

ASTR 620: Planetary Processes  
Professor Eric Nielsen

Lecture 16: Atmospheres

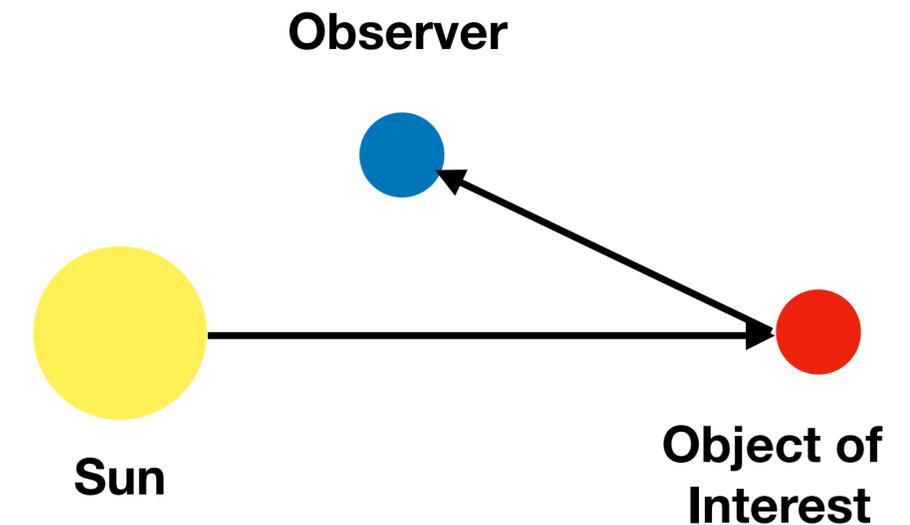


# Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Homework 4 due on Wednesday, October 19 at 11:59pm

# Review of the last class

- What is the scattering angle and phase angle in this image?
  - (A) — Scattering angle = 30 degrees, Phase angle = 70 degrees
  - (B) — Scattering angle = 30 degrees, Phase angle = 150 degrees
  - (C) — Scattering angle = 150 degrees, Phase angle = 30 degrees
  - (D) — Scattering angle = Phase angle = 30 degrees
  - (E) — Scattering angle = Phase angle = 150 degrees



# Review of the last class

- The albedo of a certain planet is 0.4 at 5000 Angstroms. What is the emissivity of that planet at 5000 Angstroms?
  - (A) — 0.0
  - (B) — 0.4
  - (C) — 0.6
  - (D) — 1.0
  - (E) — No way to tell

# Review of the last class

- The albedo of a certain planet is 0.4 at 5000 Angstroms. What is the emissivity of that planet at 20000 Angstroms?
  - (A) — 0.0
  - (B) — 0.4
  - (C) — 0.6
  - (D) — 1.0
  - (E) — No way to tell

# Review of the last class

- Typical surface temperature on Earth is about 300K. What's Earth's equilibrium temperature?
- (A) — 400 K
- (B) — 350 K
- (C) — 300 K
- (D) — 250 K
- (E) — 200 K

# Review of the last class

- An atmosphere with significant greenhouse gasses, like Earth's, is:
  - (A) — mostly transparent in the visible, mostly opaque in the mid infrared
  - (B) — mostly opaque in the visible, mostly transparent in the mid infrared
  - (C) — mostly opaque in both the visible and mid infrared
  - (D) — mostly transparent in both the visible and mid infrared
  - (E) — Completely opaque in both the visible and mid infrared

