

ASTR 620: Planetary Processes
Professor Eric Nielsen

Lecture 13: Atmospheres



Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Midterm in 2 days: Wednesday, October 5th (here in class)

No calculators are allowed on this exam. Instead, for problems that ask for a numerical answer, fill in all numbers at the end of your solution, and indicate the units the answer will have. For example:

Example: A right triangle has sides 9 km and 500 m. What is the length of the hypotenuse?

$$\text{Answer: } \sqrt{(9)^2 + \left(500 * \frac{1}{1000}\right)^2} \text{ km}$$

Finally, for each numerical problem (unless the instructions say otherwise), give the order-of-magnitude answer:

$$\begin{aligned} \text{Order of magnitude answer: } & \sqrt{(9)^2 + \left(500 * \frac{1}{1000}\right)^2} \text{ km} = \sqrt{81 + \frac{1}{4}} = \sqrt{100 + 0} \text{ km} \\ & = 10 \text{ km} \end{aligned}$$

Equations:

$$P^2 = \frac{a^3}{M} = \frac{4\pi^2 a^3}{GM}$$

$$Ap = a(1 + e)$$

$$Peri = a(1 - e)$$

$$v_c = \sqrt{\frac{GM}{R}}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$v = \sqrt{GM\left(\frac{1}{2r} - \frac{1}{a}\right)}$$

$$F_t = \frac{2GMmr}{d^3}$$

Review of the last class

- Put these layers of Earth's atmosphere in order, from lowest (closest to ground) to highest
 - (A) — (lowest) troposphere, mesosphere, stratosphere, thermosphere, exosphere (highest)
 - (B) — (lowest) troposphere, mesosphere, stratosphere, exosphere, thermosphere (highest)
 - (C) — (lowest) troposphere, stratosphere, mesosphere, thermosphere, exosphere (highest)
 - (D) — (lowest) troposphere, stratosphere, mesosphere, exosphere, thermosphere (highest)

Review of the last class

- The layers of Earth's atmosphere where temperature increases as we go up in height are:
 - (A) — troposphere, stratosphere, mesosphere, thermosphere
 - (B) — troposphere, stratosphere, mesosphere
 - (C) — troposphere, mesosphere
 - (D) — stratosphere, thermosphere
 - (E) — troposphere, thermosphere

Review of the last class

- The ozone layer is the main heating source in the:
 - (A) — troposphere
 - (B) — stratosphere
 - (C) — mesosphere
 - (D) — thermosphere
 - (E) — exosphere

Review of the last class

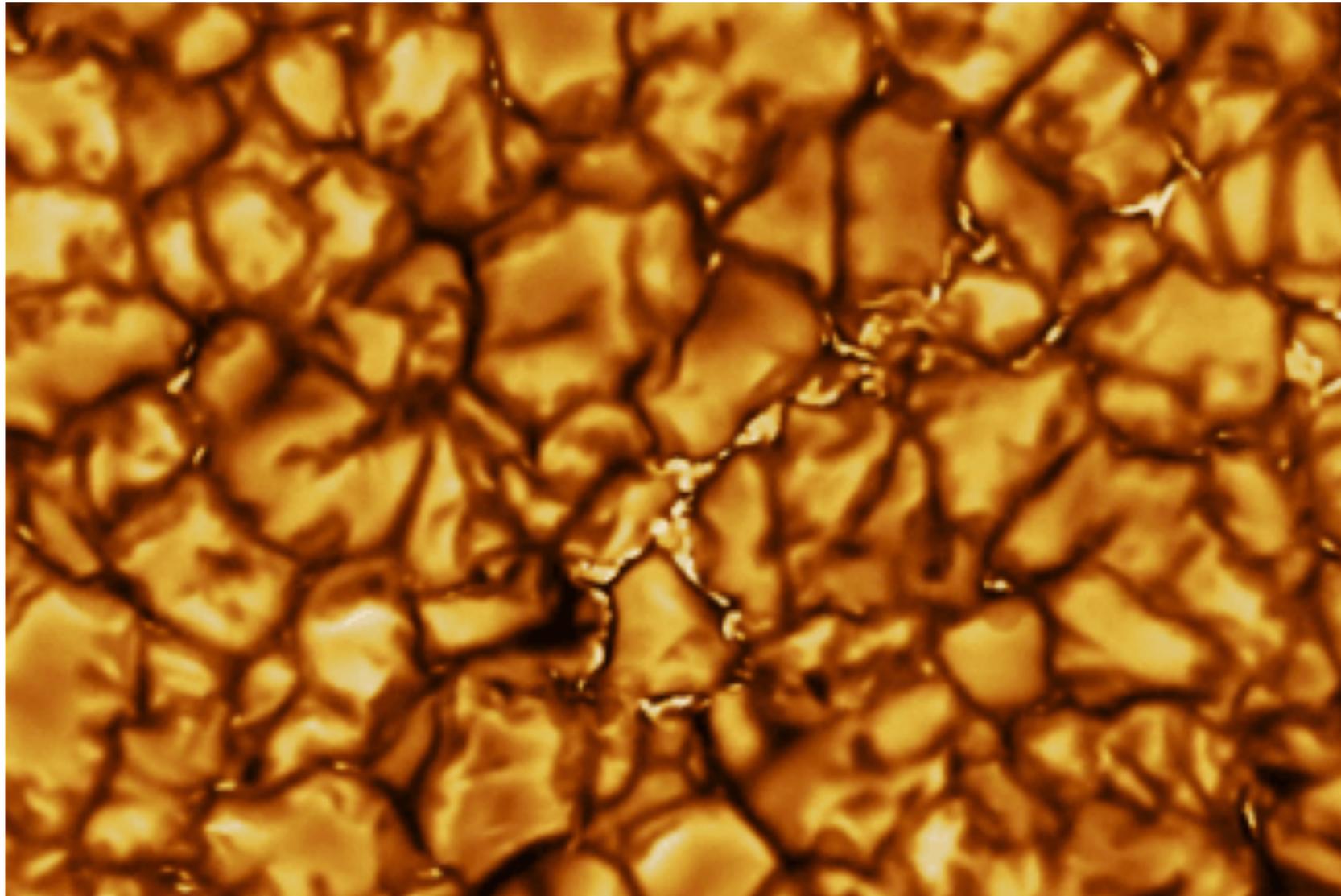
- If I go up 3 scale heights in Earth's atmosphere, the atmospheric pressure:
 - (A) — Drops by e^3
 - (B) — Drops by $3e$
 - (C) — Stays the same
 - (D) — Increases by $3e$
 - (E) — Increases by e^3

