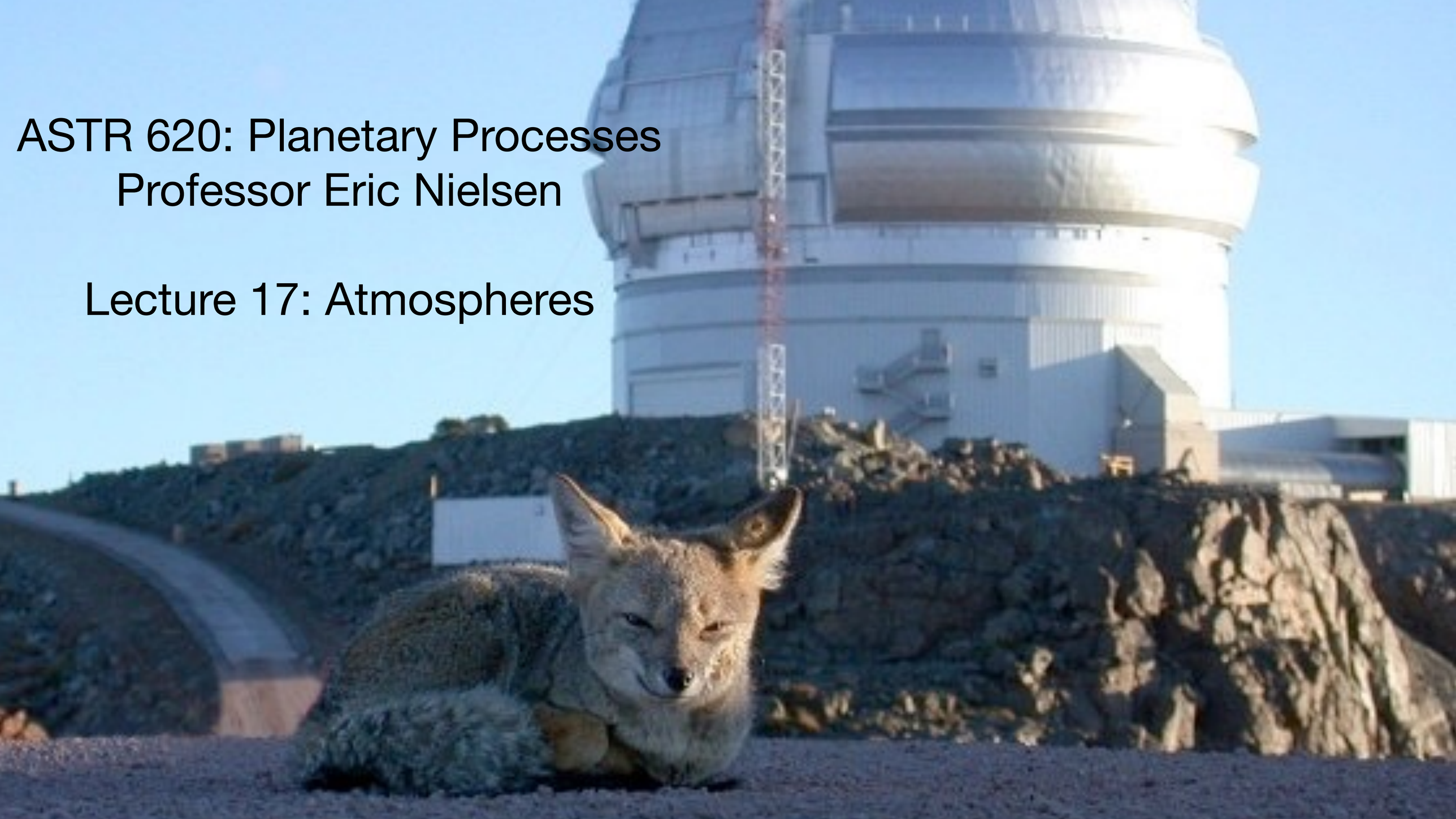


ASTR 620: Planetary Processes

Professor Eric Nielsen

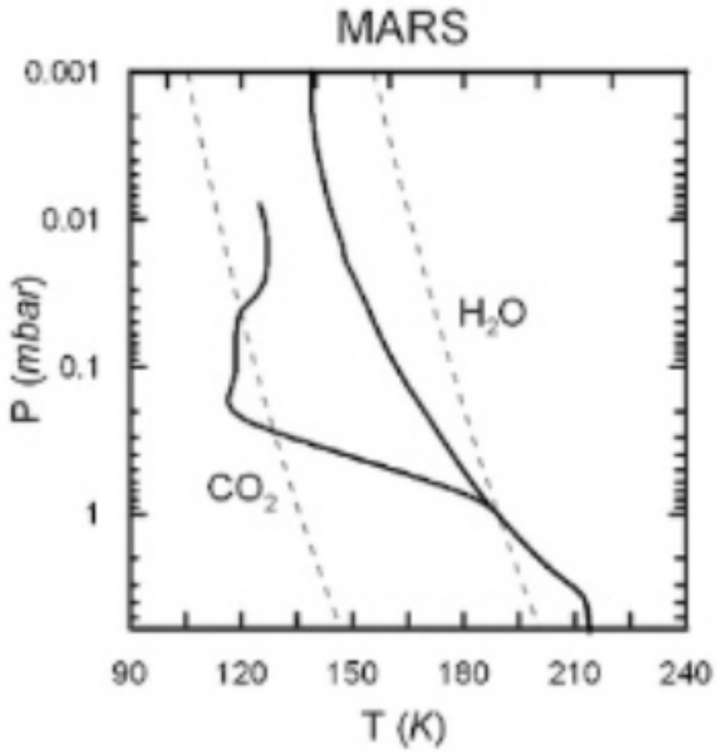
Lecture 17: Atmospheres

Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Homework 4 due tonight at 11:59pm

Review of the last class

- On a given day on Mars, the temperature-pressure profile follows the solid line on the left. On that day, carbon dioxide clouds:
  - (A) — Can form at altitudes below 0.8 bars or altitudes above 0.06 bars
  - (B) — Can form at altitudes between 0.06 and 0.8 bars
  - (C) — Can form anywhere at altitudes below 0.8 bars
  - (D) — Can form anywhere at altitudes above 0.06 bars
  - (E) — Cannot form at all



Review of the last class

- Zonal, meridional, and vertical winds travel:
  - (A) — North/South, East/West, and Up/Down
  - (B) — North/South, Up/Down, and East/West
  - (C) — East/West, North/South, and Up/Down
  - (D) — East/West, Up/Down, and North/South
  - (E) — Up/Down, East/West, and North/South

Review of the last class

- Earth's atmosphere is in:
  - (A) — Geostrophic balance, so winds flow parallel to isobars
  - (B) — Geostrophic balance, so winds flow perpendicular to isobars
  - (C) — Cyclostrophic balance, so winds flow parallel to isobars
  - (D) — Cyclostrophic balance, so winds flow perpendicular to isobars
  - (E) — Earth does not have an atmosphere

Review of the last class

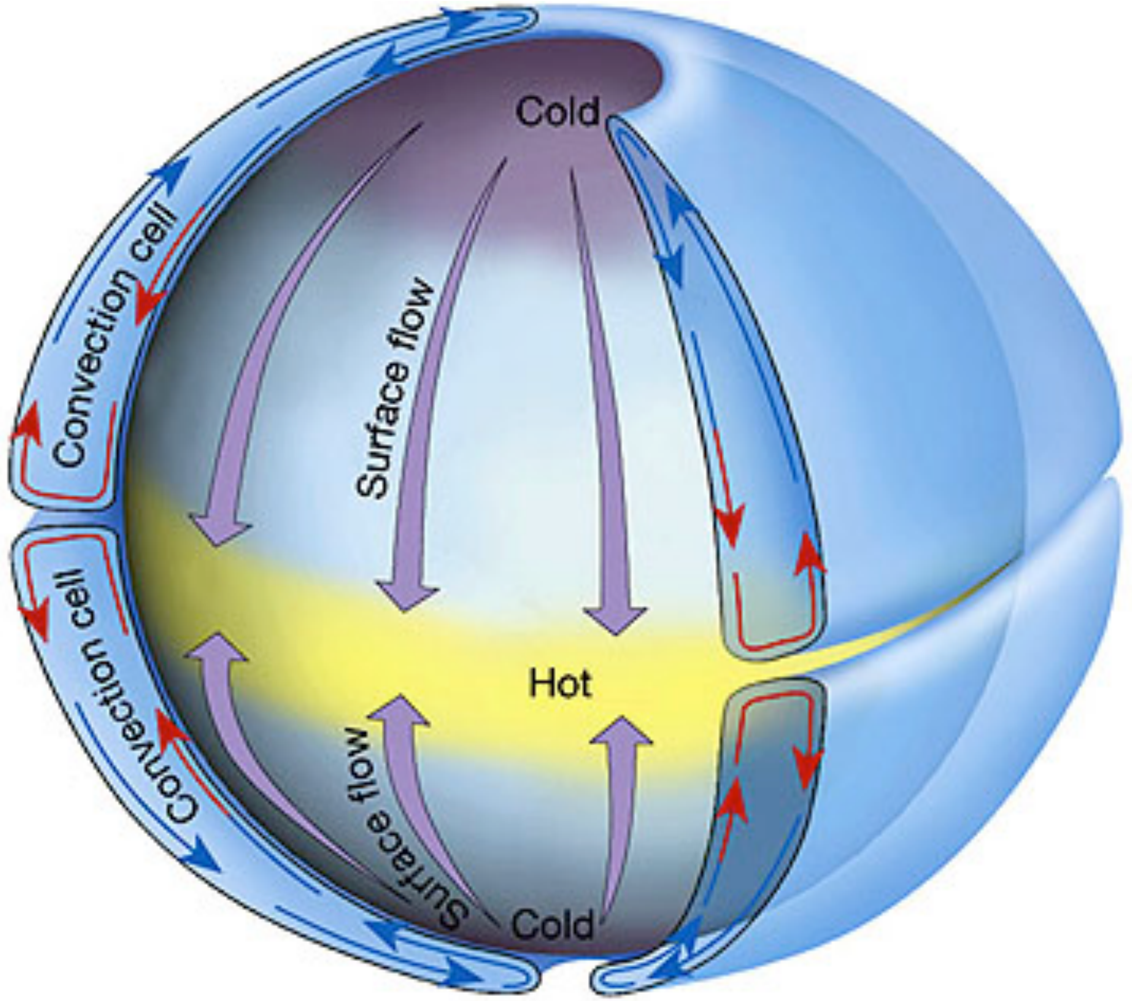
- In geostrophic balance:
  - (A) — The pressure gradient balances the centrifugal force
  - (B) — The pressure gradient balances the Coriolis force
  - (C) — The centrifugal force balances gravity
  - (D) — The centrifugal force balances the Coriolis force
  - (E) — The pressure gradient balances gravity

