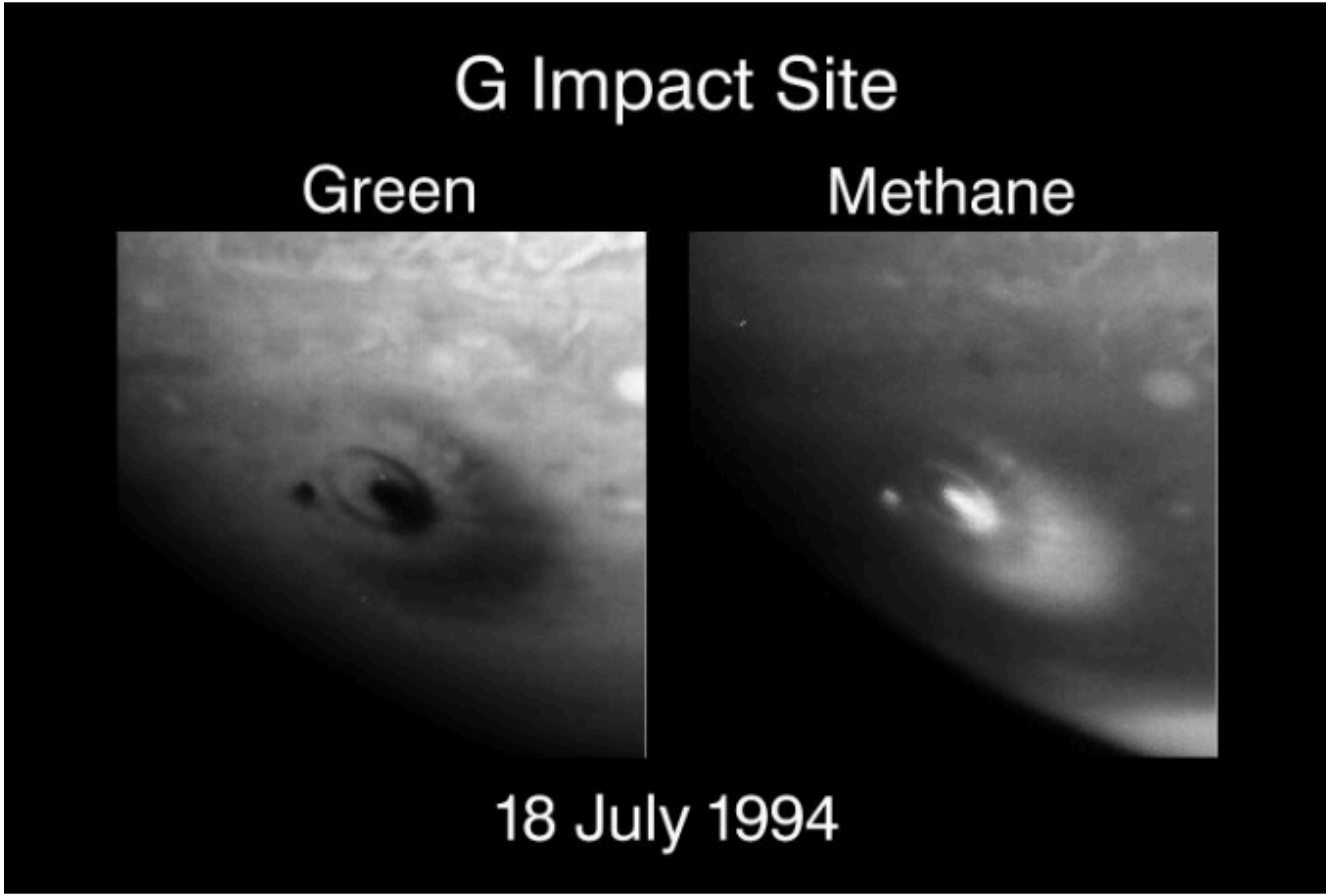
New Horizons



Galileo

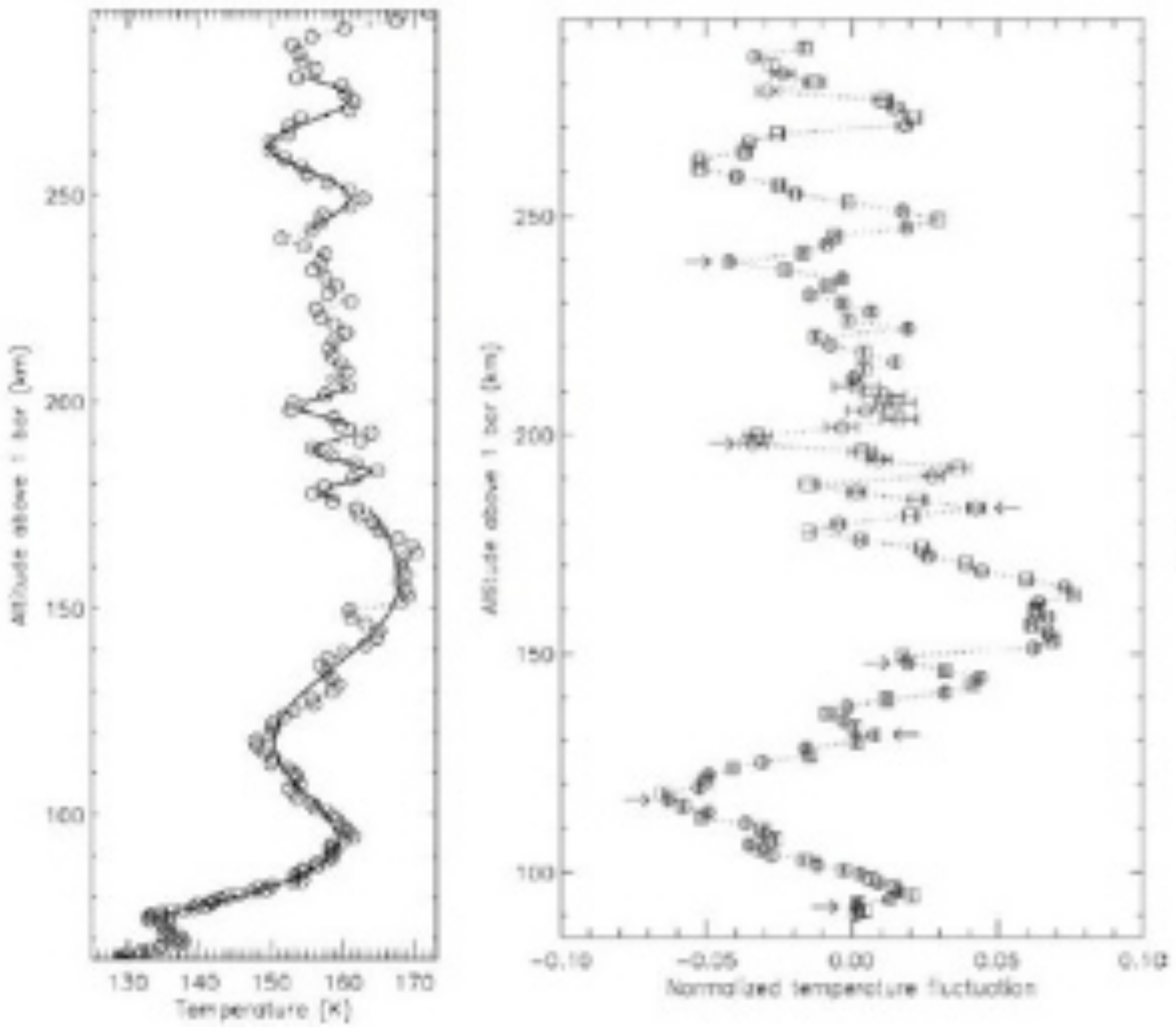
Jupiter

- Ring-like gravity wave traced by debris resulting