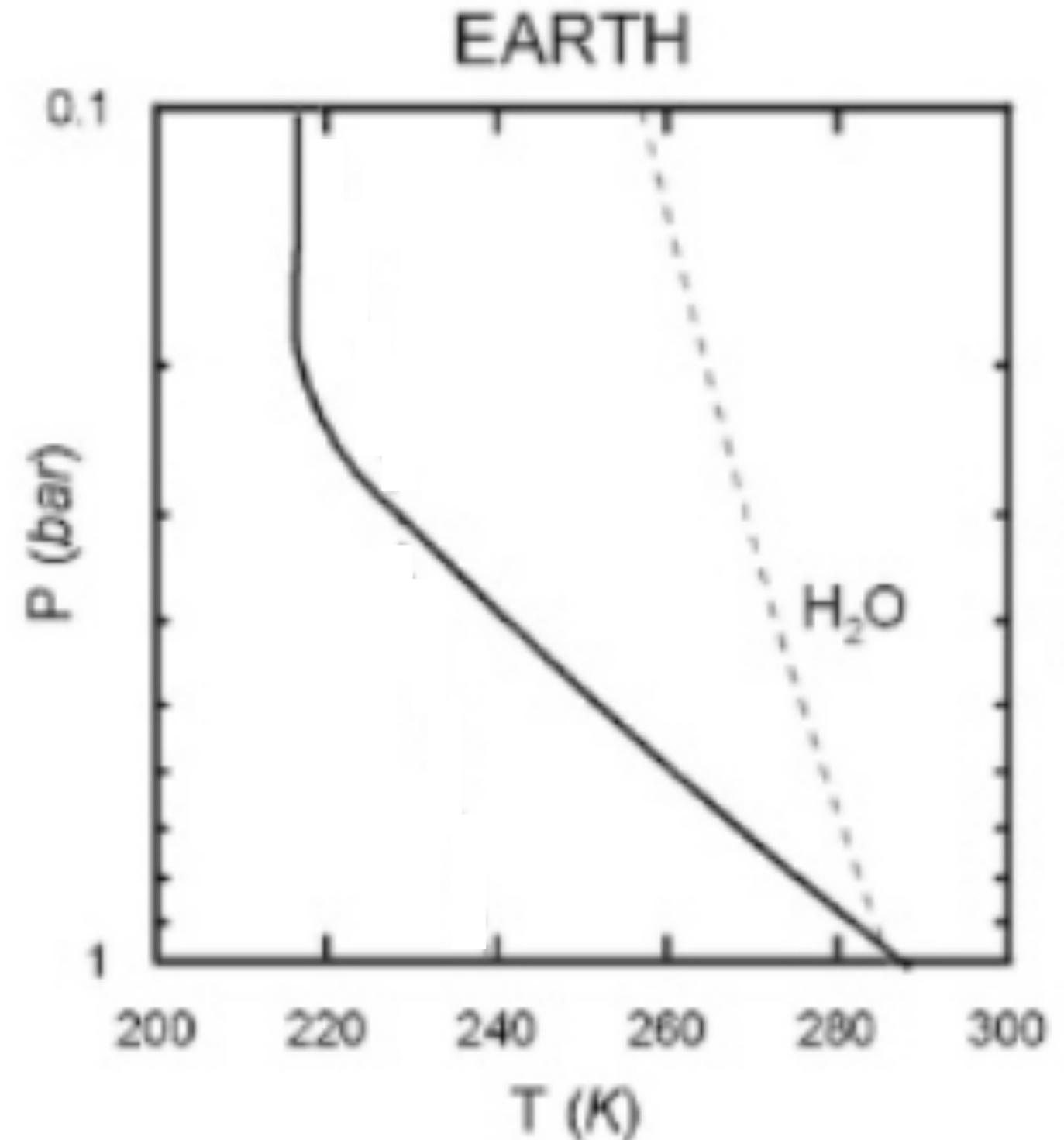
# Clouds

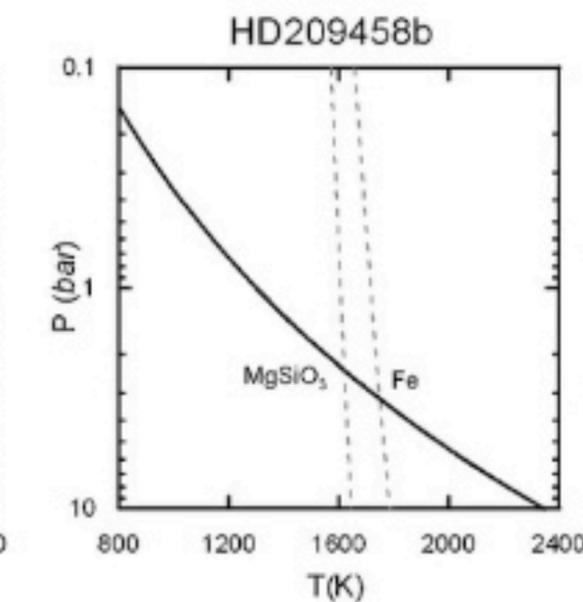
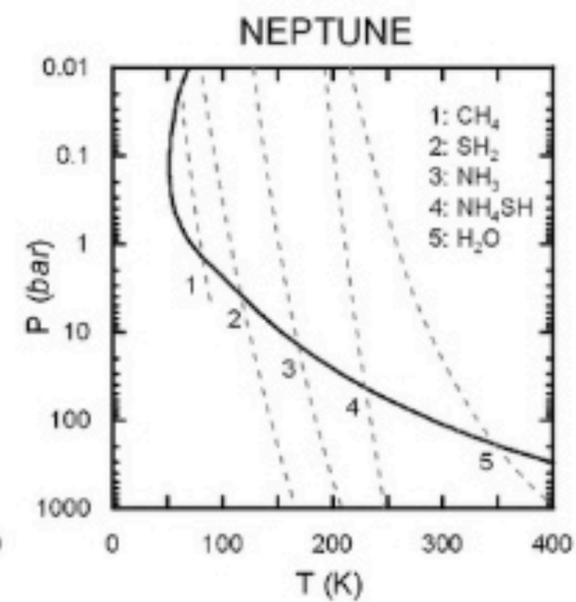
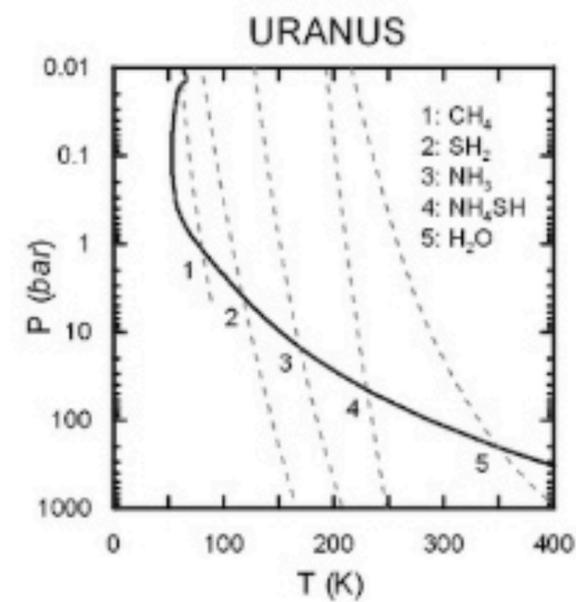
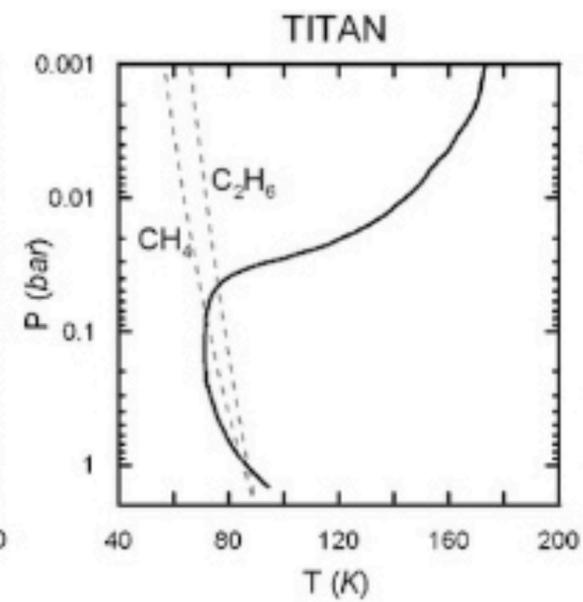
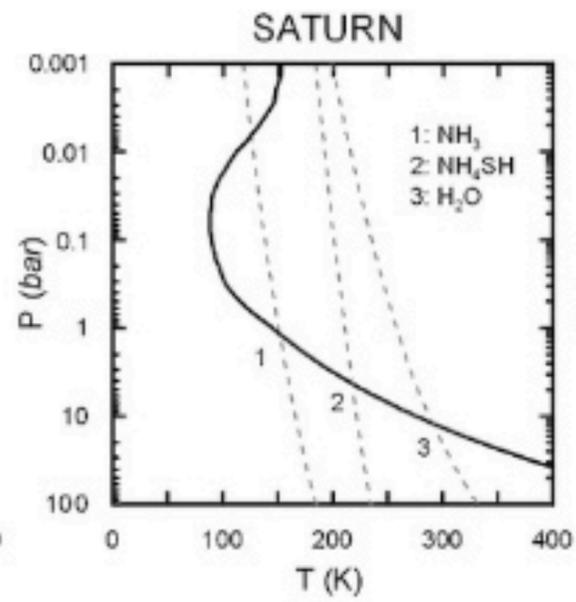
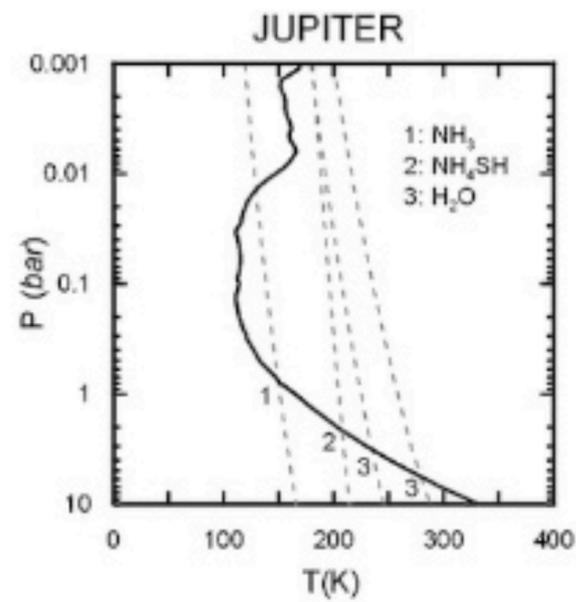
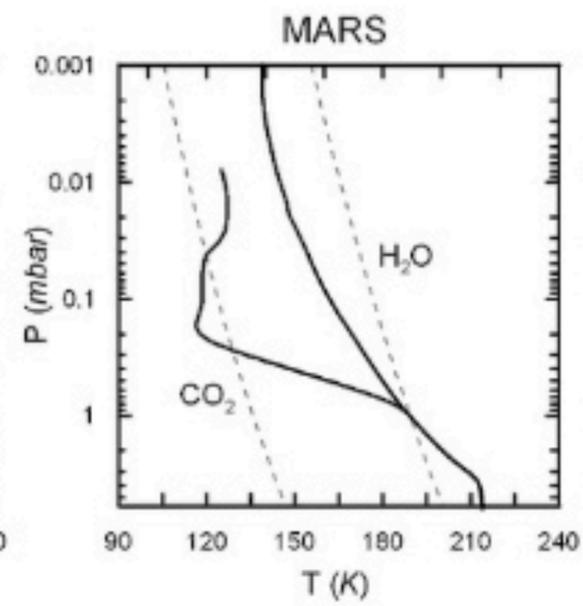
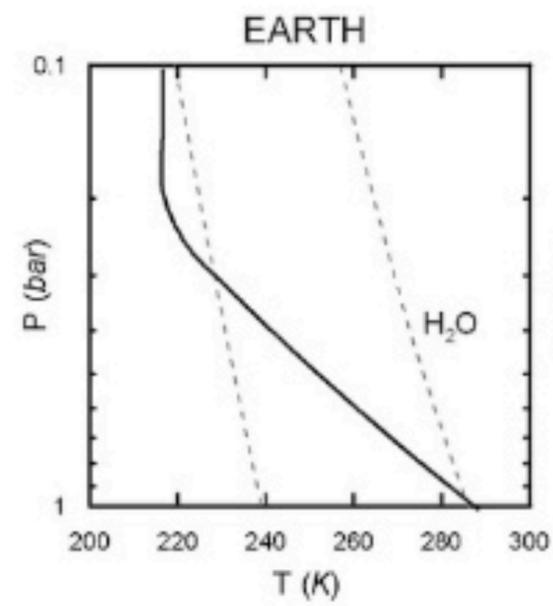
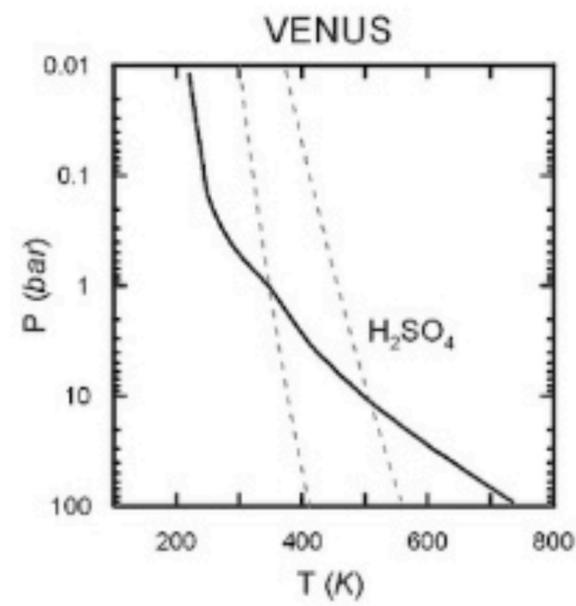
- Clouds play an important role in atmospheric heat balance and thermal structure
- The formation of clouds involves the fields of microphysics and thermodynamics
- Suspended particles can originate from:
  - condensation
  - chemical/photochemical reactions
  - outgassing
  - lifting from the surface
  - particle bombardment of atoms and molecules in upper atmosphere