Review of the last class

- The scale height of Earth's atmosphere (to order of magnitude) is:
 - (A) — 100 km
 - (B) — 10 km
 - (C) — 1 km
 - (D) — 100 m
 - (E) — 10 m

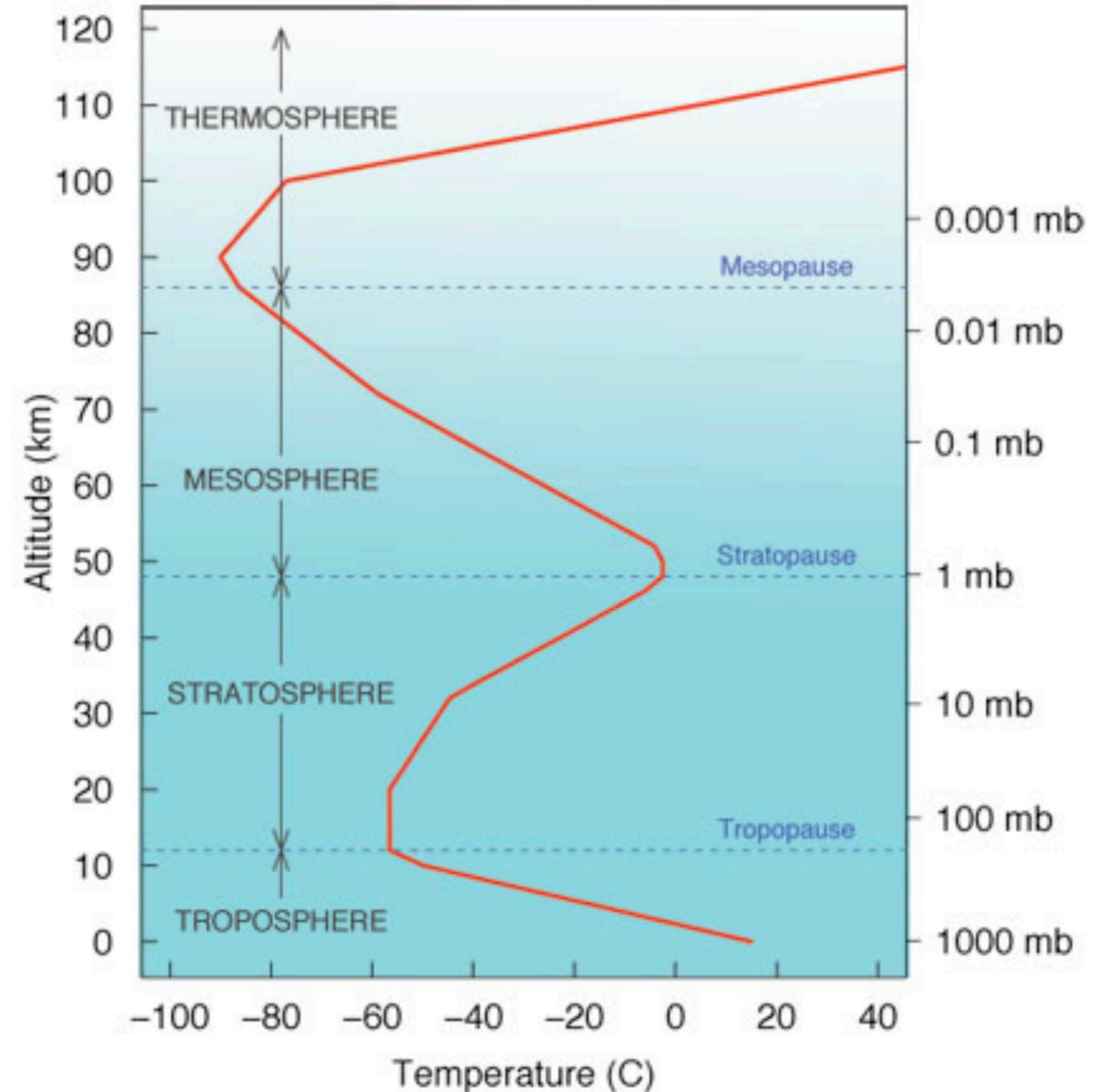
Convection

- Transports heat from lower layers to upper layers
- If the temperature gradient is sufficient: air parcel low down heats up, becomes less dense, moves upward
 - Opposite happens to an air parcel higher up
- Transports material vertically



Air Parcels

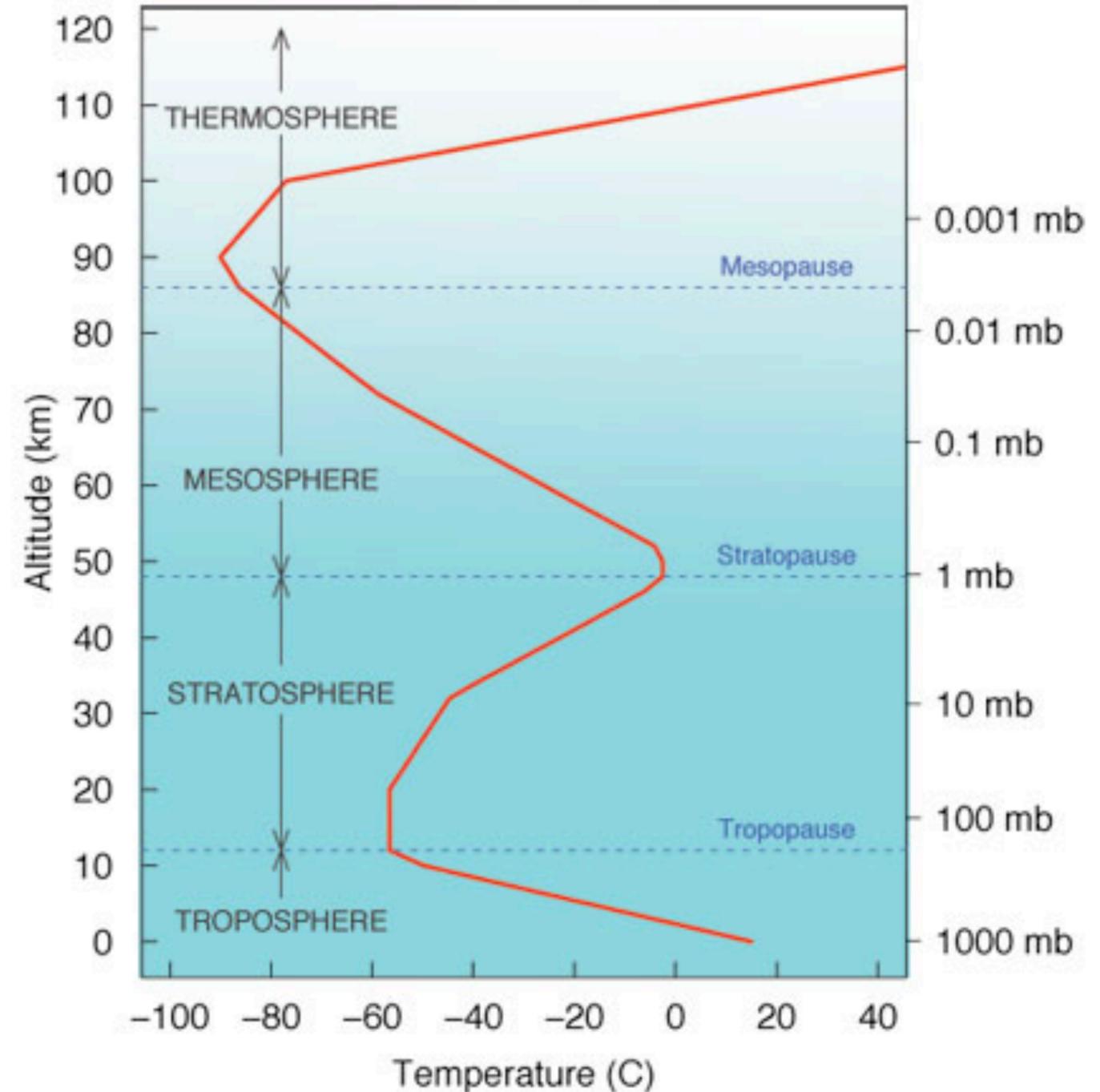
- Air parcels are macro scale: few mm to planetary scale
- Air parcel is assumed to:
 - Be thermally insulated from its environment, Temperature changes adiabatically as it rises or sinks
 - Remain at same pressure as environment air at that level (environment assumed to be in hydrostatic equilibrium)
 - Moving slowly enough that macroscopic KE of a particle inside the parcel is a negligible fraction of its total energy