from comet fragment impact

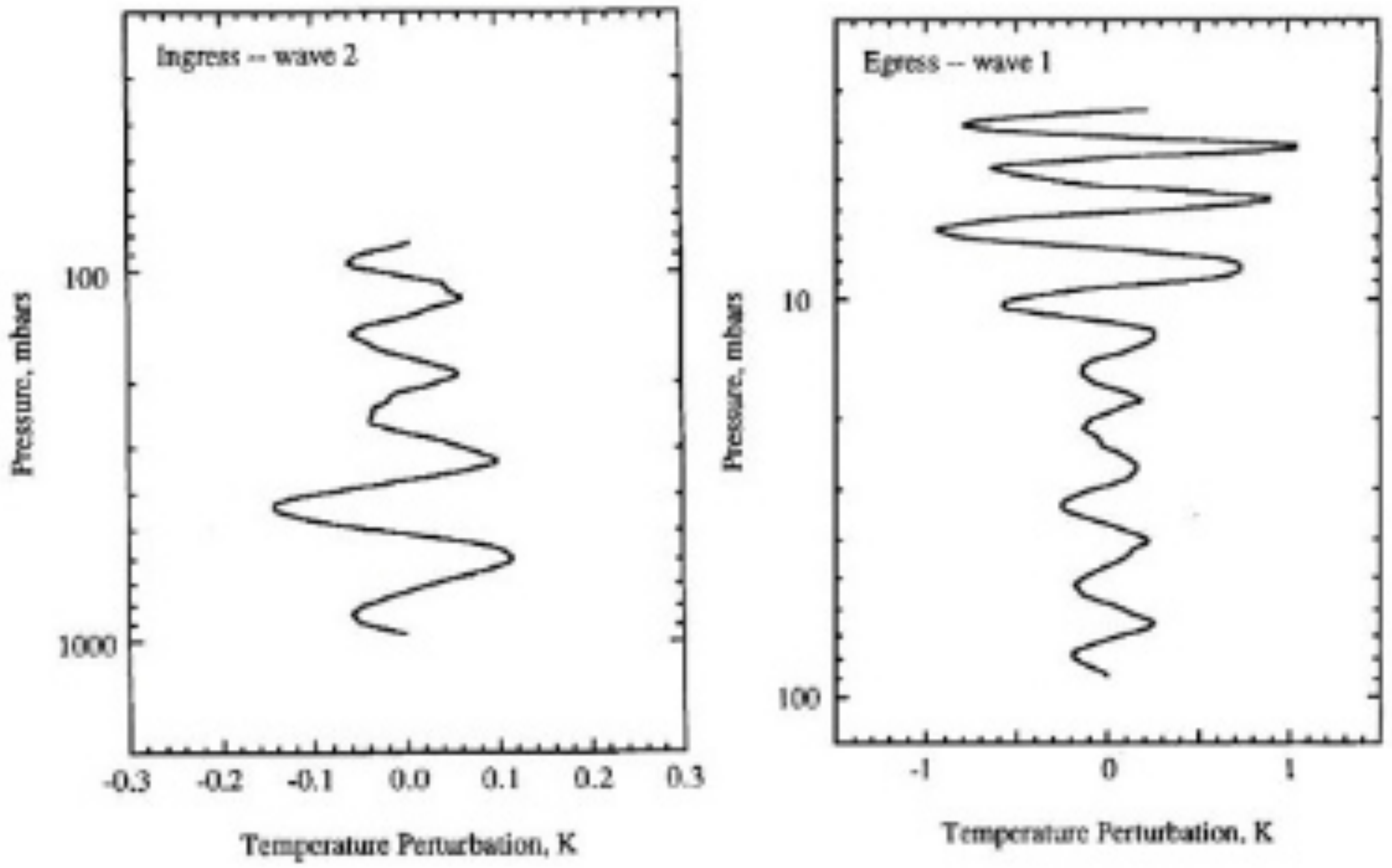


Gravity Waves

- See gravity waves in upper atmospheres of planets as oscillations in the vertical temperature profiles
  - subtract off mean temperature to see it
- Manifestations of upward vertically propagating gravity waves generated close to the tropopause