Review of the last class

- Earth's atmosphere is in geostrophic balance while Venus' is not because:
  - (A) — Venus is much hotter than Earth
  - (B) — Earth has more water vapor in its atmosphere than Venus
  - (C) — Earth has a stronger magnetic field than Venus
  - (D) — Earth rotates faster than Venus
  - (E) — Both planets' atmospheres are in geostrophic balance

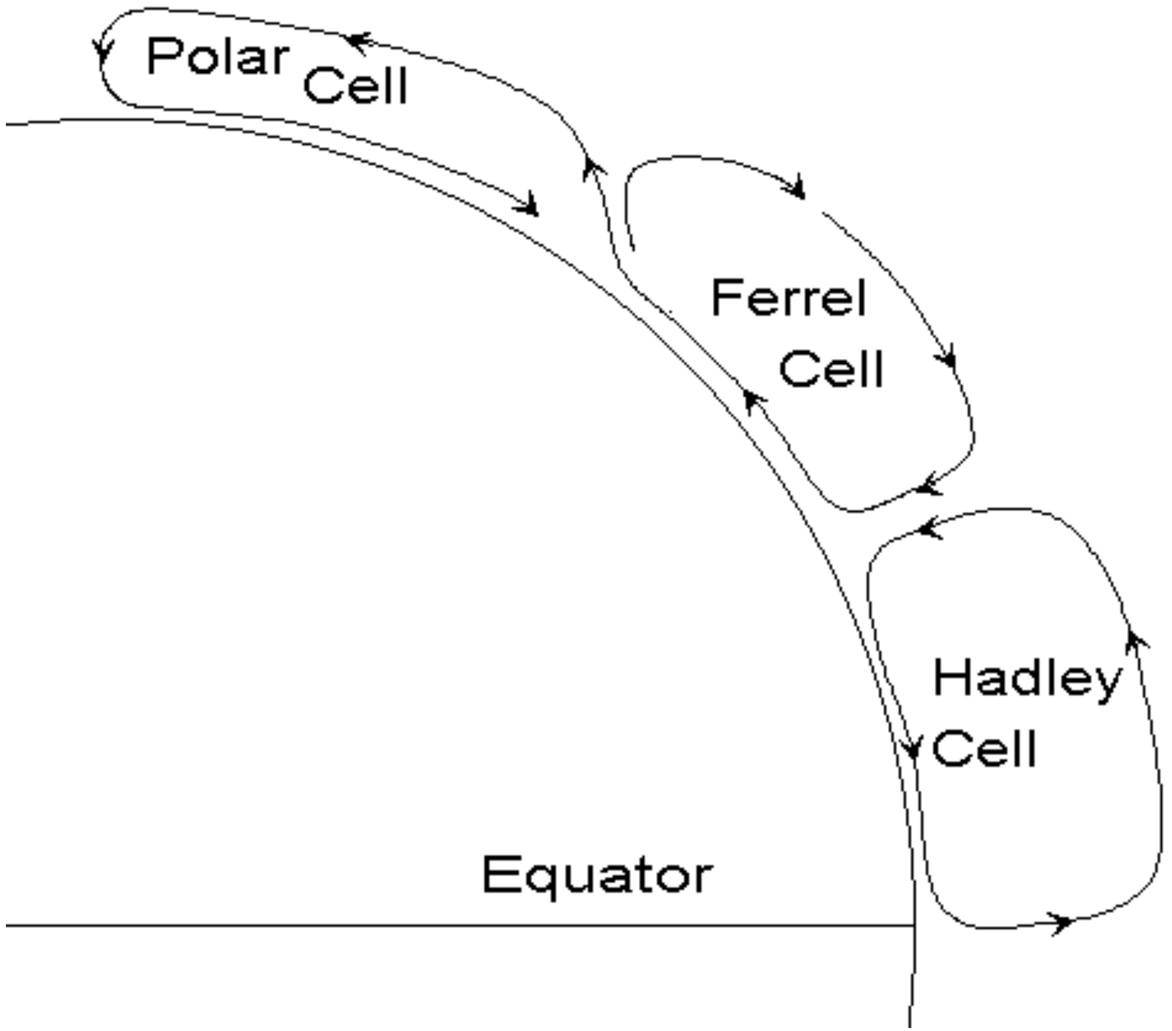
Hadley Circulation

- For no/slow rotation



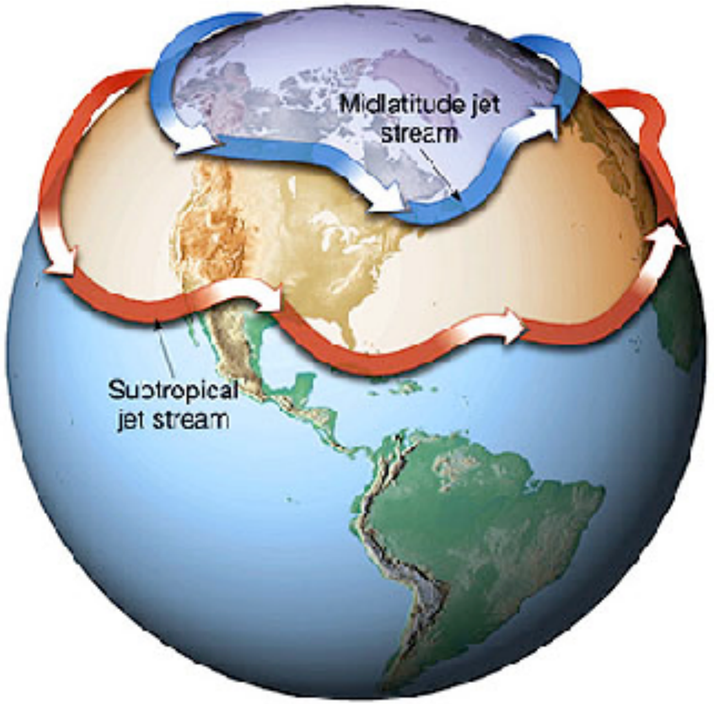
Hadley Circulation

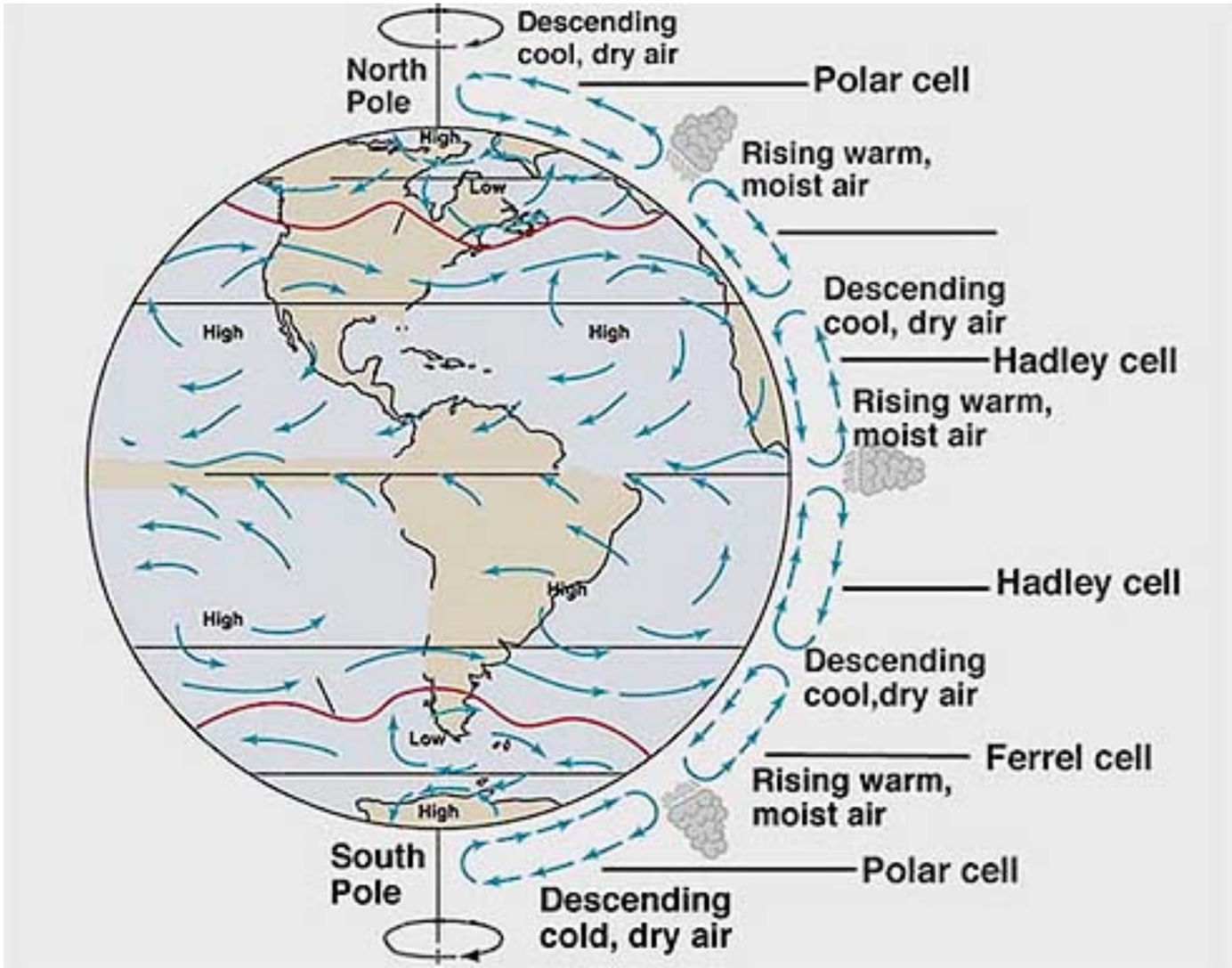
- For faster rotation (like on Earth), have interaction of different amounts of sunlight (“insolation”) and Coriolis force as a function of latitude
- Get 3 convective cells per hemisphere:
  - Hadley cells
  - Polar cells
  - Ferrel cells