Air Parcels

- If an air parcel rises with no heat being added (latent heat, radiation) or subtracted (radiation), it will expand and cool
- If we give a parcel of gas a small quantity of heat (energy) dq , it will change temperature, from T to $T+dt$

• We define the specific heat as $\frac{dq}{dT}$

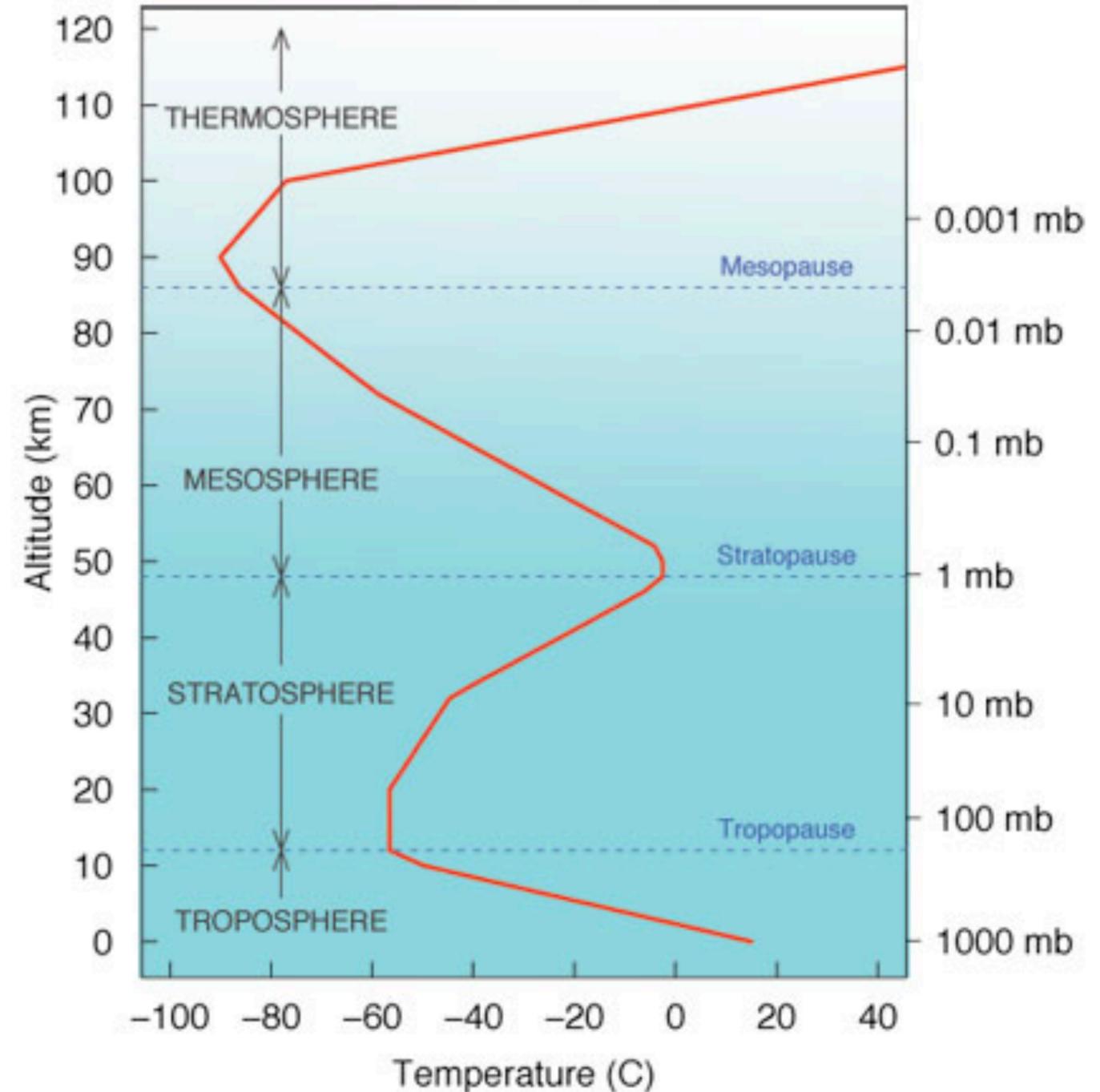


Air Parcels

- Specific heat: $\frac{dq}{dT}$, depends on whether pressure and/or volume changes over the heating
- We define two flavors of the specific heat:
- The specific heat if volume is kept constant during the heating:
- The specific heat if pressure is kept constant during the heating:

$$C_V = \left(\frac{dq}{dT} \right)_{\text{constant } V}$$

$$C_P = \left(\frac{dq}{dT} \right)_{\text{constant } P}$$



Air Parcels

- First law of thermodynamics for an ideal gas:

$$dq = C_V dT + PdV$$

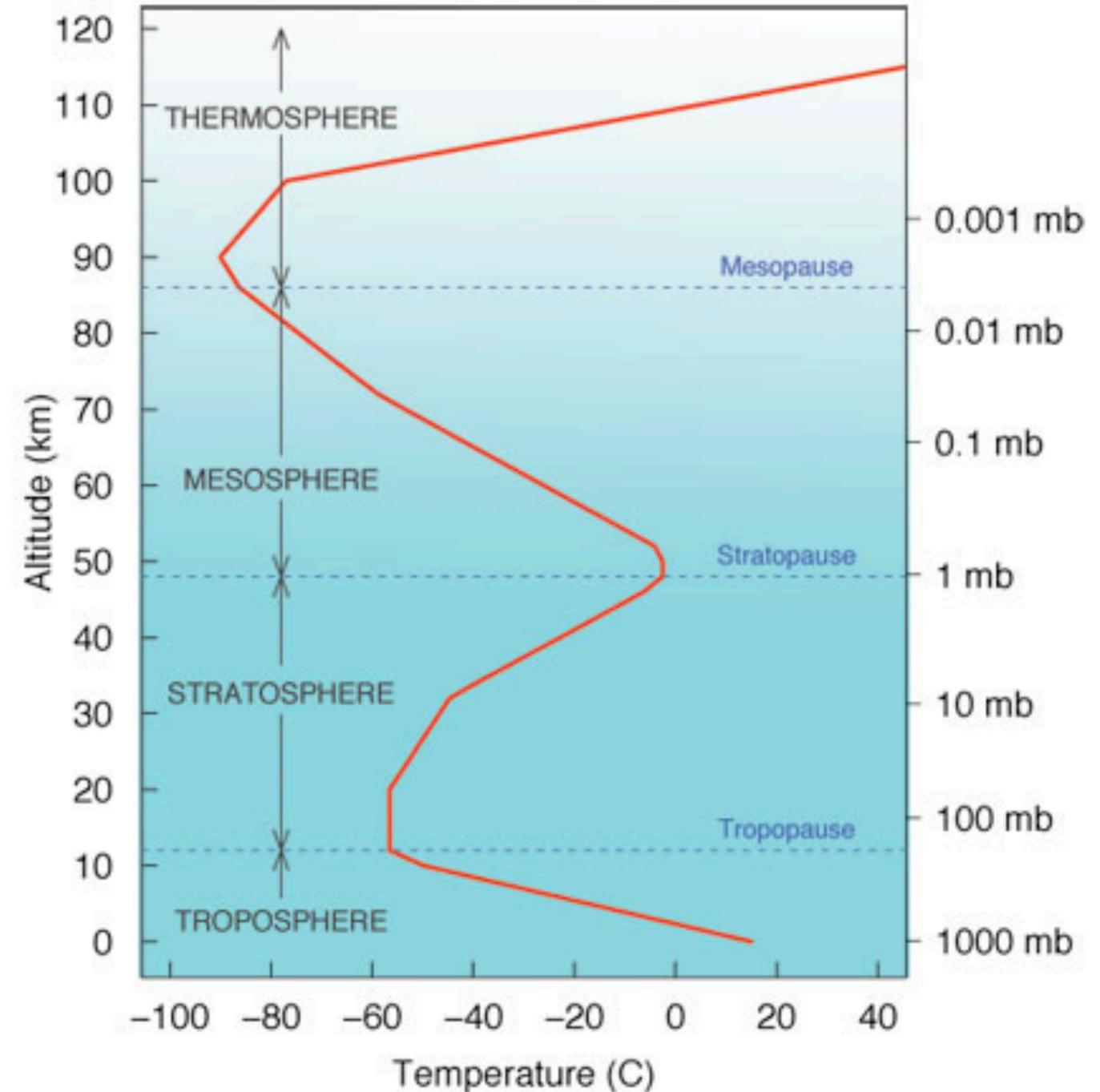
- Since we're assuming the process is adiabatic (no energy enters or leaves the parcel), $dq = 0$:

$$C_V dT = -PdV$$

$$C_V = -P \frac{dV}{dT}$$

- For an ideal gas, the difference between these two specific heats is the universal gas constant R :

$$R = C_P - C_V$$



Air Parcels

- The ideal gas law relates these quantities:

$PV = RT$ (assuming V is the specific volume, containing one gram of molecules)