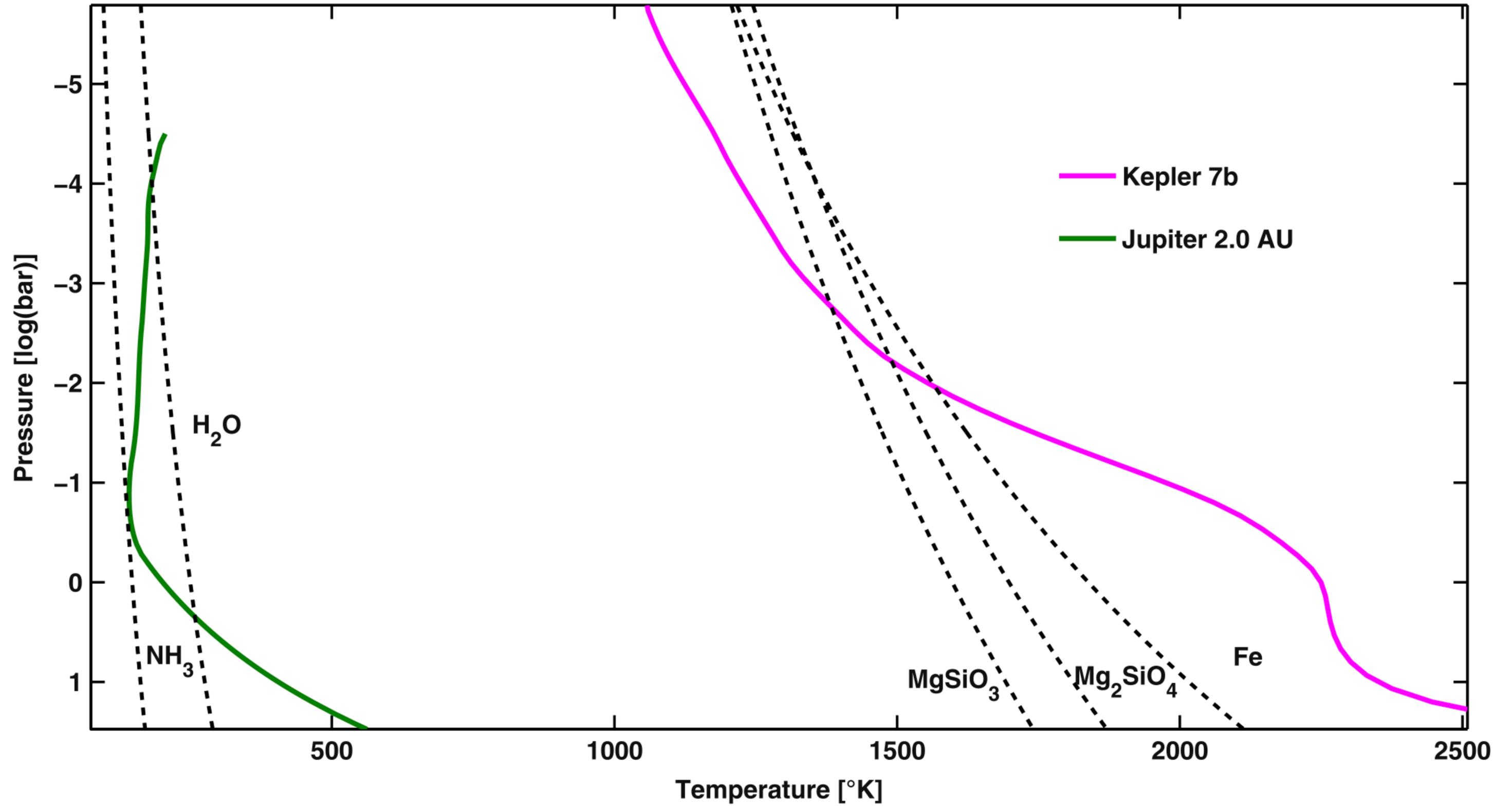


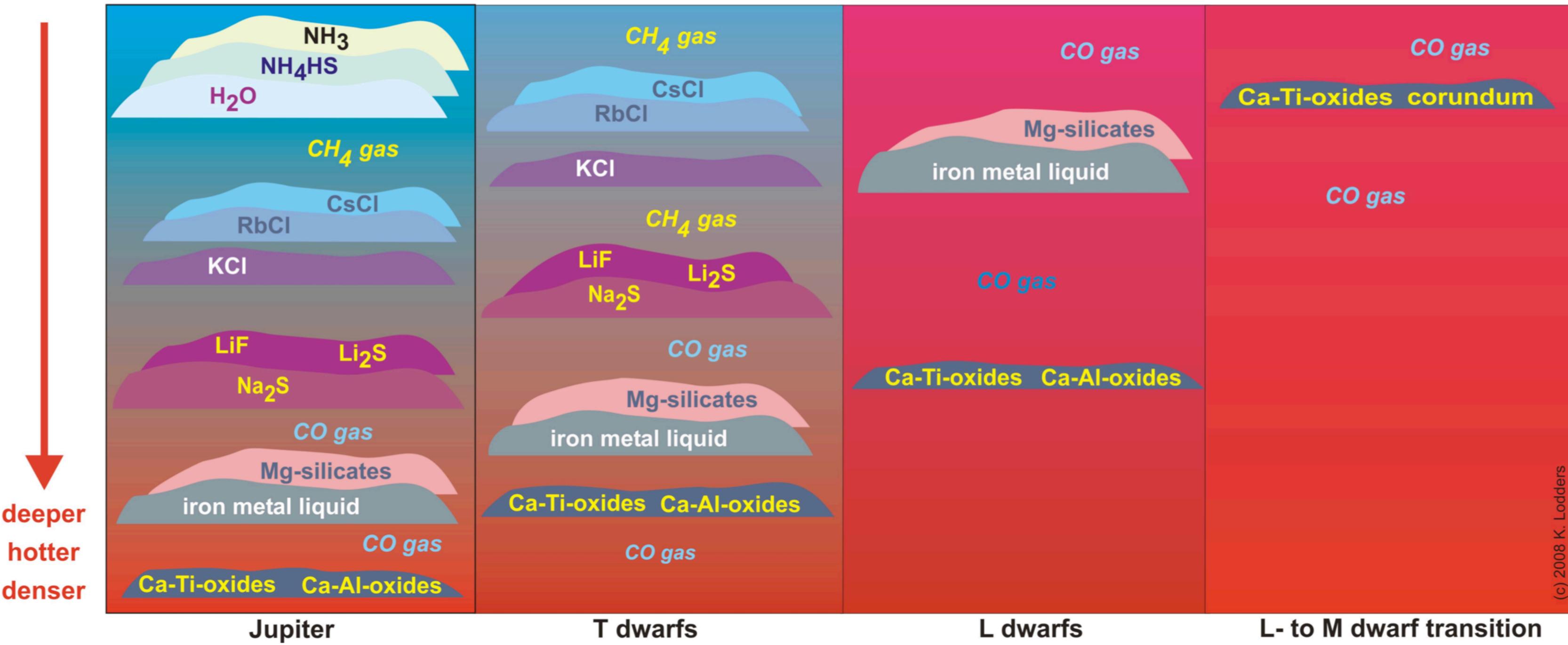
# Cloud formation

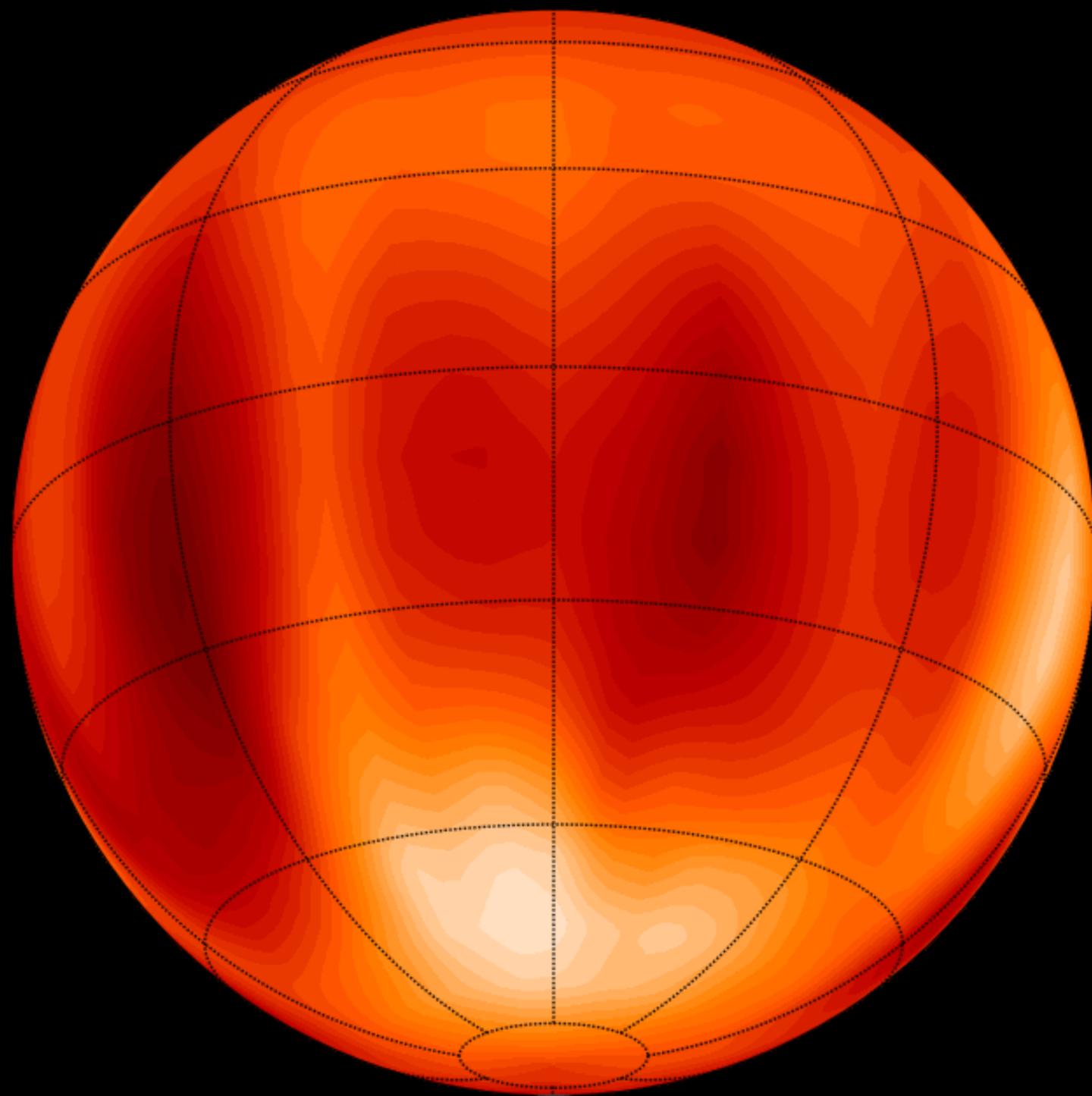
- Saturation vapor pressure: pressure (at a given temperature) where water (or another condensible) condenses
  - dashed line represents the saturation vapor pressure for water
- Clouds form where the saturation vapor pressure curve is at a higher temperature than the environmental lapse rate
- The two curves cross at the cloud base