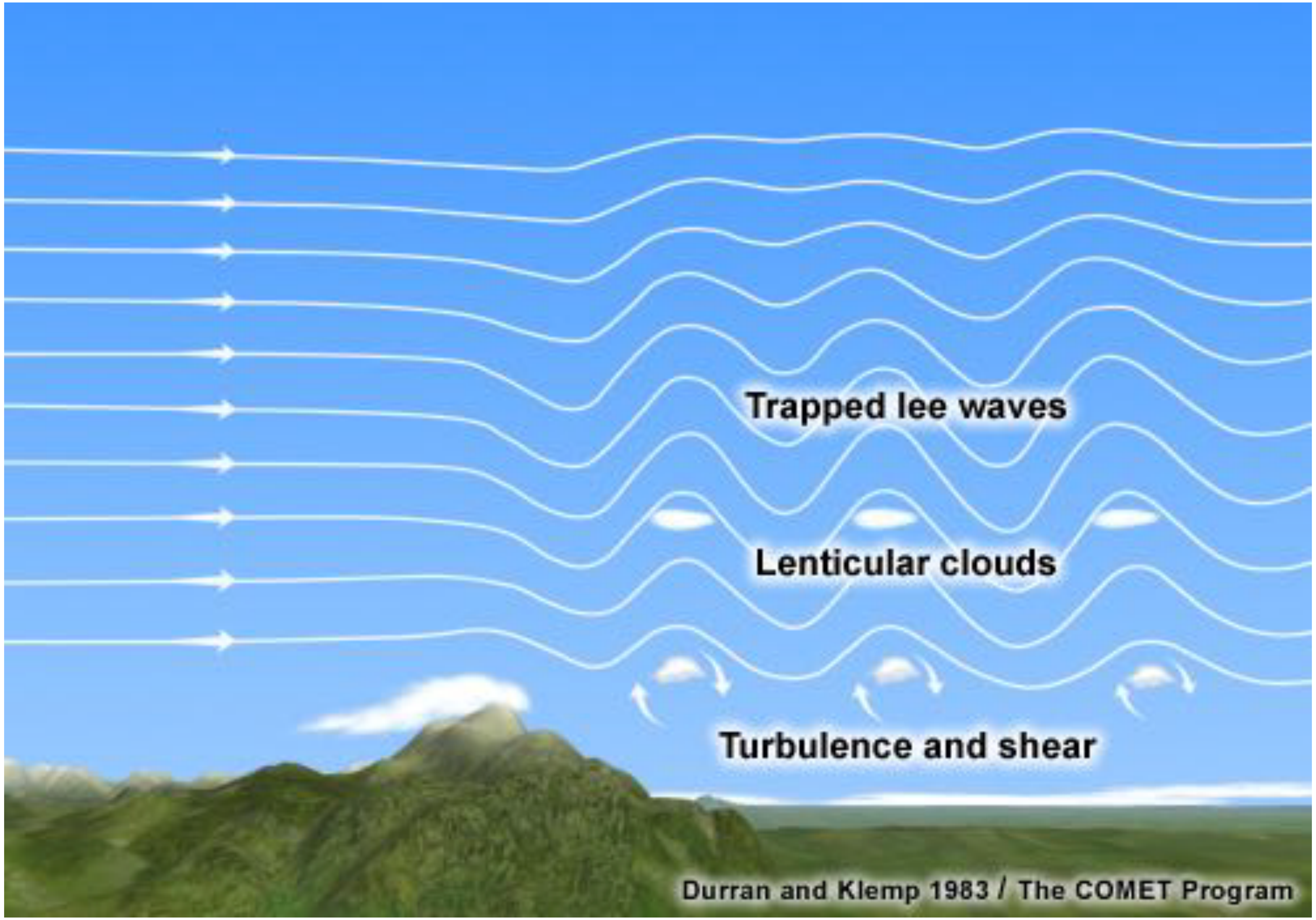
Jupiter



Neptune

Trapped Gravity Waves

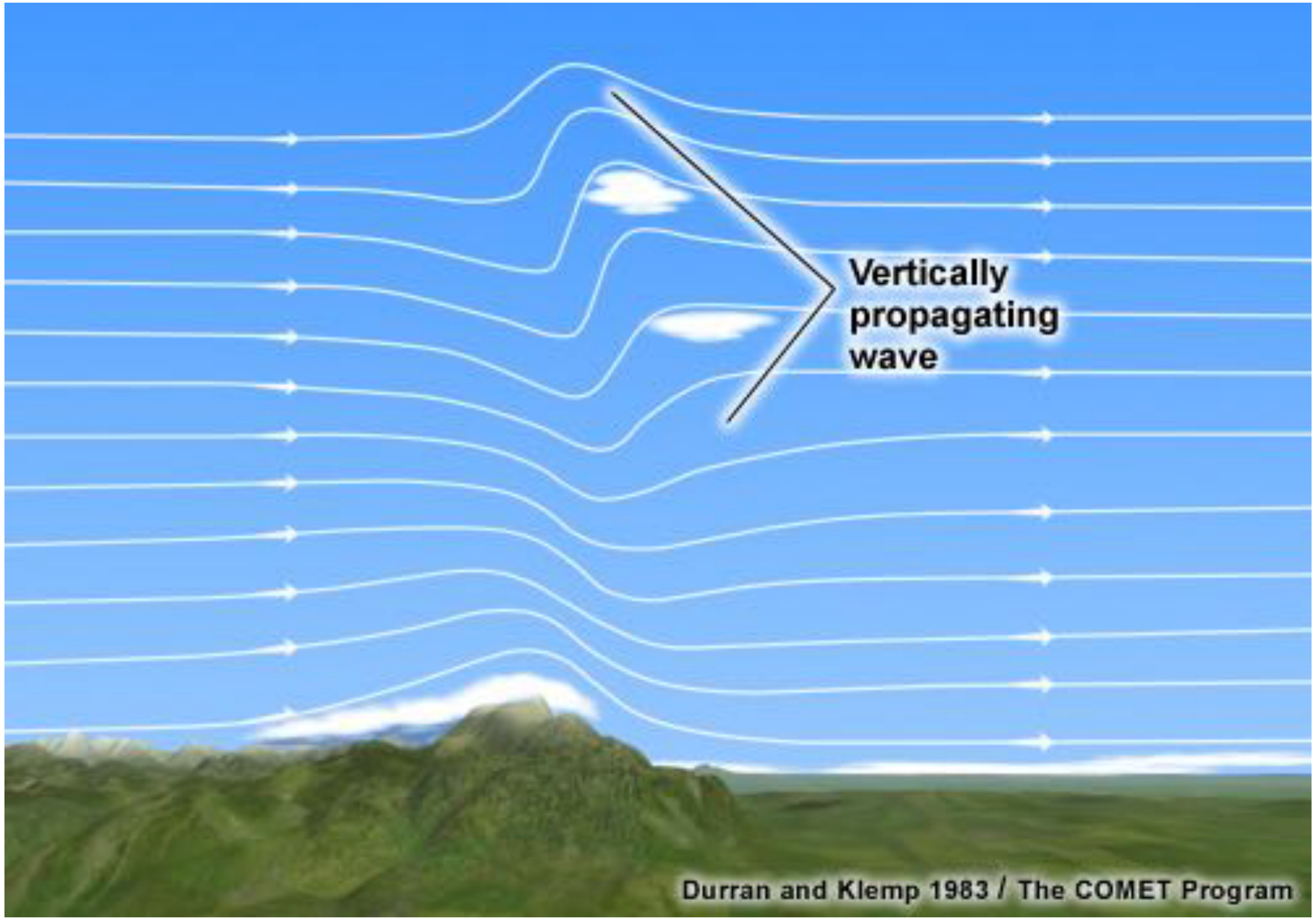
- Trapped in a layer with high static stability and moderate wind speeds
  - Usually in the lowest 1-5 km of the troposphere
- Trapped waves occur when wind speed above the mountain increase sharply with height, and when stability decreases in the layer just above the mountain top



Vertically Propagating Gravity Waves

- Occur when static stability increases above the mountain peak and when the wind speed does not increase significantly with height