Hadley Circulation

- Cells meet at ~30 and ~60 degrees latitude
  - (locations of the tropical and polar jet streams)





Hadley cell (thermal direct)

Air rises near equator and descends near 30º where a belt of high pressure is found (subtropical high)

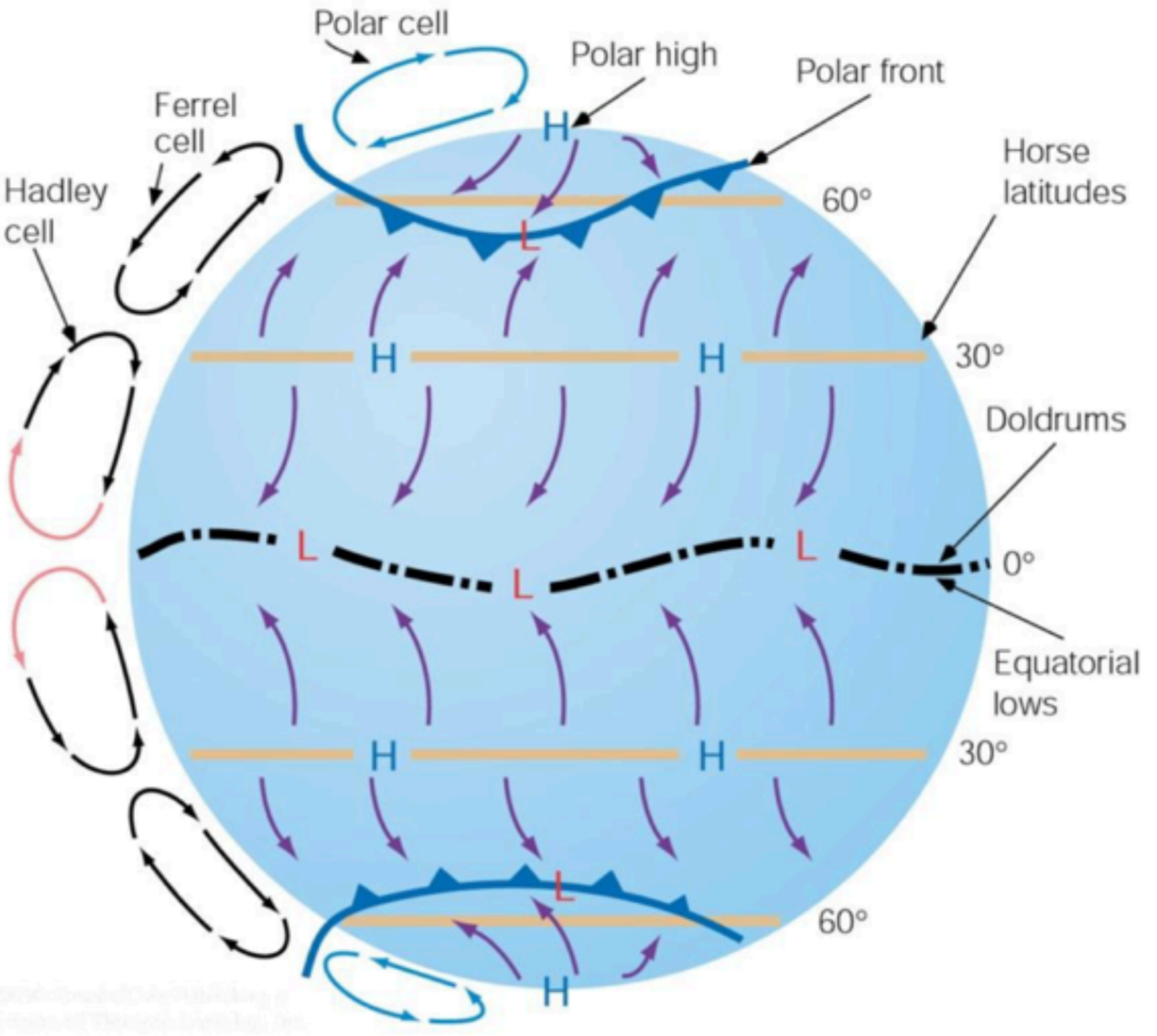
Polar Cell (thermal direct)

Cold air moving southward from the pole rises when it reaches the ***polar front*** near 60º. The rising air aloft returns to the north where it sinks back to the pole.

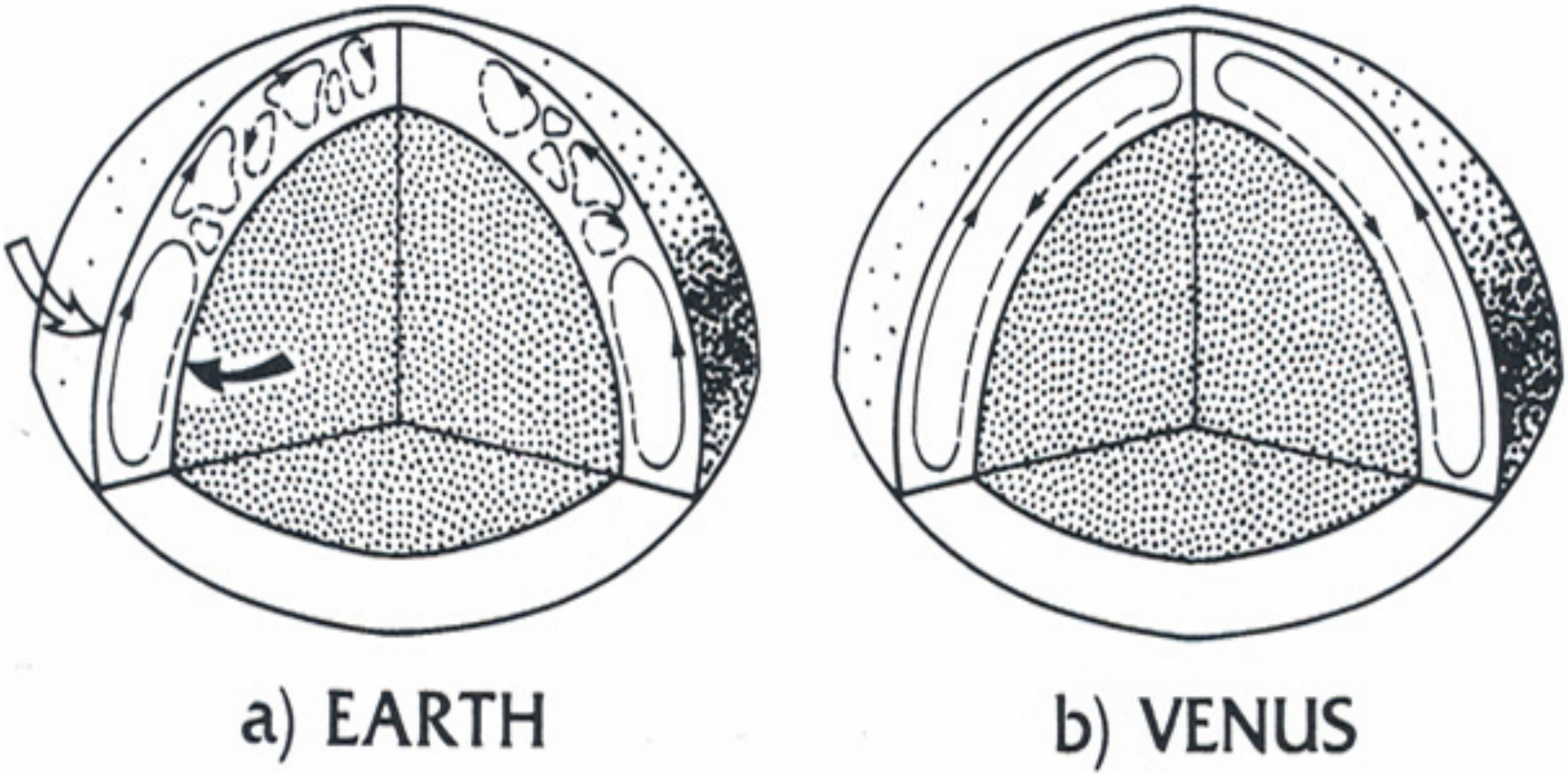
Ferrel Cell (thermal indirect)

A reverse circulation between the Hadley and Polar Cell (between 30º and 60º) that carries air northward near the surface and southward aloft