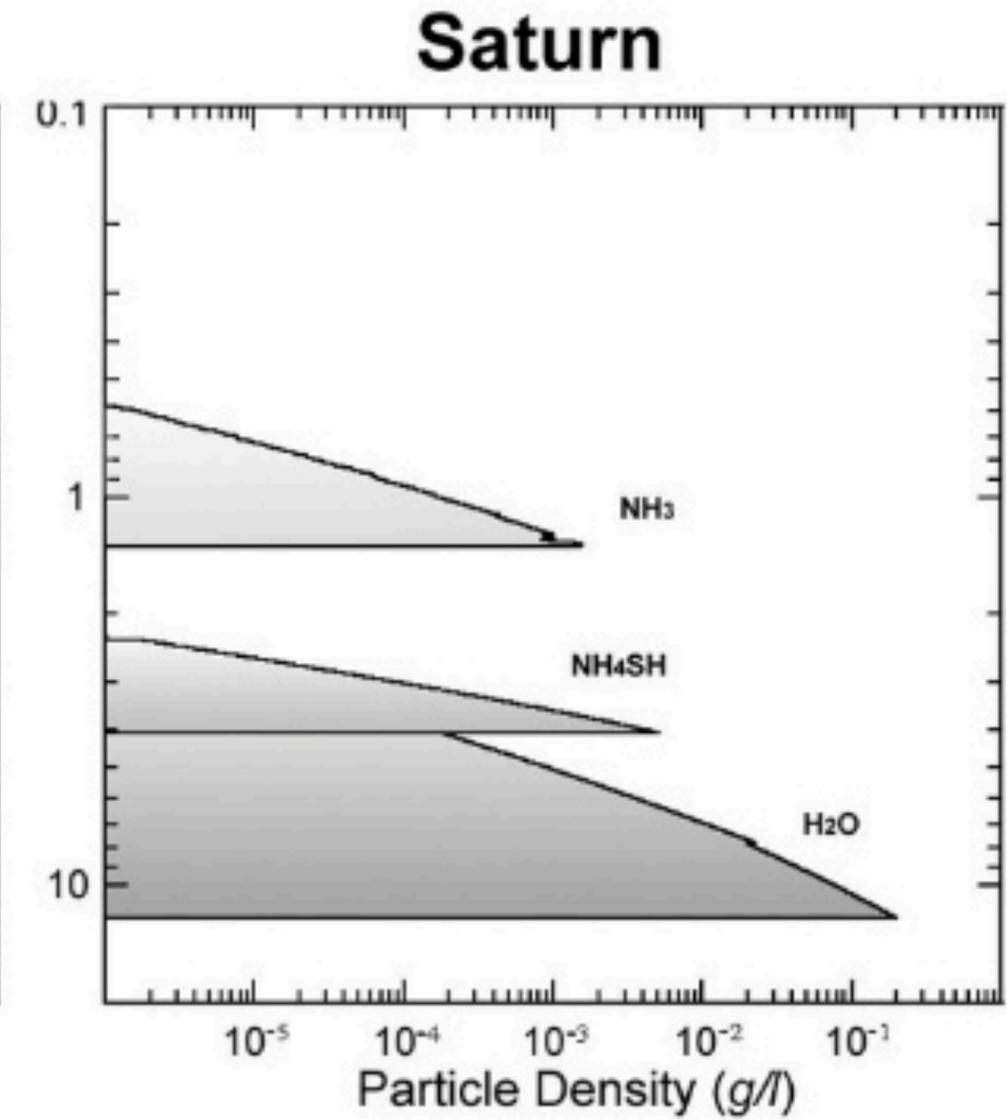
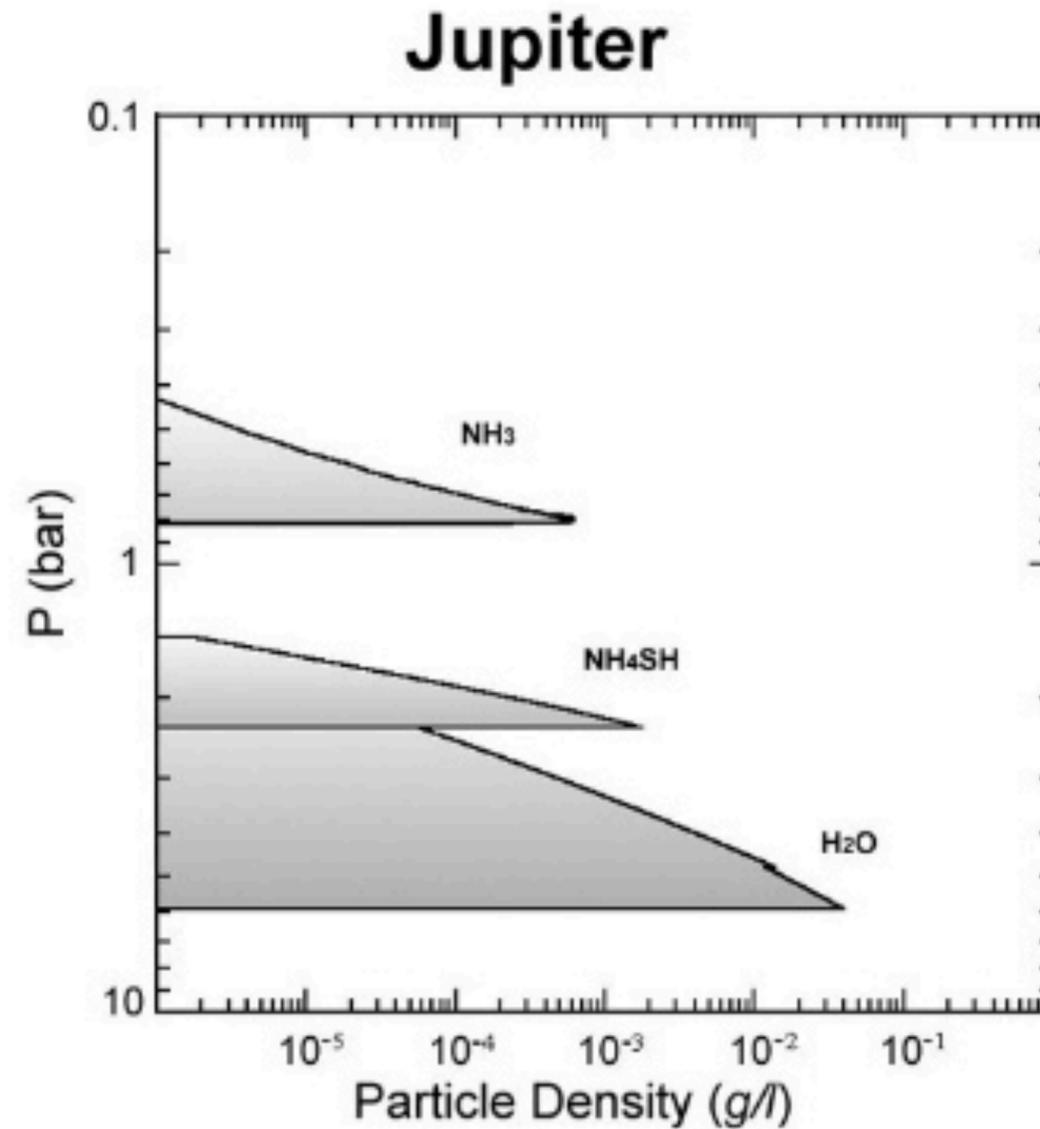




**Lehman-16**  
**Crossfield et al. 2014**

# Giant Planets

- Cloud composition and vertical structure for gas giants
  - Assuming solar abundances of reference elements



# Atmospheric Motions

- Atmospheres obey laws of hydrodynamics valid for any fluid under:
  - spherical geometry
  - planetary rotation
- Open boundary to space at top, lower boundary at surface (sometimes) or “deep atmosphere”



# Atmospheric Motions

- External and internal energy sources can drive motions
- Gaseous, compressible fluids that generally obey the ideal gas law
- Mostly electrically neutral



# Atmospheric Motions

- Motions in planetary atmospheres are described by:
  - Newton's second law (called the Navier-Stokes equation for continuous fluid motions)
  - laws of thermodynamics
  - continuity equation (mass conservation)
  - equation of state (ideal gas law)
- Generally use spherical coordinates, but sometimes cartesian (where appropriate)



# Atmospheric Motions

- Atmospheric motions are described using two classical views:
  - Eulerian: flow is studied (measured) in a fixed point in space relative to a reference frame
    - Example: a Martian lander that is measuring wind speeds, temperature, pressure, etc
  - Lagrangian: motions follow atmospheric flow
    - Example: a small fluid parcel marked with a dye



# Atmospheric Motions

- We can relate the rate of change of some field variable ( $A$ ) following the motion (Lagrangian view) to its rate of change at a fixed point (Eulerian view)
- Take the “total derivative” (also called the “substantive derivative” or “advective derivative” or “material derivative”)
  - Accounts for both changes in the fluid at a location, and movement of material (advection)

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \left( v_x \frac{\partial A}{\partial x} + v_y \frac{\partial A}{\partial y} + v_z \frac{\partial A}{\partial z} \right) = \frac{\partial A}{\partial t} + \vec{v} \cdot \nabla A$$

$$\vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$$

$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt}$$



# Atmospheric Motions

- $\frac{dA}{dt} = \frac{\partial A}{\partial t} + \vec{v} \cdot \nabla A$

- $\frac{dA}{dt}$ : Rate of change with respect to time following motion

- $\frac{\partial A}{\partial t}$ : Local rate of change with respect to time at a fixed point