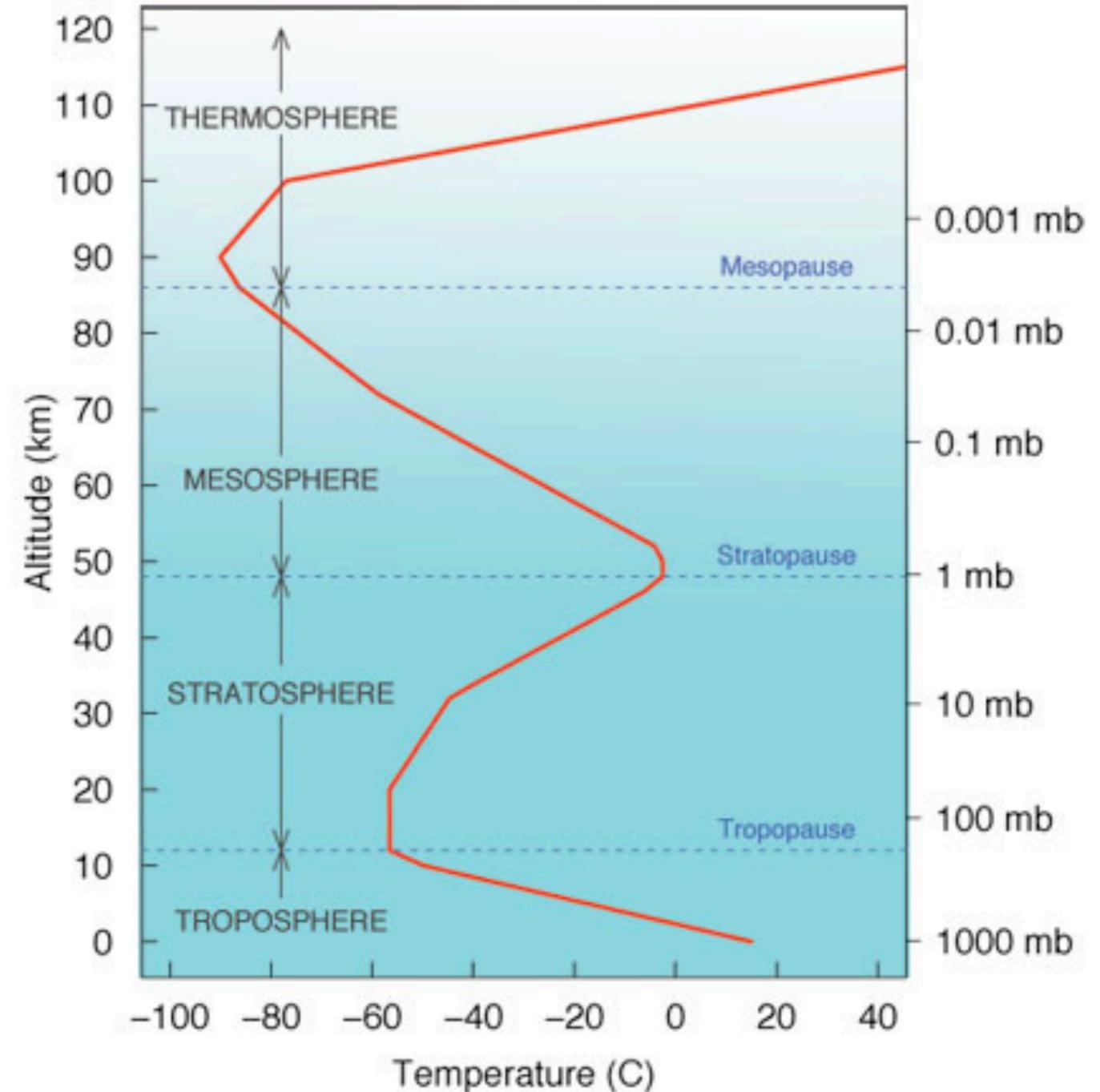
$$V = \frac{1}{\rho} = \frac{RT}{P}$$

- If we differentiate, and assume the number of particles (N) in the parcel is constant:

$$dV = \frac{RdT}{P} - \frac{RTdP}{P^2}$$

- Plugging into $dq = C_V dT + PdV$:

$$dq = C_V dT + RdT - \frac{RT}{P} dP$$



Air Parcels

$$\bullet \quad dq = C_V dT + R dT - \frac{RT}{P} dP$$

$$dq = (C_V + R) dT - \frac{RT}{P} dP$$

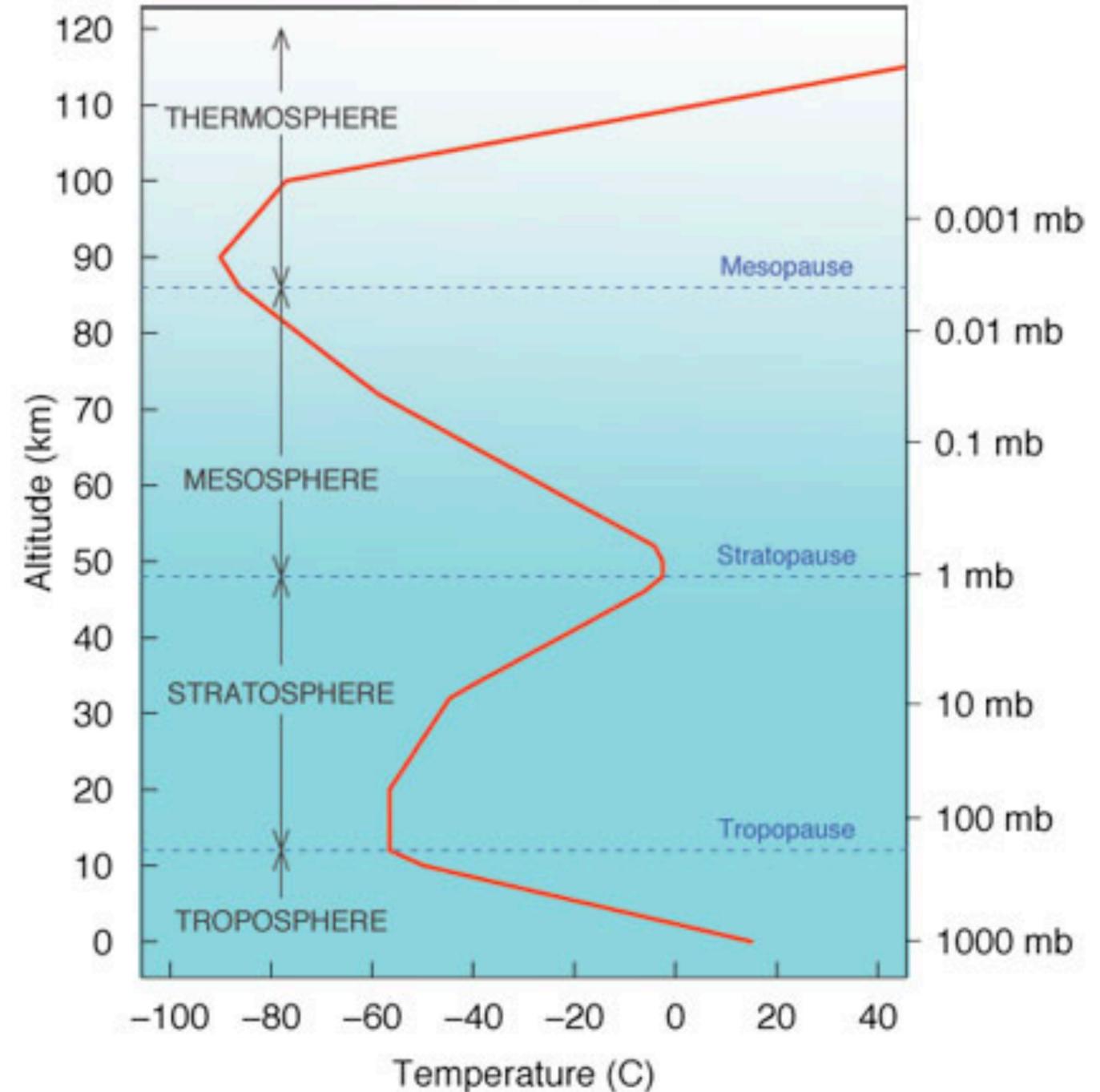
$$\bullet \quad C_P - C_V = R \quad \text{so} \quad C_V + R = C_P$$