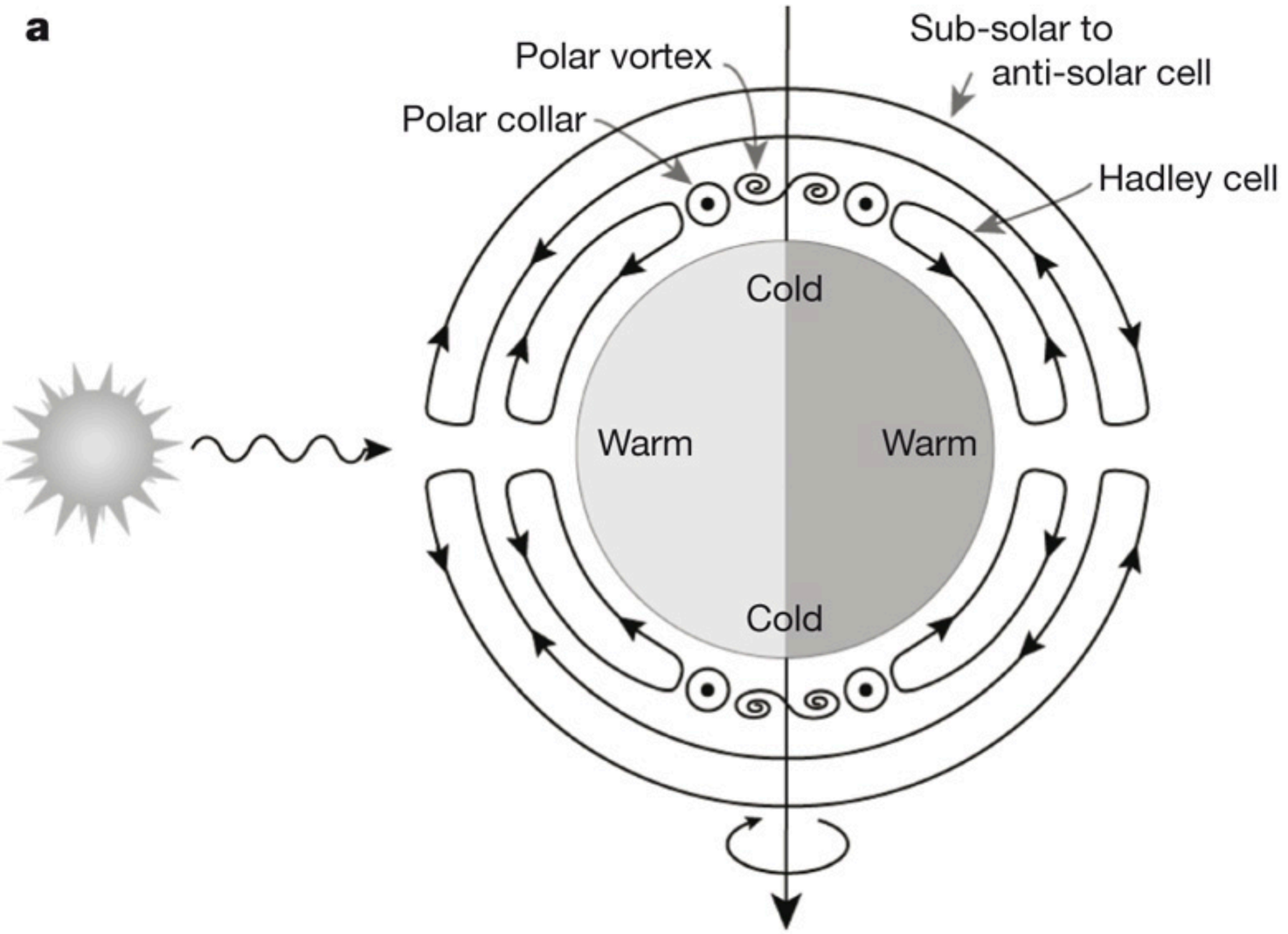
A rotating earth break the single cell into Three Cells



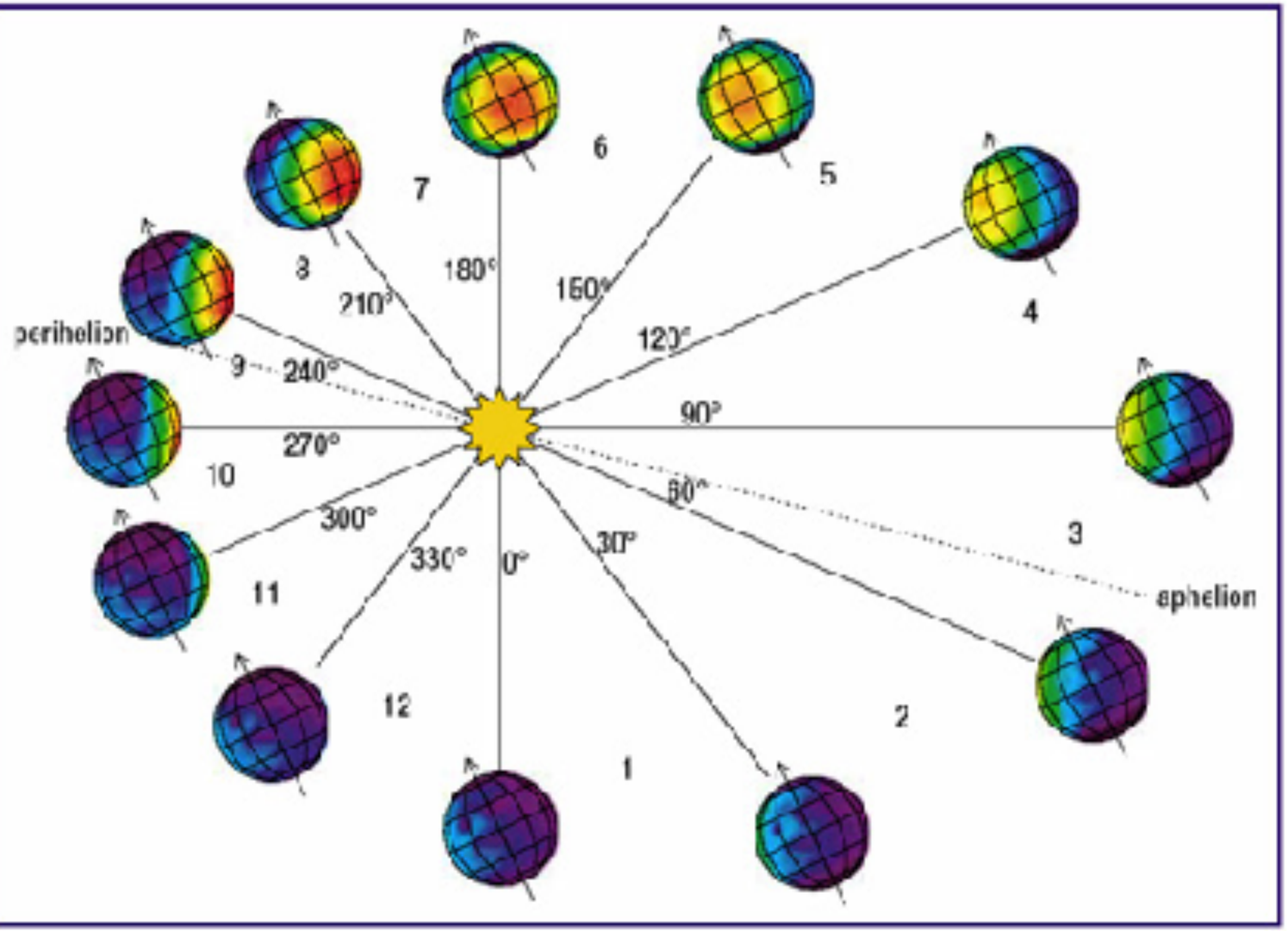
Earth and Venus



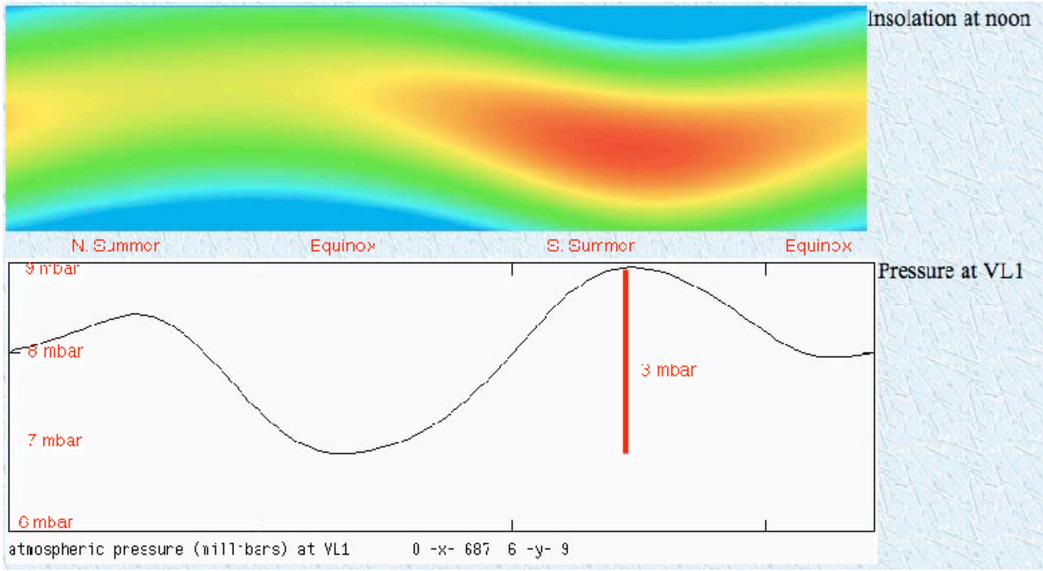
Venus


a
Polar vortex
Polar collar
Sub-solar to
anti-solar cell
Hadley cell
Cold
Warm
Warm
Cold