- $\vec{v} \cdot \nabla A$ : Advection term



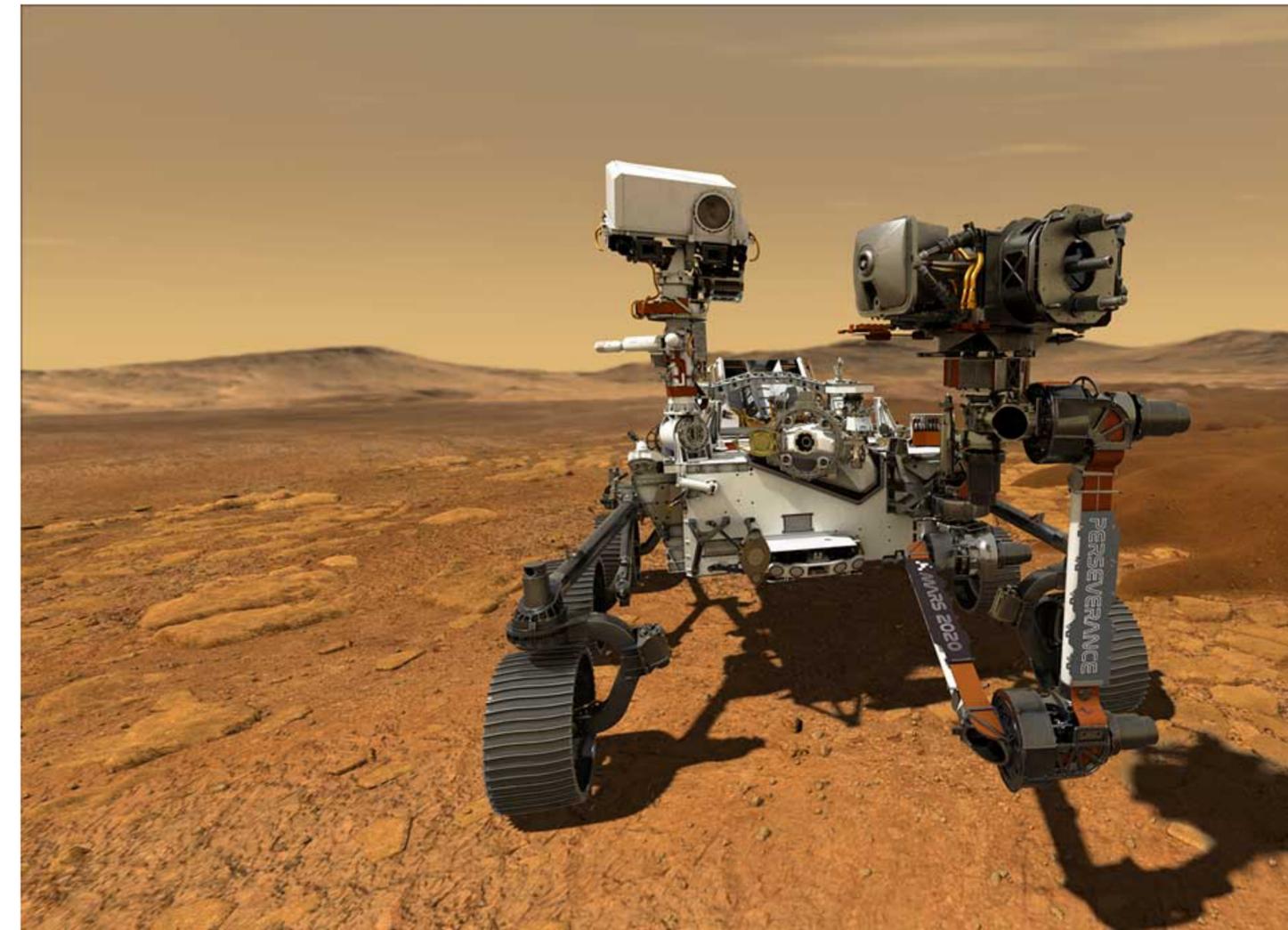
# In-class activity:

## Temperature Change on Mars

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \left( v_x \frac{\partial A}{\partial x} + v_y \frac{\partial A}{\partial y} + v_z \frac{\partial A}{\partial z} \right) = \frac{\partial A}{\partial t} + \vec{v} \cdot \nabla A$$
$$\vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$$

$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt}$$

- The temperature on the surface of Mars decreases by 3K per 100 km in the eastward direction. A rover moving eastward at 1 km/hr measures a temperature decrease of 1 K/hr. What is the temperature change (in K/hr) measured by a fixed Mars lander that the rover is passing by?
- Hint: pay attention to sign (+ or -)



# In-class activity:

## Temperature Change on Mars

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \left( v_x \frac{\partial A}{\partial x} + v_y \frac{\partial A}{\partial y} + v_z \frac{\partial A}{\partial z} \right) = \frac{\partial A}{\partial t} + \vec{v} \cdot \nabla A$$

- The temperature at a point on the surface of Mars decreases by 3K per 100 km in the eastward direction. A rover moving eastward at 1 km/hr measures a temperature decrease of 1 K/hr. What is the temperature change (in K/hr) measured by a fixed Mars lander that the rover is passing by?

- Luckily this problem is 1-dimensional, so let's just define x as increasing to the East. Also, the quantity we care about is temperature, so let's set  $A=T$ :

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x}$$

# In-class activity:

## Temperature Change on Mars

$$\bullet \frac{dT}{dt} = \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x}$$

- The first term is how temperature changes as a function of time following the flow, so that's what the rover measures (-1 K/hr) — note the temperature decreases!
- The second term is how temperature changes as a function of time at a fixed point in space: that's what the stationary lander measures, and what we want to solve for.
- Finally, the last term includes the velocity of the flow (the 1 km/hr eastward motion of the rover) and the change in temperature as a function of distance (-3K / 100 km)
- Let's solve for the change in temperature at the stationary lander, and plug in numbers:

$$\frac{\partial T}{\partial t} = \frac{dT}{dt} - v_x \frac{\partial T}{\partial x} = \left(-1 \frac{K}{hr}\right) - \left(1 \frac{km}{hr}\right) \frac{-3K}{100km} = -1 \frac{K}{hr} + 0.03 \frac{K}{hr} = -0.97 \frac{K}{hr}$$

# Break

**05:00**

# Momentum Conservation

- Use the equation of motion for fluids:
  - Force per unit volume on fluid is equal to sum of external force ( $\rho g$ ) and pressure gradient force

$$\rho \frac{d\vec{v}}{dt} = \rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{v} \right] = -\nabla P - \rho g$$

- For a fluid with viscosity the equation of motion becomes the Navier-Stokes Equation:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{1}{\rho} \nabla P - \vec{g} + \frac{\eta}{\rho} \nabla^2 \vec{v}$$

$\eta$ : dynamic viscosity (cm<sup>2</sup>/s)

$\nu$ : kinematic viscosity (g/cm/s)

$$\nu = \eta / \rho$$



# Momentum Conservation

- $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{1}{\rho} \nabla P - \vec{g} + \frac{\eta}{\rho} \nabla^2 \vec{v}$$

- For an incompressible fluid,  $\nabla \cdot \vec{v} = 0$

- In this case, the Navier Stokes equation becomes:

$$\frac{\partial \vec{v}}{\partial t} = -\frac{\nabla P}{\rho} - \vec{g} + \frac{\eta}{\rho} \nabla^2 \vec{v}$$



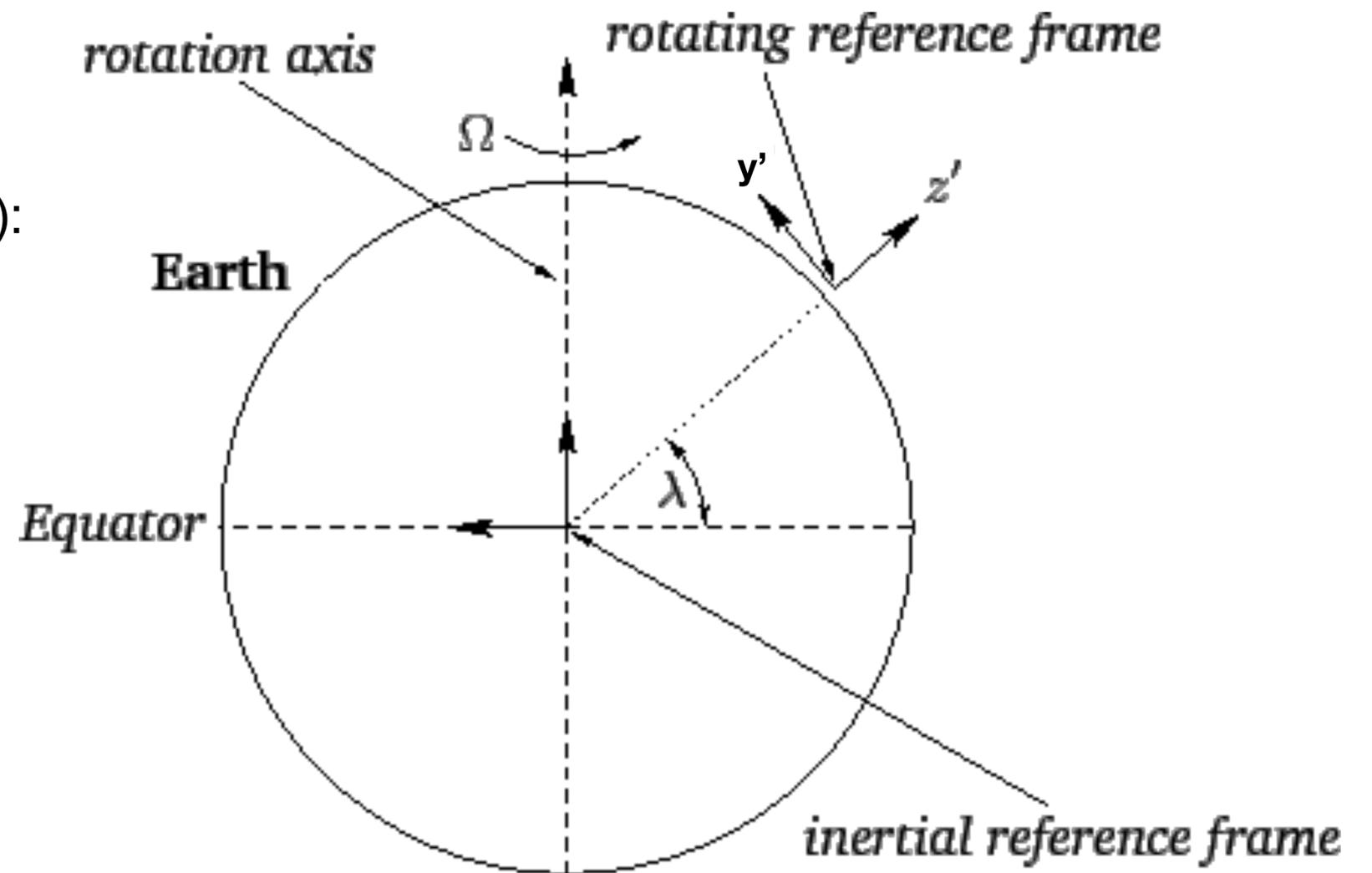
# Rotating Reference Frame

- The rotation of the planet ( $\Omega$  is angular velocity) is significant for large-scale dynamics
- We need to account for this by transforming to a rotating reference frame
- Align rotating coordinates  $(x', y', z')$  with (east, north, up):