And we assume adiabatic, so $dq=0$:

$$C_P dT = \frac{RT}{P} dP$$

$$\bullet \quad \text{Going back to the ideal gas law, } V = \frac{1}{\rho} = \frac{RT}{P}:$$

$$C_P dT = V dP = \frac{dP}{\rho}$$



Air Parcels

- $C_p dT = V dP = \frac{dP}{\rho}$

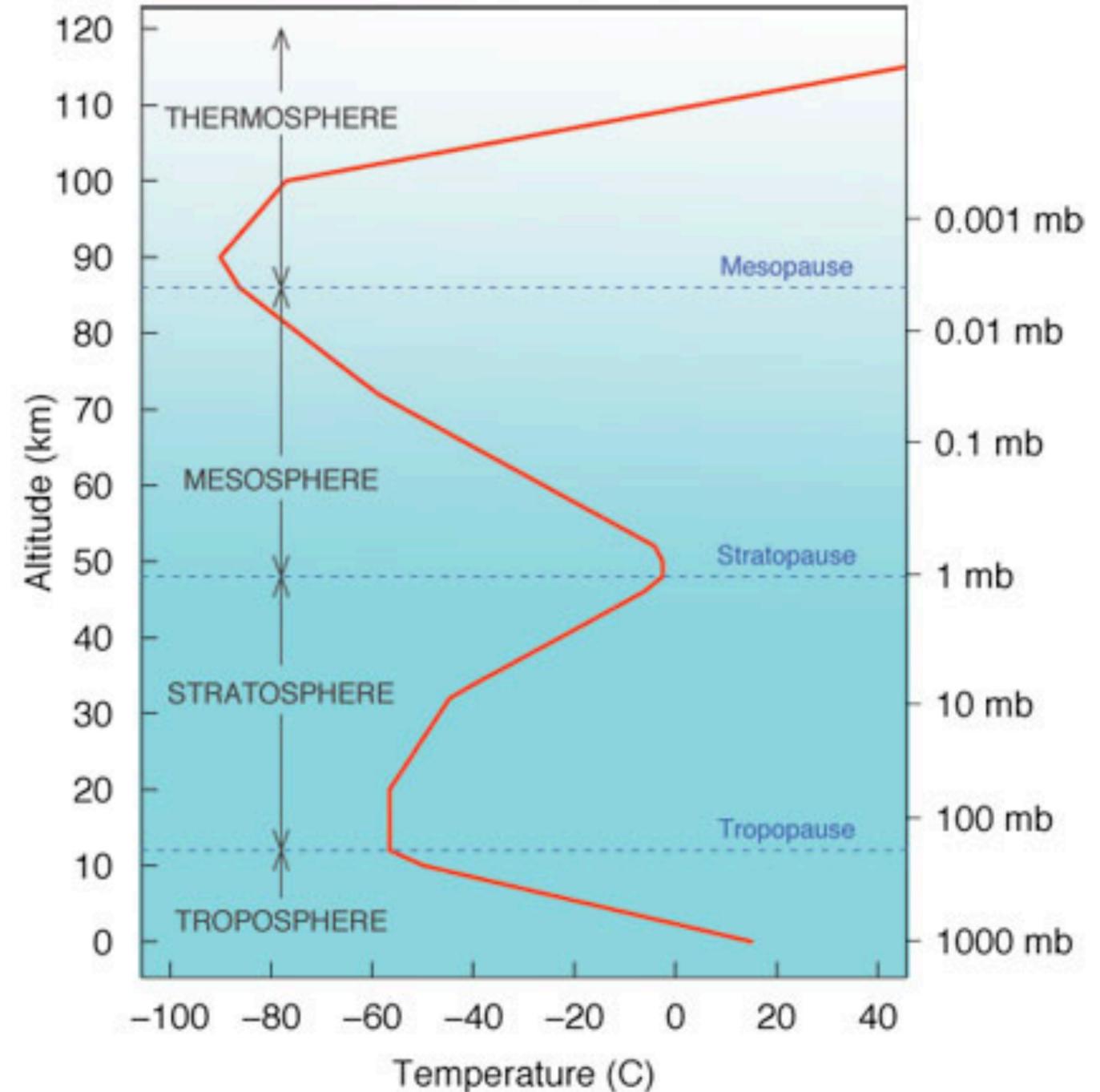
- From hydrostatic equilibrium, we have an expression that relates pressure gradient and density:

$$\frac{dP}{dz} = -\rho g$$

- $\frac{dP}{\rho} = -g dz = C_p dT$

- This gives us the dry adiabatic lapse rate:

$$\Gamma_d = \frac{dT}{dz} = -\frac{g}{C_p}$$



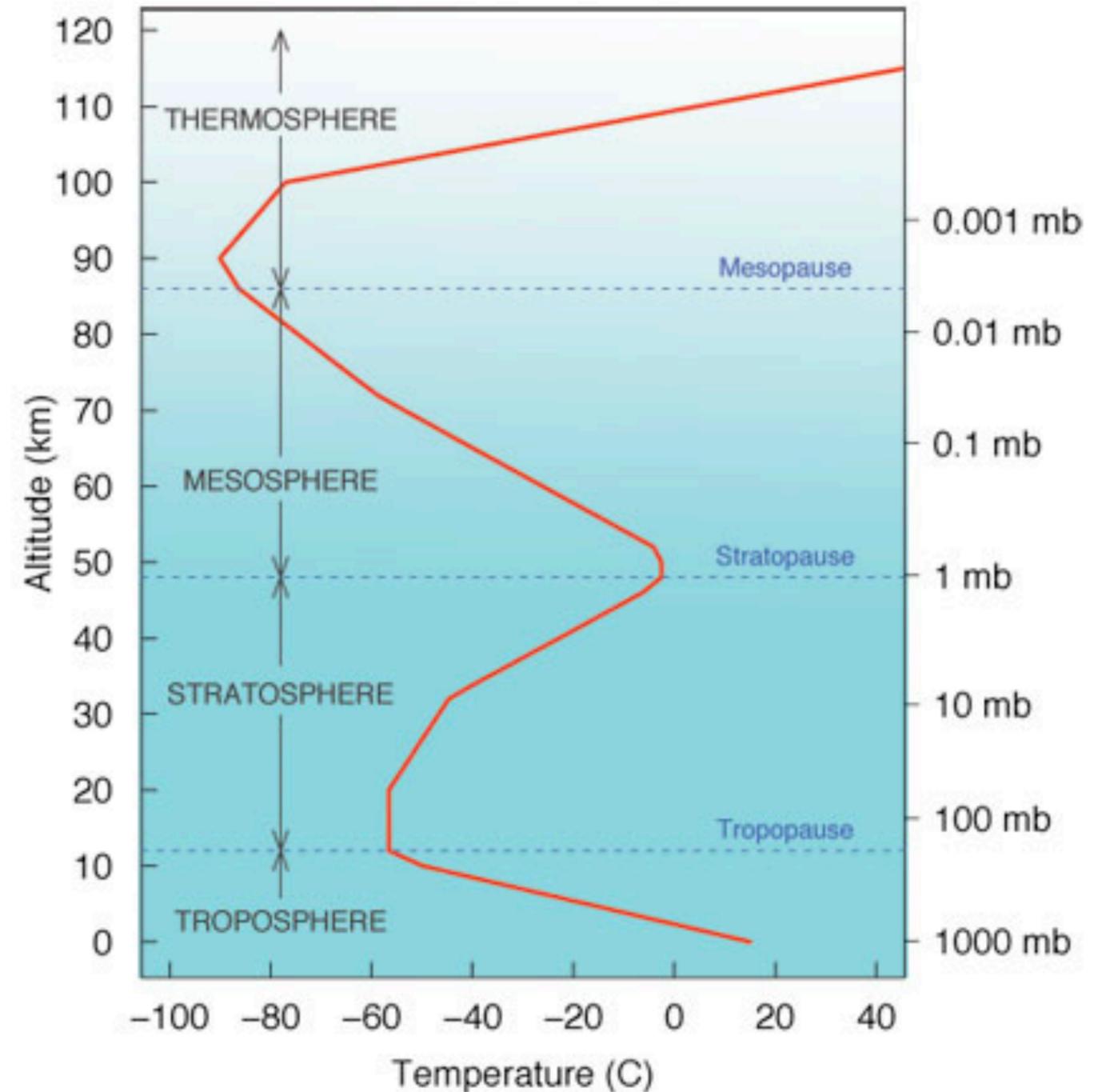
Dry Adiabatic Lapse Rate

- $$\Gamma_d = \frac{dT}{dz} = -\frac{g}{C_P}$$

- Rewriting in terms of γ , the ratio of $\frac{C_P}{C_V}$:

- $$\Gamma_d = \frac{dT}{dz} = -\frac{\gamma - 1}{\gamma} g \frac{\mu}{k}$$

- How temperature changes as a function of height depends on local gravity, and gas properties (mean molecular weight μ and γ)



Dry Adiabatic Lapse Rate

- Typical values of γ :