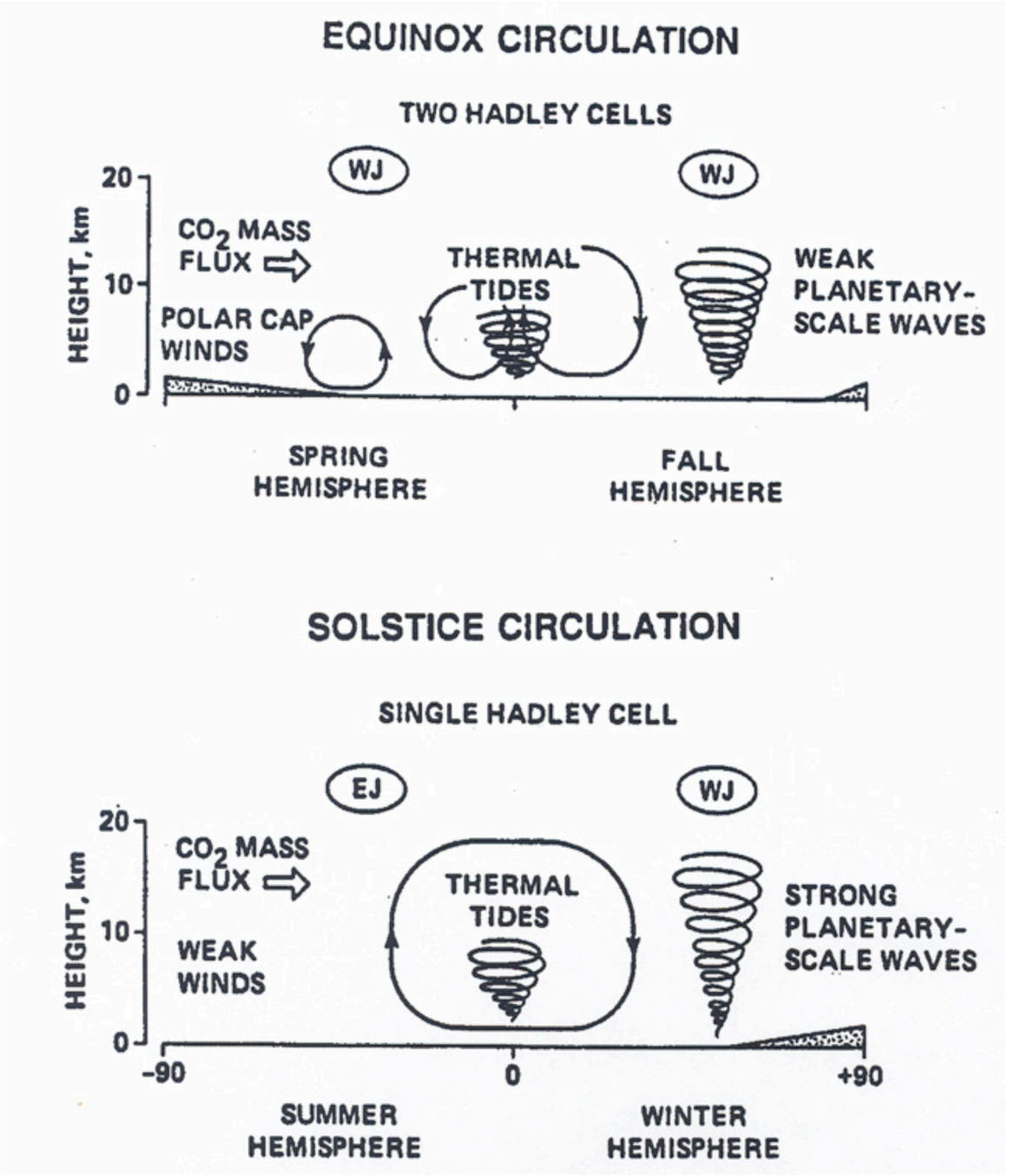
Mars



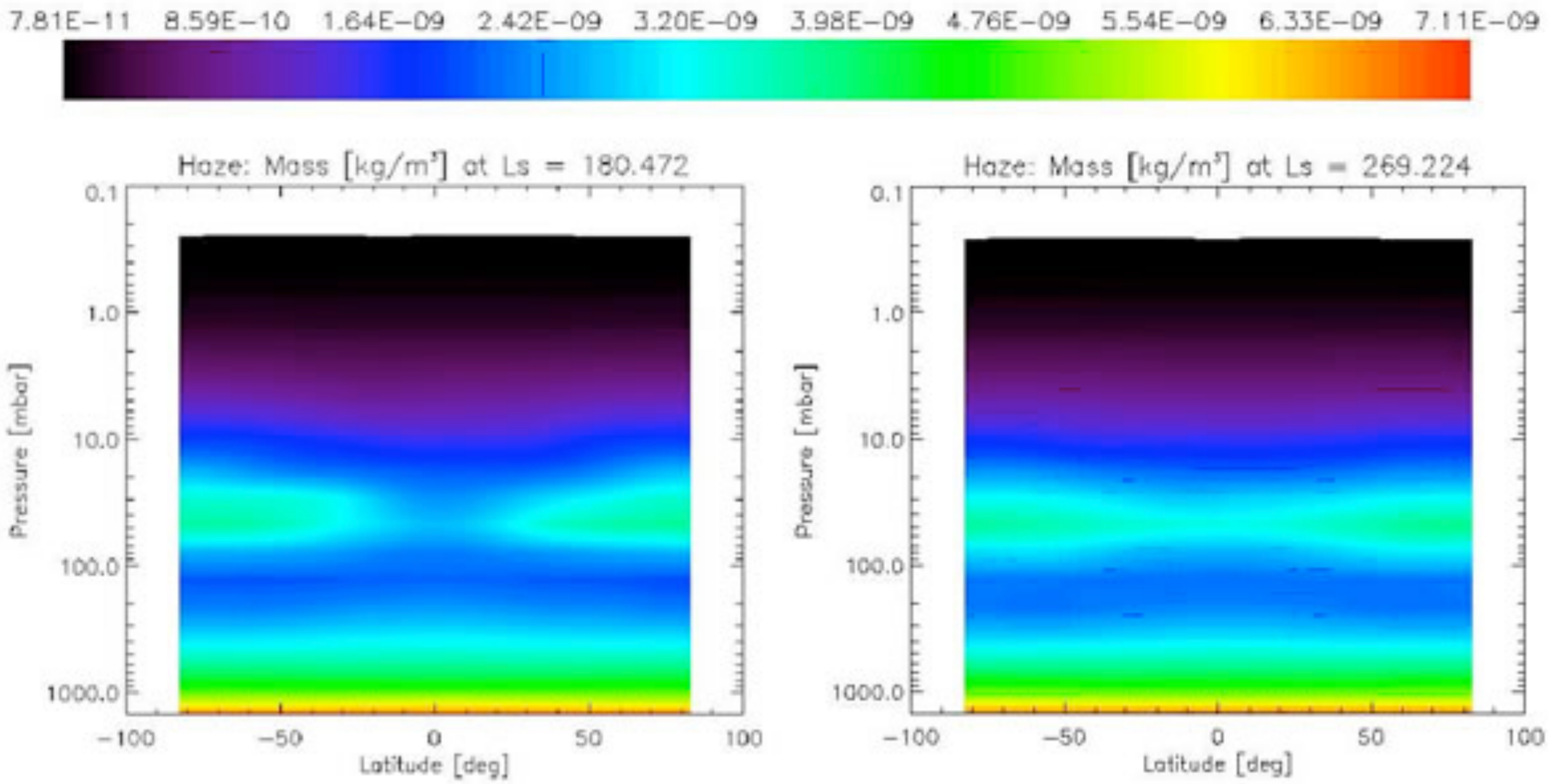
- Seasonal changes very important
- e=0.0934, obliquity = 25.2 degrees
  - (For Earth, e=0.017, obliquity = 23.5)



Mars


EQUINOX CIRCULATION
TWO HADLEY CELLS
WJ
WJ
20
10
0
HEIGHT, km
CO_2 MASS FLUX
POLAR CAP WINDS
THERMAL TIDES
WEAK PLANETARY-SCALE WAVES
SPRING HEMISPHERE
FALL HEMISPHERE
SOLSTICE CIRCULATION
SINGLE HADLEY CELL
EJ
WJ
20
10
0
HEIGHT, km
CO_2 MASS FLUX
WEAK WINDS
THERMAL TIDES
STRONG PLANETARY-SCALE WAVES
-90
0
+90
SUMMER HEMISPHERE
WINTER HEMISPHERE

Titan



- Start of northern fall (equinox) Northern Winter
- Equator-to-pole Hadley circulation pole-to-pole circulation
  results in accumulation of haze at poles

Break

Atmospheric Waves

- Role of waves in atmospheric dynamics:
  - Modify mean atmospheric structure (temperature, composition)
  - Alter mean atmospheric flow (e.g. accelerate it)
- Waves are characterized by amplitude, wavelength, and frequency (and also phase speed, group speed when propagating)

Atmospheric Waves

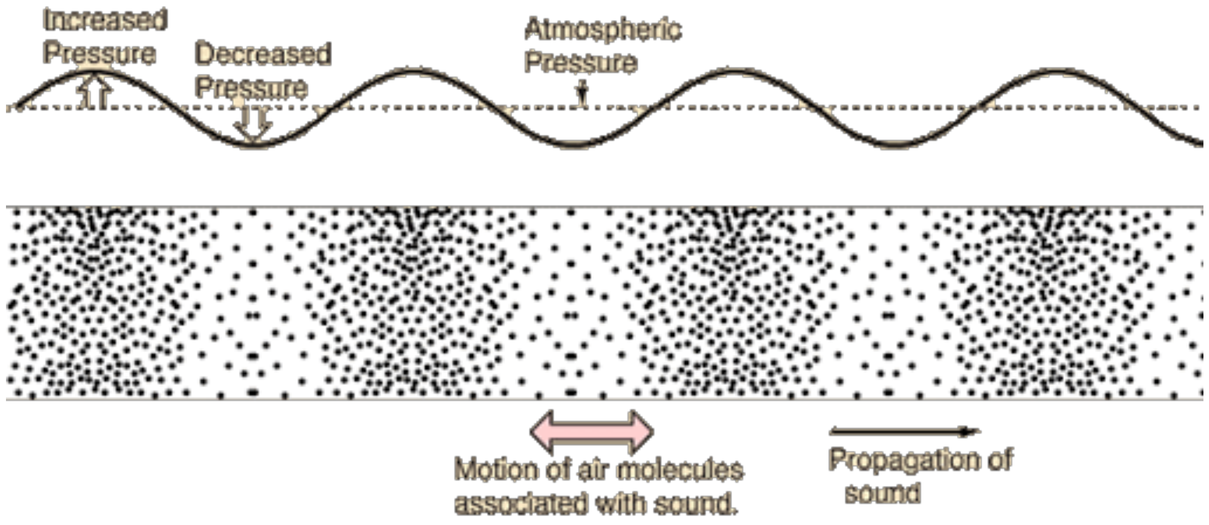
- Classified in various ways:
  - restoring mechanism (e.g. gravity, Coriolis force)
  - forced vs. free modes (e.g. thermal tides)
  - traveling vs. stationary (trapped by topography?)