- $u = \frac{dx'}{dt}$ : zonal (east-west) winds

- $v = \frac{dy'}{dt}$ : meridional (north-south) winds

- $w = \frac{dz'}{dt}$ : vertical winds



# Rotating Reference Frame

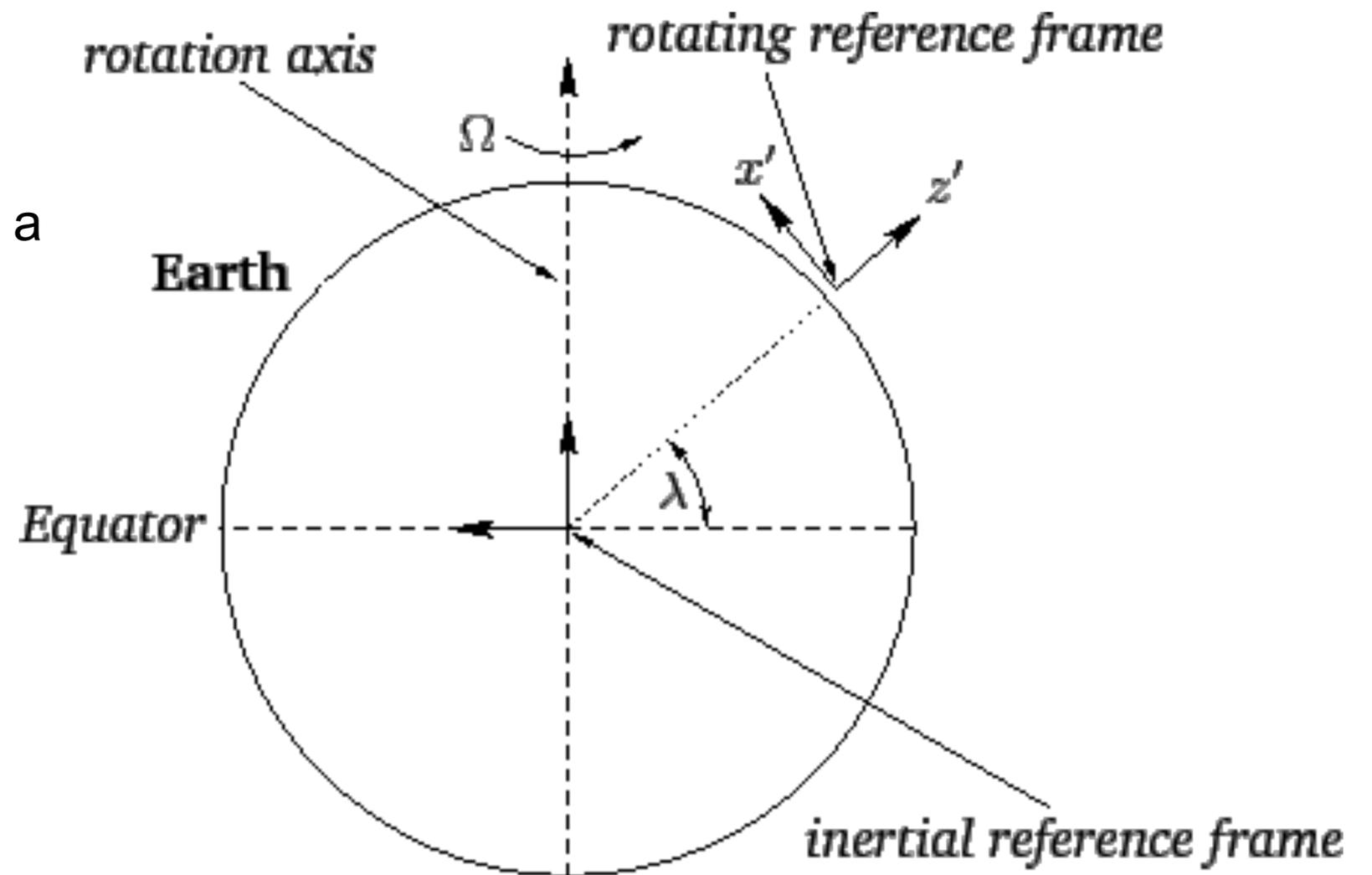
- One more transformation to spherical coordinates:

$R_p$ : planetary radius

$\phi$ : latitude

- We can get simpler versions of these equations after a bit of math, and making some simplifying assumptions:

- No longitudinal variation in wind speeds
- vertical wind  $\sim 0$
- focus on mid-latitudes
- frictionless atmosphere



# Scale Analysis

- $$\frac{du}{dt} = 2\Omega v \sin \phi - \frac{1}{\rho} \frac{\partial P}{\partial x}$$

- $$\frac{dv}{dt} = -2\Omega u \sin \phi - \frac{1}{\rho} \frac{\partial P}{\partial y}$$

- Define the Coriolis parameter,  $f = 2\Omega \sin \phi$

- $$\frac{du}{dt} = fv - \frac{1}{\rho} \frac{\partial P}{\partial x}$$

- $$\frac{dv}{dt} = -fu - \frac{1}{\rho} \frac{\partial P}{\partial y}$$



# Scale Analysis

- Can define characteristic scales based on observations.
- For example, for Earth at latitude of 45 degrees:
  - $U \approx 10m/s$ : Horizontal velocity scale (u or v)
  - $W \approx 1cm/s$ : Vertical velocity scale
  - $L \approx 10^6m$ : Length scale
  - $\Delta P/\rho \approx 10^3m^2/s^2$ : horizontal pressure fluctuation scale
  - $L/U \approx 10^5s$ : time scale



# Scale Analysis

- For disturbances at the characteristic (synoptic) scale of 1000 km:

- $\frac{du}{dt} \approx \frac{dv}{dt} \approx 0$

- $-fv \approx \frac{1}{\rho} \frac{\partial P}{\partial x}$

- $fu \approx \frac{1}{\rho} \frac{\partial P}{\partial y}$

- This is known as the geostrophic relationship
  - approximates the relationship between the pressure field and the horizontal wind field
  - implies that the wind speed is proportional to horizontal pressure gradient
  - As isobars come together, wind intensifies



# Rossby Number

- The Rossby number is the ratio between the characteristic horizontal wind speed ( $U$ ) and the Coriolis force:

- $$R_0 = \frac{U}{fL}$$

- In the geostrophic approximation,  $R_0 \ll 1$



# On Friction and Turbulence

- In the lab, energy is dissipated by transfer from laminar flow to molecules (heat): molecular viscosity
- In a turbulent atmosphere, have analogous situations on a larger scale: ordered flow  $\rightarrow$  turbulent eddies
- Big eddies  $\rightarrow$  small eddies  $\rightarrow$  molecular motion (chain of energy dissipation)



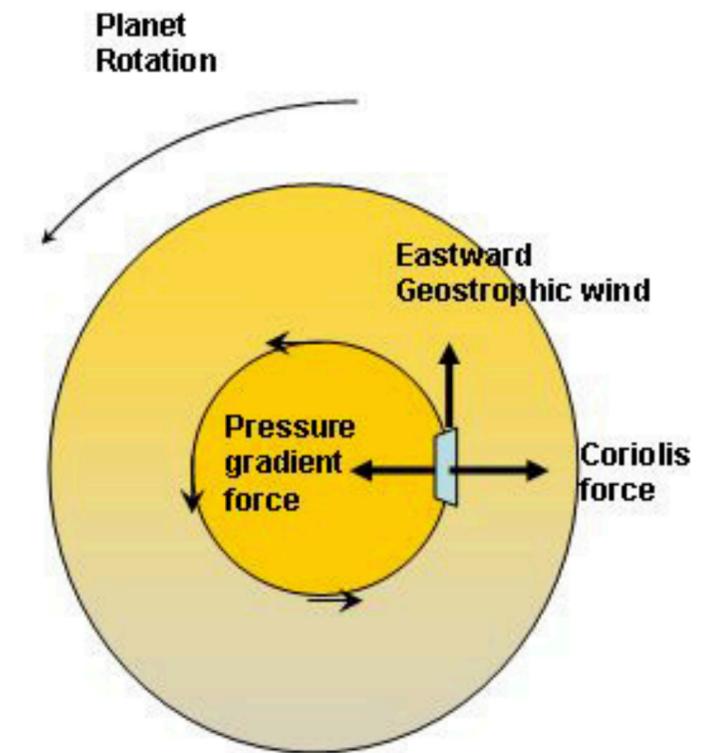
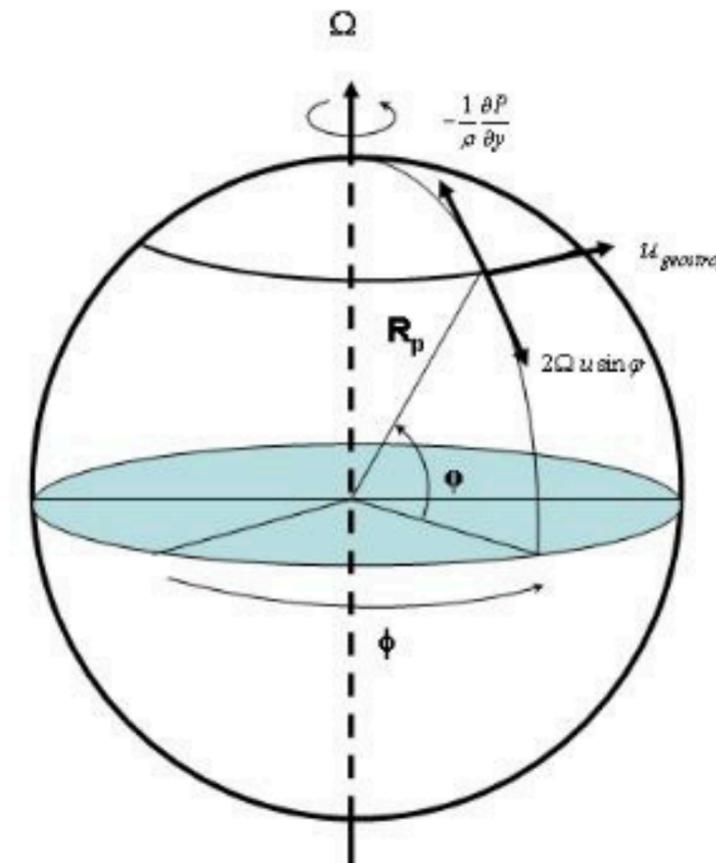
# On Friction and Turbulence