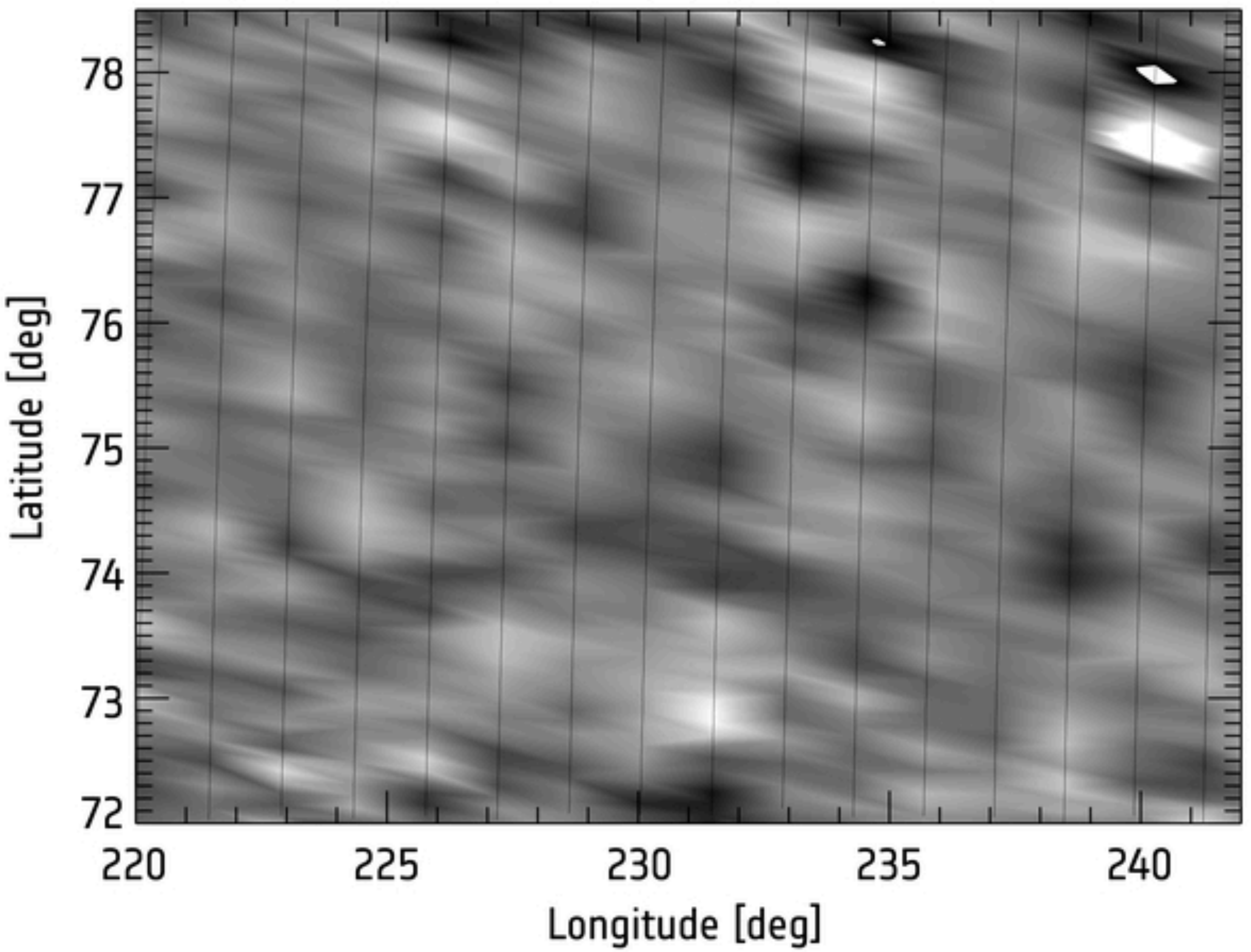
- Typically extend vertically up to the higher troposphere and are tilted backward with height toward the mountain chain



Venus

- Venus Explorer aerobraking provided in situ measurements of upper atmosphere
- Discovered “atmospheric gravity waves” and “planetary waves” in Venus’ polar atmosphere

- black: less dense
  white: more dense
  average density perturbation: 10% of mean background density



For next time

- Reading: Planetary Science, 13.1, 13.2