Perturbation Theory

- $u(x,t) = \bar{u} + u'(x,t)$
  - 1) Determine which terms in motion and thermodynamic equations are relevant for particular types of wave
  - 2) Use perturbation theory to make magnitudes linear
  - 3) Derive wave equations for appropriate for field variable
  - 4) Introduce harmonic solution containing amplitudes, wavenumbers, frequency (oversimplification: really need Fourier series)
  - 5) Derive dispersion relation (phase speed vs. wavenumber) and group speed where appropriate

Acoustic Waves

- Result from restoration effect of gradient pressure force
- Oscillating pressure (compression, rarefaction) can be considered adiabatic
- longitudinal waves (particle oscillation is parallel to propagation direction)



Acoustic Waves

- Relevant equations:

  (v = w = 0)

- Momentum equation:

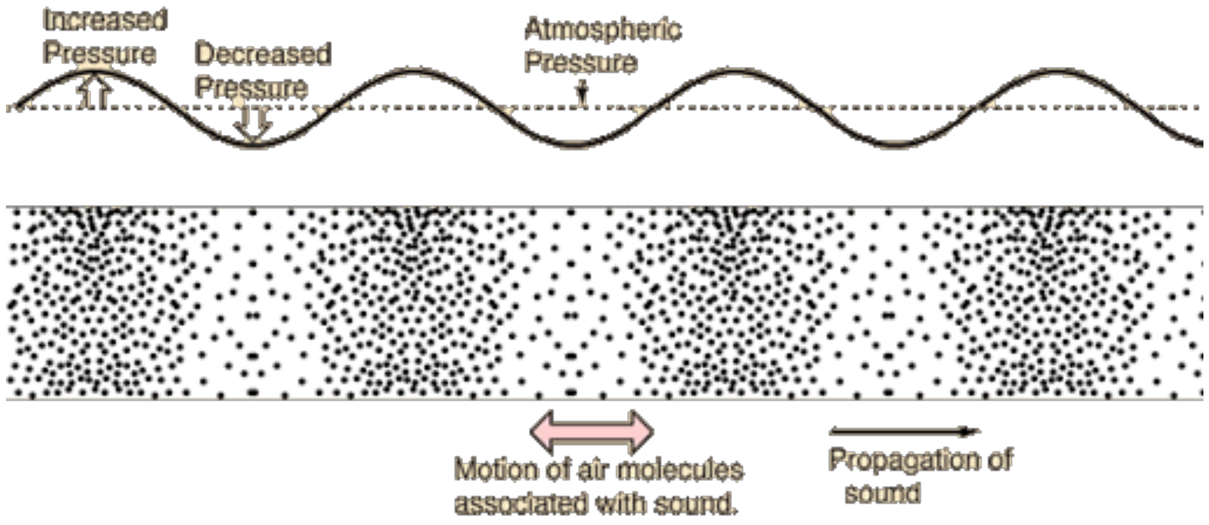
$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial x} = 0$$

- Continuity equation:

$$\frac{d\rho}{dt} + \frac{1}{\rho}\frac{\partial u}{\partial x} = 0$$

- Thermodynamics:

$$P\rho^{-\gamma} = \text{constant}$$



Acoustic Waves

- Use perturbation theory to expand variables:

$$u(x,t) = \bar{u} + u'(x,t)$$

$$P(x,t) = \bar{P} + P'(x,t)$$

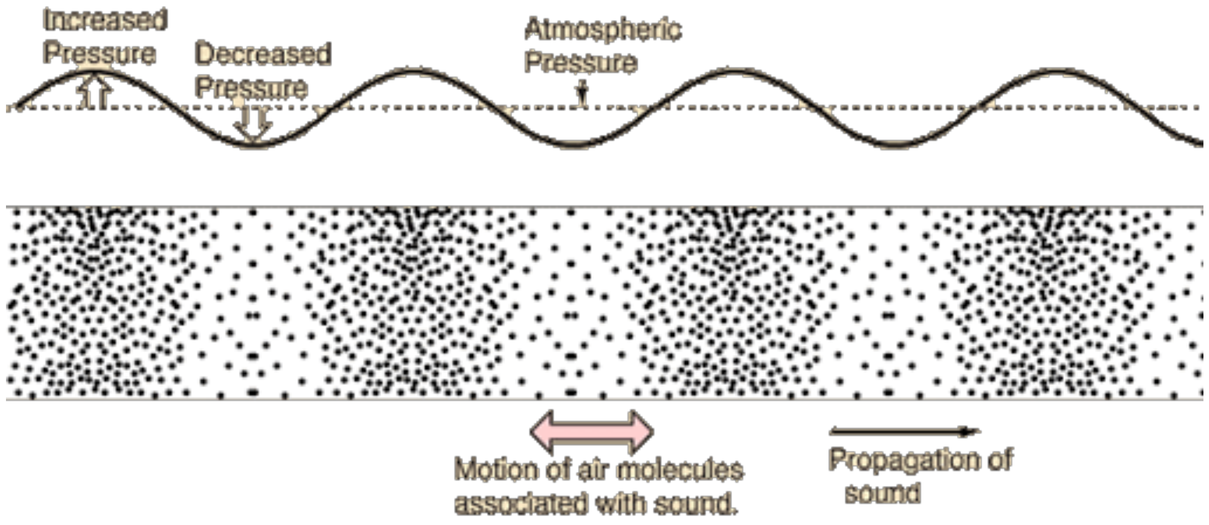
$$\rho(x,t) = \bar{\rho} + \rho'(x,t)$$

- After a bit of math, we get:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 P' - \frac{\gamma \bar{P}}{\bar{\rho}}\frac{\partial^2 P'}{\partial x^2} = 0$$

- This has a solution of the form:

$$P' = P_0 e^{ik(x - c_w t)}$$



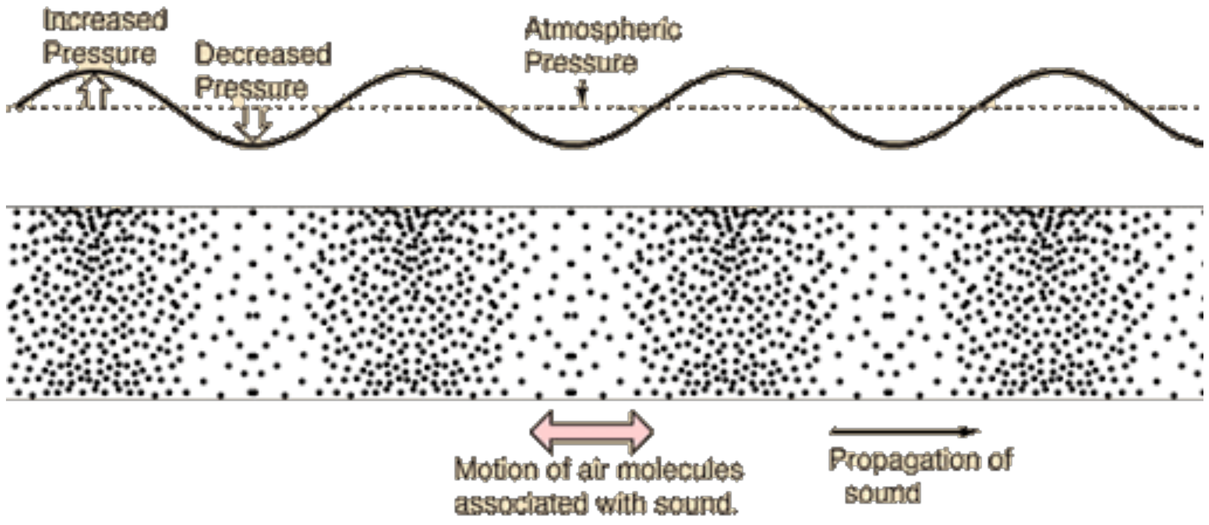
Acoustic Waves

- $P' = P_0 e^{ik(x - c_w t)}$
- Plugging in this solution and solving for the constant c_w:

$$c_w = \bar{u} \pm \left(\frac{\gamma \bar{P}}{\bar{\rho}} \right)^{\frac{1}{2}} = \bar{u} \pm \sqrt{\gamma R_g T} = \bar{u} + c_s$$

- Where c_s is the speed of sound relative to the background flow:

$$c_s = \sqrt{\gamma R_g T}$$



Acoustic Waves

- Amplitude of sound wave is related to its intensity (W/m^2) by:

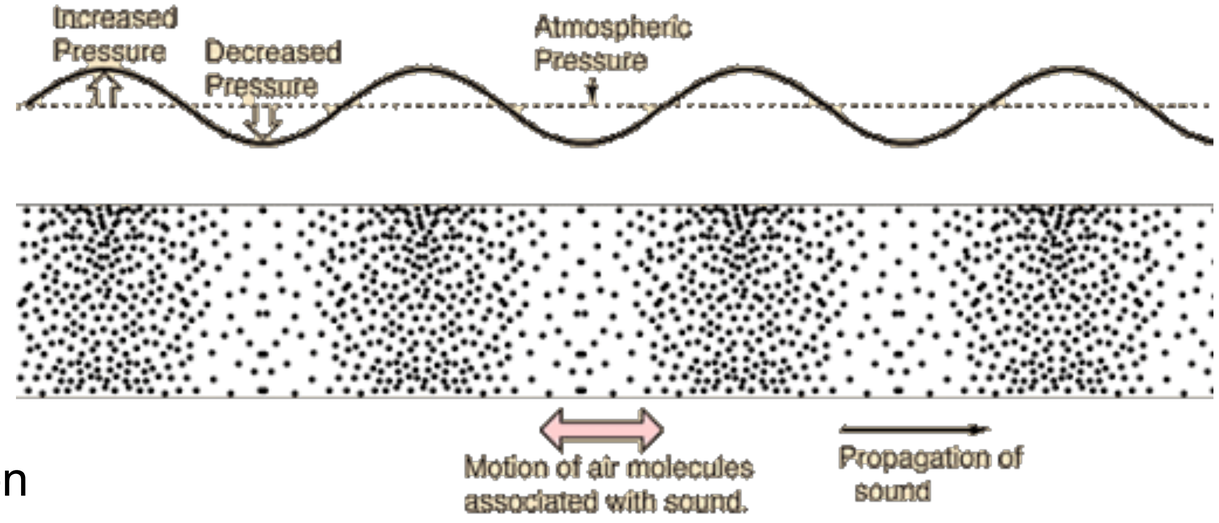
$$I_s = \frac{1}{2}\rho\omega^2(\Delta l)^2 c_s = \frac{P_0^2}{2\rho c_s}$$

- Where:

$\omega = 2\pi\nu =$ angular frequency

$\rho =$ air density

$\Delta l =$ displacement of air particles due to presure perturbation



Acoustic Waves

- Amplitude of sound wave is related to its intensity (W/m^2) by:

$$I_s = \frac{1}{2}\rho\omega^2(\Delta l)^2 c_s = \frac{P_0^2}{2\rho c_s}$$

- Pressure amplitude:

$$P_0 = (2I_s\rho c_s)^{\frac{1}{2}}$$

- Sound intensity:

$$L = 10\log_{10}(\frac{I_s}{I_0})\ dB$$

"Standard Reference sound intensity": $I_0 = 10^{-12} W/m^2$

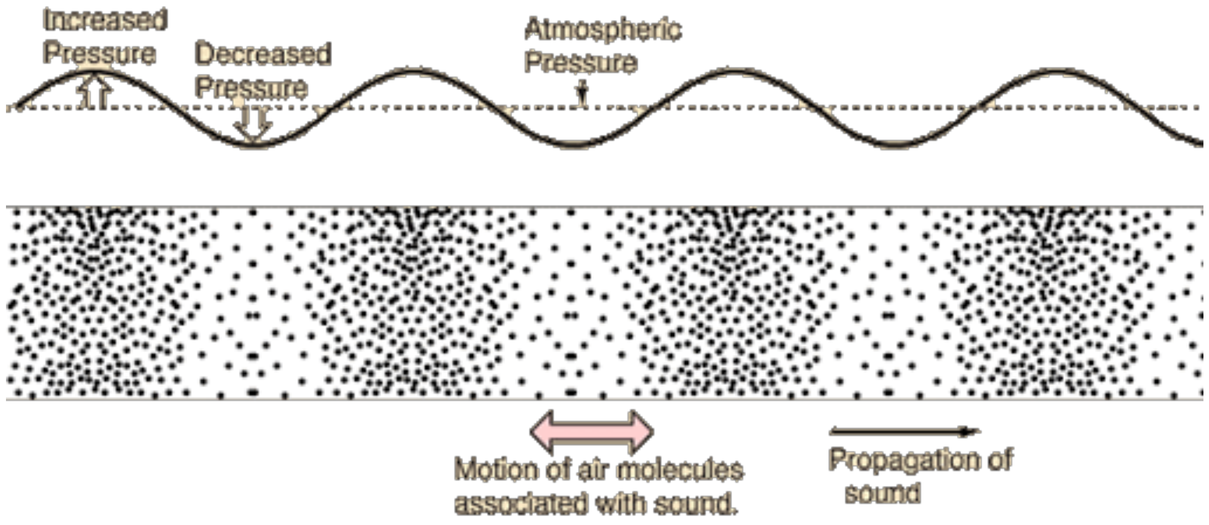


TABLE 8.1
Sound Speed in Planetary Atmospheres

Planet	Level	T (K)	P (Bar)	c_s (m s^{-1})
Venus	Surface	734	92	420
	z=60 km (clouds)	250	0.22	248
Earth	Surface	285	1	340
	z=10 km	217	0.1	295
Mars	Surface	240	0.007	229
	z=80 km	140	0.0001	185
Titan	Surface	94	1.5	194
	z=28 km	73	0.3	170
Jupiter	Ammonia cloud	127	0.4	815
	Water cloud	270	5	1200
Saturn	Ammonia cloud	107	0.5	763
Uranus	Methane cloud	85	1.4	655
Neptune	Methane cloud	81	1.4	640
HD 209458B	—	1350	0.5	2500

Source: Adapted from Sánchez-Lavega, A., *Phys. Teacher*, 40, 239, 2002.

Order of Magnitude: Acoustic Waves

- $P_0 = (2I_s\rho c_s)^{\frac{1}{2}}$

$$c_s = \sqrt{\gamma R_g T} = \sqrt{\frac{\gamma P}{\rho}}$$

1 bar = 10^5 Pa

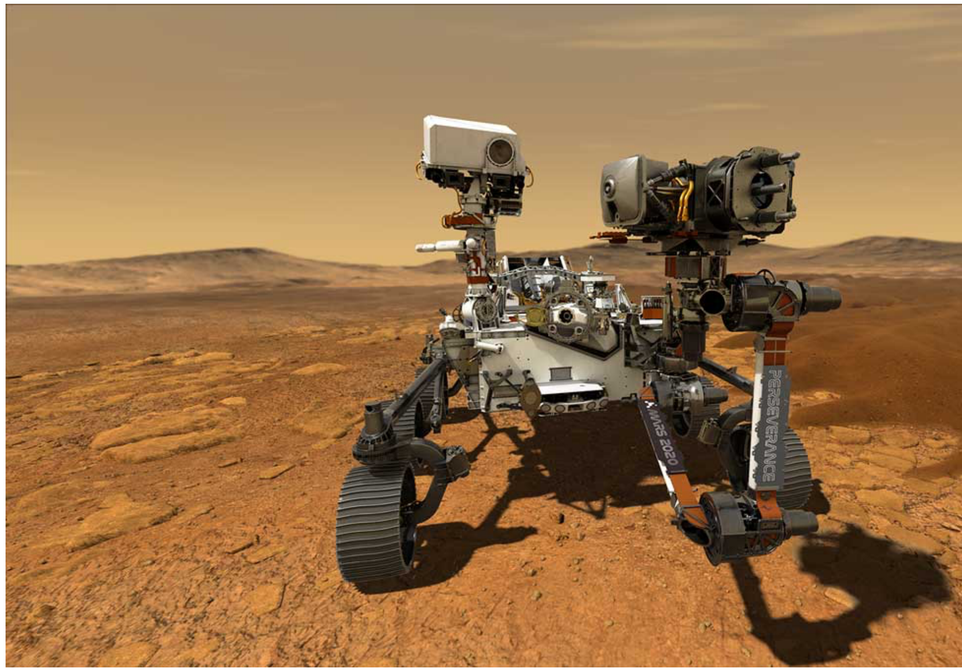
$$L = 10\log_{10}(\frac{I_s}{I_0})\ dB$$

Monatomic: $\gamma = \frac{5}{3}$

Diatomic: $\gamma = \frac{7}{5}$

$$I_0 = 10^{-12} W/m^2$$

Polyatomic: $\gamma = \frac{4}{3}$



- (1) Thunder in Earth's atmosphere produces a sound wave with an intensity of 80 dB. What is the pressure amplitude considering the conditions at the surface?

- (2) Calculate the pressure amplitude of a sound wave in Mars' atmosphere corresponding to a sound level of 60 dB being detected by a microphone on a Martian probe.

Order of Magnitude: Acoustic Waves

- $P_0 = (2I_s\rho c_s)^{\frac{1}{2}}$ $\quad c_s = \sqrt{\gamma R_g T} = \sqrt{\frac{\gamma P}{\rho}}$

1 bar = 10^5 Pa

$L = 10\log_{10}(\frac{I_s}{I_0})\ dB$ $\quad$ Monatomic: $\gamma = \frac{5}{3}$

Diatomic: $\gamma = \frac{7}{5}$

$I_0 = 10^{-12} W/m^2$ $\quad$ Polyatomic: $\gamma = \frac{4}{3}$

- (1) Thunder in Earth's atmosphere produces a sound wave with an intensity of 80 dB. What is the pressure amplitude considering the conditions at the surface?
- Ok, let's first solve the second equation for the intensity:

$\log_{10}(\frac{I_s}{I_0}) = \frac{L}{10}$ $\quad I_s = I_0 10^{L/10}$

- Then we plug that into the pressure amplitude equation:

$P_0 = (2I_0 10^{L/10}\rho c_s)^{\frac{1}{2}}$

Order of Magnitude: Acoustic Waves

- $P_0 = (2I_0 10^{L/10} \rho c_s)^{\frac{1}{2}}$
- Almost done, now we also need to replace density with something we have, pressure and sound speed:

$c_s = \sqrt{\gamma R_g T} = \sqrt{\frac{\gamma P}{\rho}}$ Solving for density: $\rho = \frac{\gamma P}{c_s^2}$

- Plugging in: $P_0 = \sqrt{\frac{2I_0 10^{L/10} \gamma P}{c_s}}$ Great, we can use this equation for both problems.
- On Earth's surface, P is 1 bar (cgs units) = 10^5 Pa (mks units), and speed of sound is 340 m/s (mks)
- The atmosphere is mainly molecular nitrogen and molecular oxygen, both diatomic, so $\gamma = \frac{7}{5}$
- Finally, the problem asks us to solve for 80 dB, and the constant $I_0 = 10^{-12} W/m^2$

Order of Magnitude: Acoustic Waves

- $P_0 = \sqrt{\frac{2I_0 10^{L/10}\gamma P}{c_s}} = \sqrt{\frac{2(10^{-12}W/m^2)10^{(80)/10}(7/5)(10^5 Pa)}{(340m/s)}} = \sqrt{\frac{3x10^{-12+8+5}}{3x10^2}} = \sqrt{10^{-1}} = \frac{1}{3}Pa$

- (2) Calculate the pressure amplitude of a sound wave in Mars' atmosphere corresponding to a sound level of 60 dB being detected by a microphone on a Martian probe.