- Process of eddy cascade removes kinetic energy from winds and converts it to local heating (described by eddy viscosity)
- operates on length scale  $l$ , the “mixing length” (analogous to mean free path for molecular viscosity)
- eddy viscosity is most important at boundary layer near surface, where  $u$  approaches 0



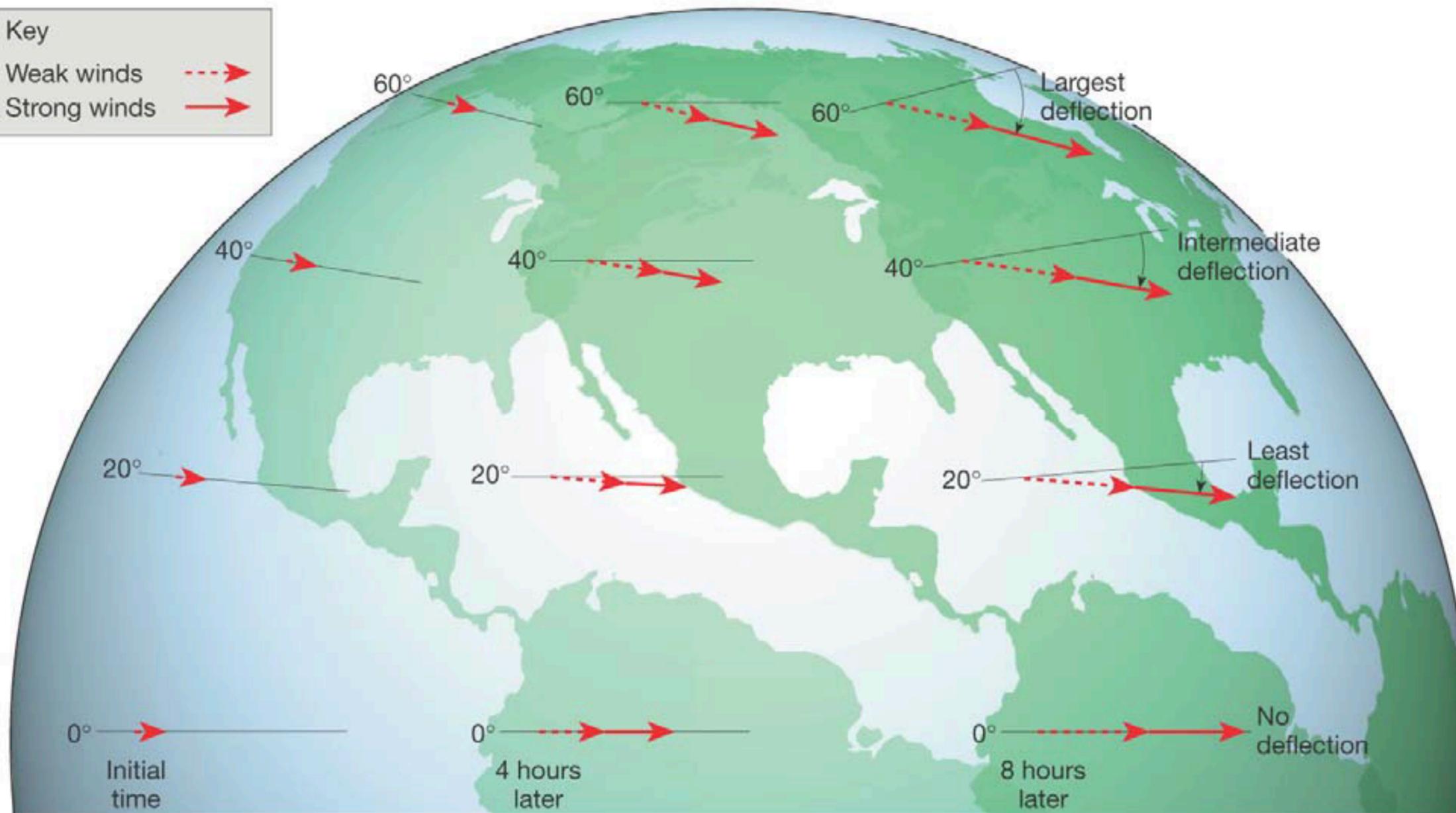
# Geostrophic Balance

- For rapidly rotating planets (like Earth), pressure gradient force and Coriolis force dominate momentum equation ( $R_0 \ll 1$ )
- (This breaks down as  $f$  goes to 0, like at equatorial latitudes)
  - Coriolis parameter,  $f = 2\Omega \sin \phi$



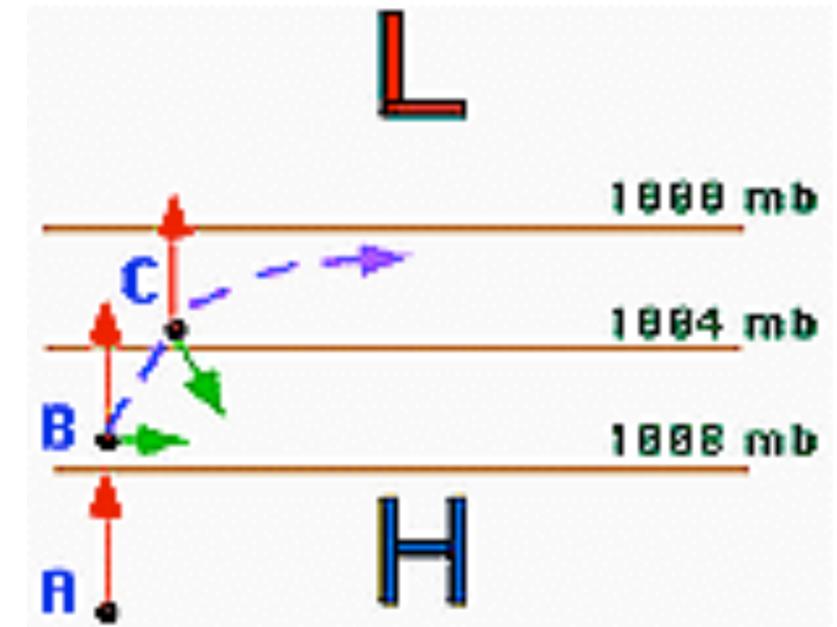
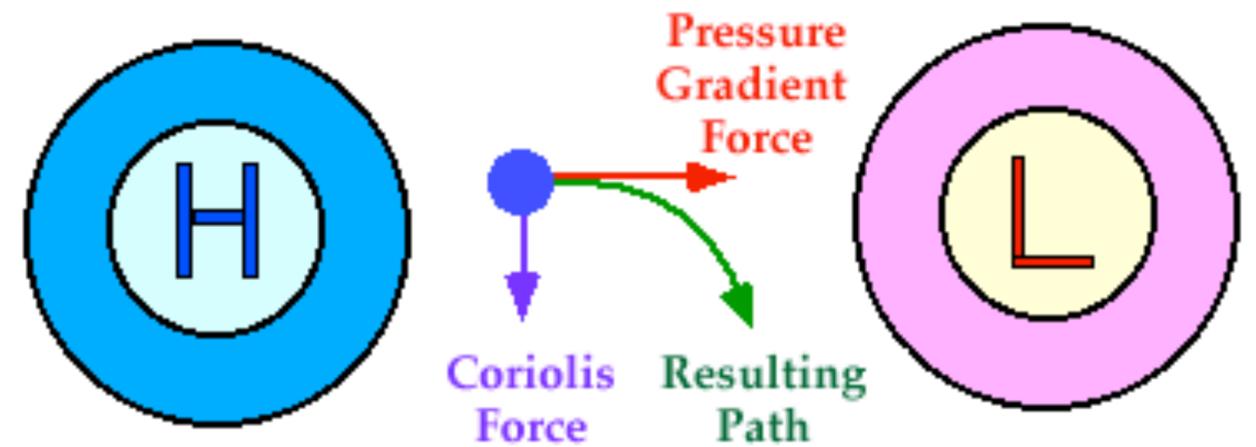
Key

Weak winds	
Strong winds	



# Geostrophic Wind

- Wind when there's a balance between Coriolis and pressure gradient forces
  - Air parcel at rest will move from high P to low P because of pressure gradient force
  - As air parcel moves, it is deflected by Coriolis force (to right in Northern hemisphere)
  - Deflection increases until Coriolis force equals pressure gradient force
  - Wind blows parallel to isobars (geostrophic)
  - Can figure out flow motion by looking at pressure distribution



# Example

- What is the magnitude of the zonal jet stream here in Las Cruces required given the observed horizontal pressure gradient?
- We know/measure:
- $\phi \approx 30^\circ$       Air density:  $\rho = 1\text{kg/m}^3$