- On Mars' surface, P is 0.007 bar (cgs units) = 700 Pa (mks units), and speed of sound is 229 m/s (mks)

- The atmosphere is mainly carbon dioxide, polyatomic, so $\gamma = \frac{4}{3}$

- $P_0 = \sqrt{\frac{2(10^{-12}W/m^2)10^{(60)/10}(4/3)(700Pa)}{(229m/s)}} = \sqrt{\frac{2x4x7x10^{-12+6+2}}{2x3x10^2}} = \sqrt{9x10^{-6}} = 3x10^{-3}Pa$

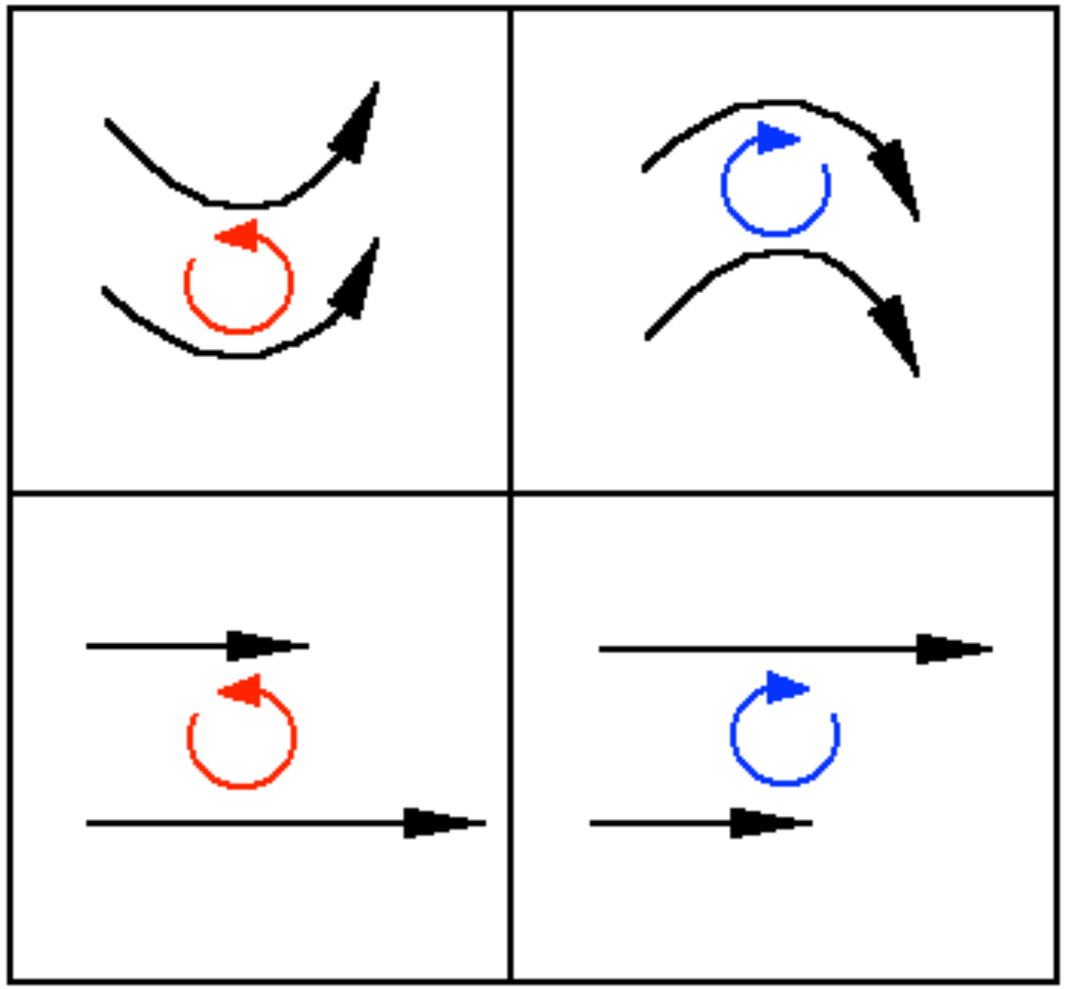
Measurements of Rotation

- Circulation: a scalar integral quantity, macroscopic measure of rotation for a finite area of the fluid

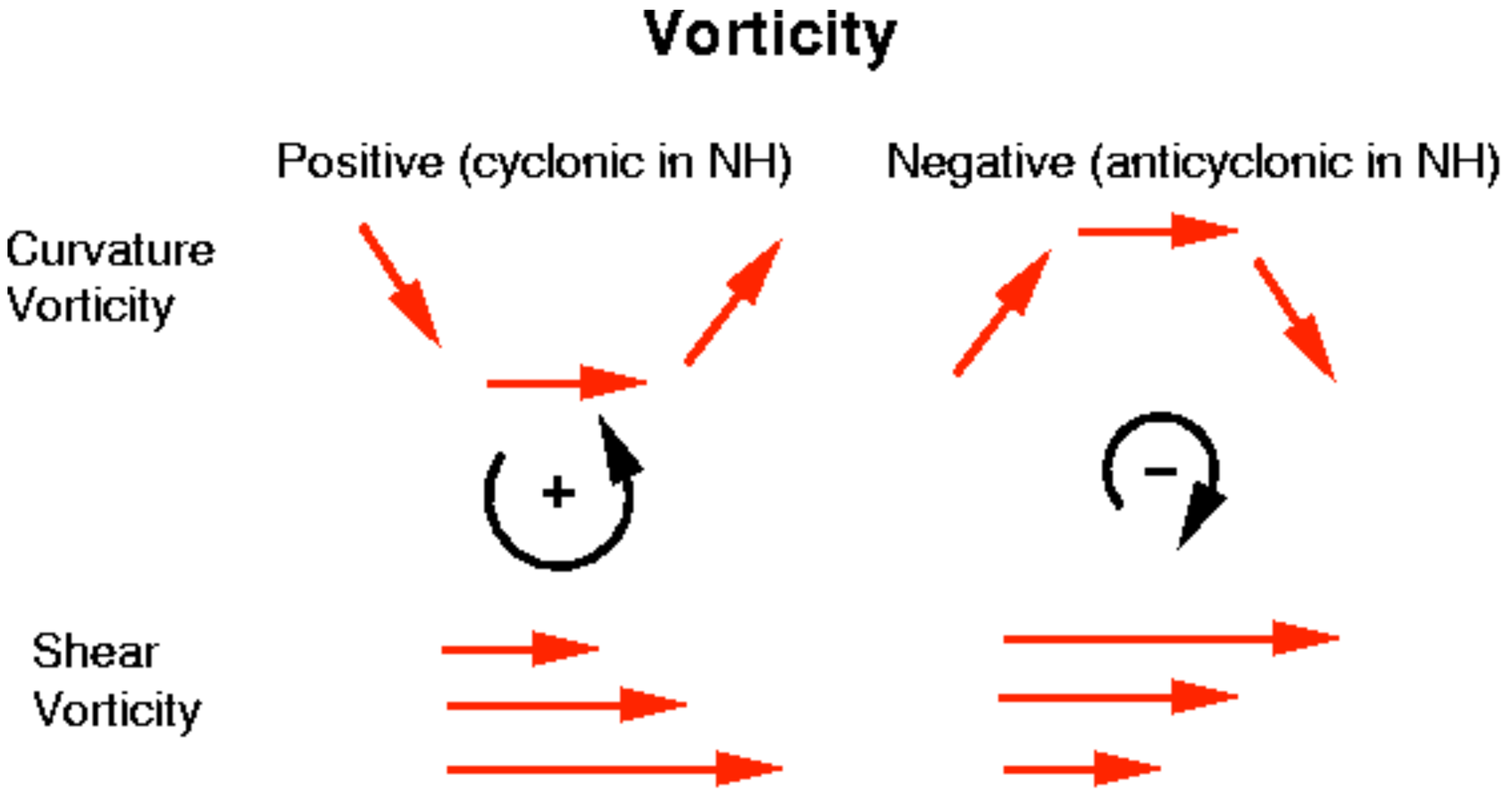
- Vorticity: a vector field, is a microscopic measure of the rotation at any point in the fluid

- The concepts of circulation, vorticity, and potential vorticity play a key role in atmospheric dynamics and meteorology
    - They characterize the rotational state of a fluid

Vorticity



Vorticity


Vorticity
Positive (cyclonic in NH)
Negative (anticyclonic in NH)
Curvature Vorticity
+
−
Shear Vorticity

Vorticity

- Defined as the curl of the velocity vector
- Measures local rotation of fluid (s^{-1})
- $\vec{\omega} = \vec{\nabla} \times \vec{u} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$
- For 2D flow in (x,y) plane, vorticity only has a z-component
- $\vec{\omega} = \omega(0,0,\xi)$

$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ This measures the “spin” of the flow

For next time

- Homework 4 due tonight at 11:59pm
- Reading: Planetary Science, 4.5.3, 4.5.4.6