Pressure decreases toward the North, pressure gradient is:

$$\frac{-10\text{mbar}}{500\text{km}} = \frac{-10^{-3}\text{Pa}}{5 \times 10^5\text{m}} = -2 \times 10^{-3}\text{Pa/m}$$

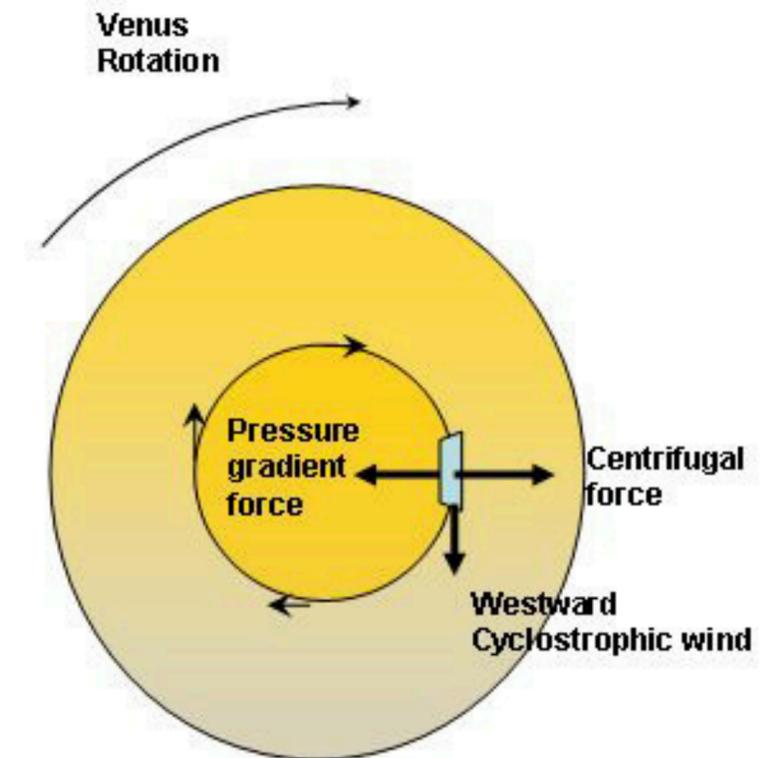
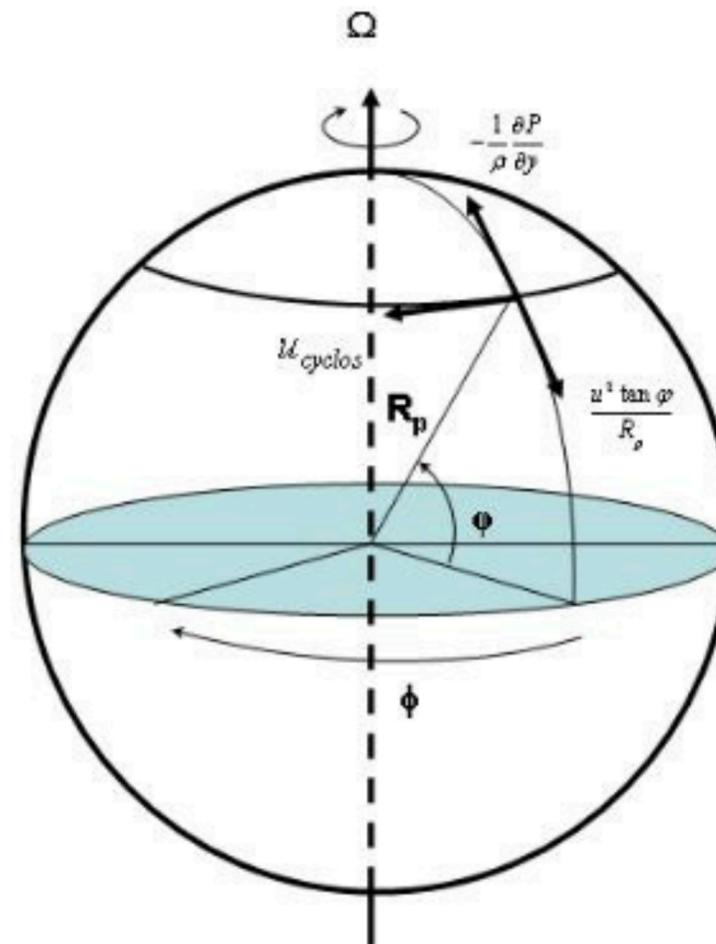
- $f(30^\circ) = 2\Omega \sin 30^\circ = 7.27 \times 10^{-5}$

- $f u = -\frac{1}{\rho} \frac{\partial P}{\partial y}$        $u = -\frac{1}{\rho f} \frac{\partial P}{\partial y} = \frac{2 \times 10^{-3}}{1 \times 7.27 \times 10^{-5}} = 27.5\text{m/s}$       About 55 mph

# Cyclostrophic Balance

- Slowly rotating bodies (like Venus) have small  $f$ , so  $R_0 > 1$ 
  - (also applies to equatorial latitude on faster-rotating bodies)
- In this case, can ignore Coriolis term in momentum equations
  - Dominant terms reflect a balance between pressure gradient force and centrifugal force

$$u = \sqrt{\frac{R_p}{\rho \tan \phi} \frac{\partial P}{\partial y}}$$



# Cyclostrophic Balance

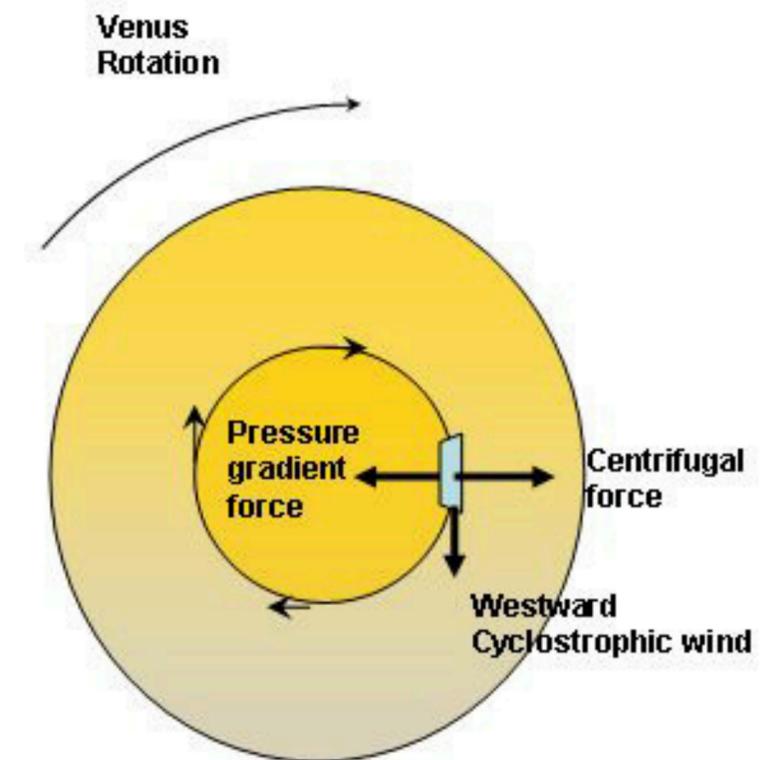
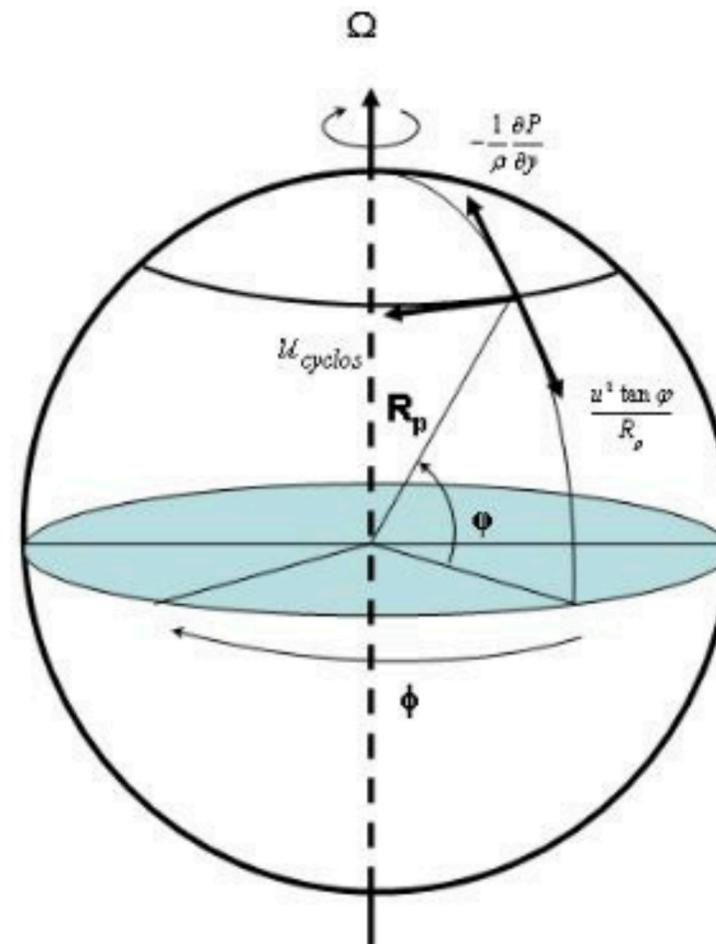
• What if  $\frac{\vec{v}^2}{r} \gg f\vec{u}$ ?

• For example, if we had large wind speeds and small radius of curvature

• Then the pressure gradient force balances the centrifugal force:

$$\frac{U^2}{R} = -\frac{1}{\rho} \frac{\partial P}{\partial y}$$

$$U = \left( \frac{R}{\rho} \frac{\partial P}{\partial y} \right)^{\frac{1}{2}}$$



# For next time

- Homework 4 due on Wednesday, October 19 at 11:59pm
- Reading: Planetary Science, 3.1.2, 4.5.1-4.5.2