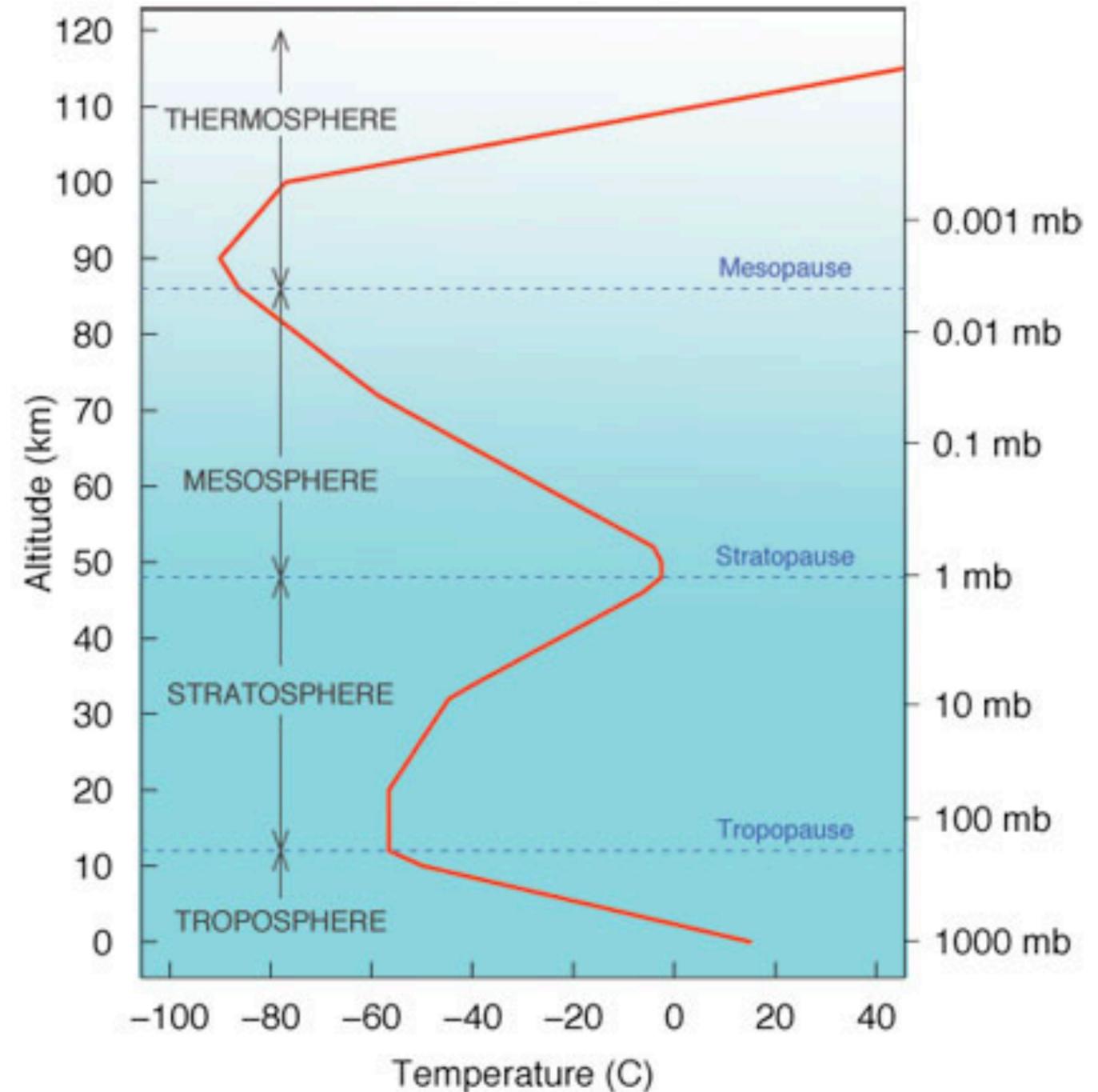
- Monatomic gas: $\frac{5}{3}$

- Diatomic gas: $\frac{7}{5}$

- Polyatomic gas: $\frac{4}{3}$

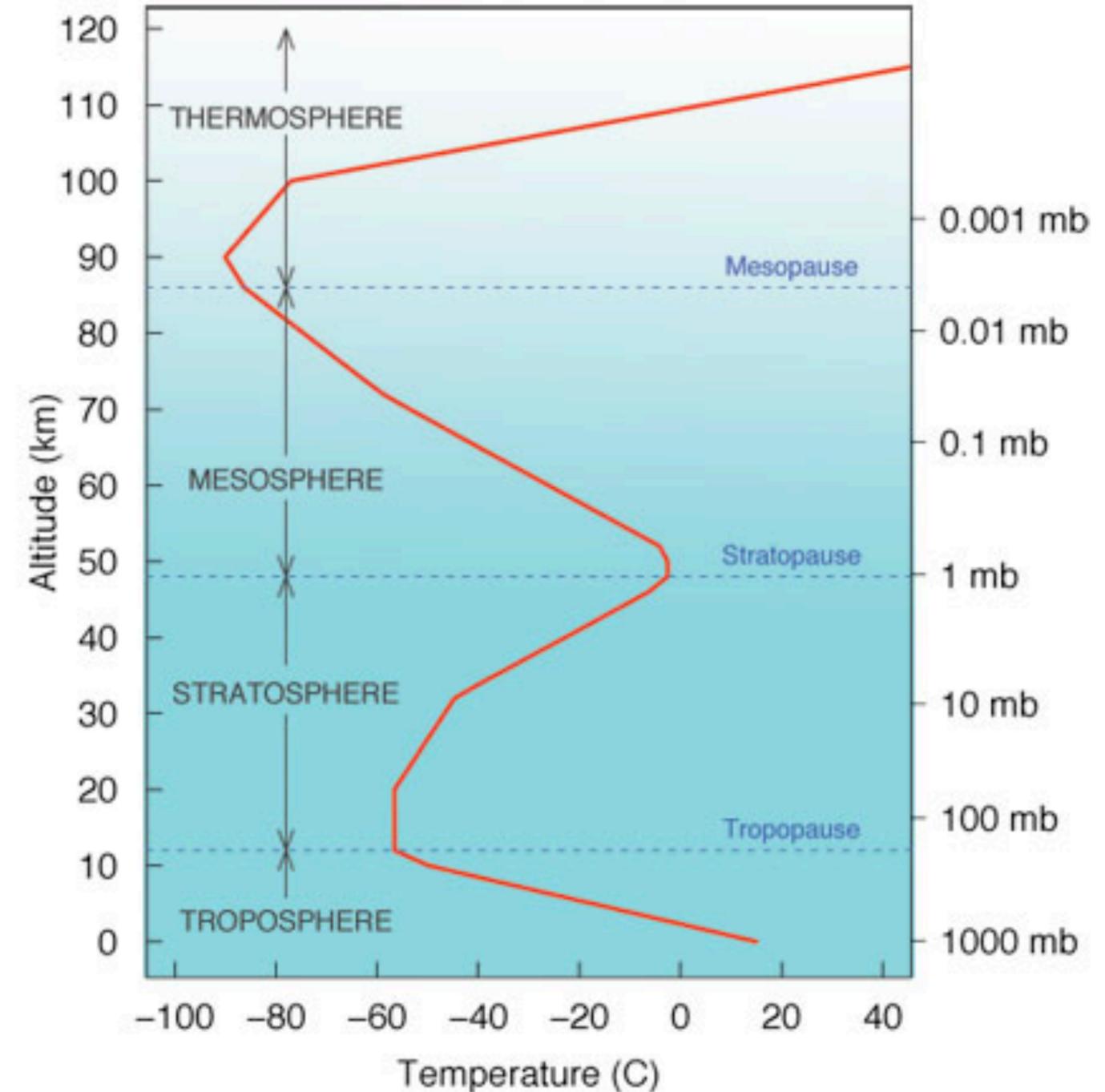
- For dry air on Earth:

- $\Gamma_d = -9.8 \text{ K/km}$



Dry Adiabatic Lapse Rate

	Earth	Venus	Mars
g (m/s ²)	9.8	8.8	3.7
C_p (J/kg/K)	1004	930	735
dT/dz (K/km)	-9.8	-9	-5



(Non-Dry) Lapse rates

- We didn't include the possibility of liquids condensing or releasing latent heat
- With water vapor in the picture, the released latent heat offsets some of the dry adiabatic cooling
 - This results in LESS of a decrease in temperature with height



Moist Lapse rates

- From our derivation of the dry adiabatic lapse rate:

$$\frac{dP}{\rho} = -gdz = C_p dT$$

- We need one more term if we consider vaporization of liquids:

$$C_p dT + gdz = -Lw_s$$

- L: Latent heat of vaporization (J/kg)
- w_s : “mixing ratio” — mass of saturated water per mass of air



Moist Lapse rates

- $C_p dT + g dz = - L w_s$

$$-\left(\frac{dT}{dz}\right)_{\text{moist adiabatic}} = \left[\frac{\frac{g}{C_p}}{1 + \left(\frac{L}{C_p}\right) \left(\frac{dw_s}{dT}\right)} \right]$$

- $\frac{dw_s}{dT}$: Change in condensible mixing ratio (mass of condensible gas to mass of air) as air cools and the condensible condenses
- C_p : specific heat for the atmosphere as a whole, not just condensing gas



Moist Lapse rates

- $C_p dT + g dz = -L w_s$

$$-\left(\frac{dT}{dz}\right)_{\text{moist adiabatic}} = \left[\frac{\frac{g}{C_p}}{1 + \left(\frac{L}{C_p}\right) \left(\frac{dw_s}{dT}\right)} \right]$$

- Since energy goes into condensation process, moist/wet lapse rate is LESS STEEP.
- Average value for Earth's troposphere is -6.5 K/km



Order of Magnitude: Venus Resort

- Venus' atmosphere:
 - Scale height: 11 km
 - Moist Adiabatic Lapse rate: 8 K/km
 - Surface conditions: 100 bars, 700K
- I want to build a floating resort on Venus, and, if possible, I'd prefer not to kill my guests (I'll assume they brought their own scuba gear to breath from)
- (1) How high above Venus' surface should I build my resort to have a pressure equal to sea level on Earth?
- (2) At this height above Venus's surface, what is the temperature?
- (3) What should the name of my resort be? (Puns encouraged)
- Hint: $\ln(10) = 2,3$



Order of Magnitude: Venus Resort

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- I want to build a floating resort on Venus, and, if possible, I'd prefer not to kill my guests (I'll assume they brought their own scuba gear to breath from)
- (1) How high above Venus' surface should I build my resort to have a pressure equal to sea level on Earth?
- In hydrostatic equilibrium, pressure goes as $-\exp(z/\text{scale_height})$. To get the pressure to decrease by a factor of 10, I need to go up 2.3 scale heights. 4.6 scale heights to get a factor of 100, which is what we need to not kill our guests.
- $h = (4.6)(11\text{km}) = 50\text{km}$

Order of Magnitude: Venus Resort

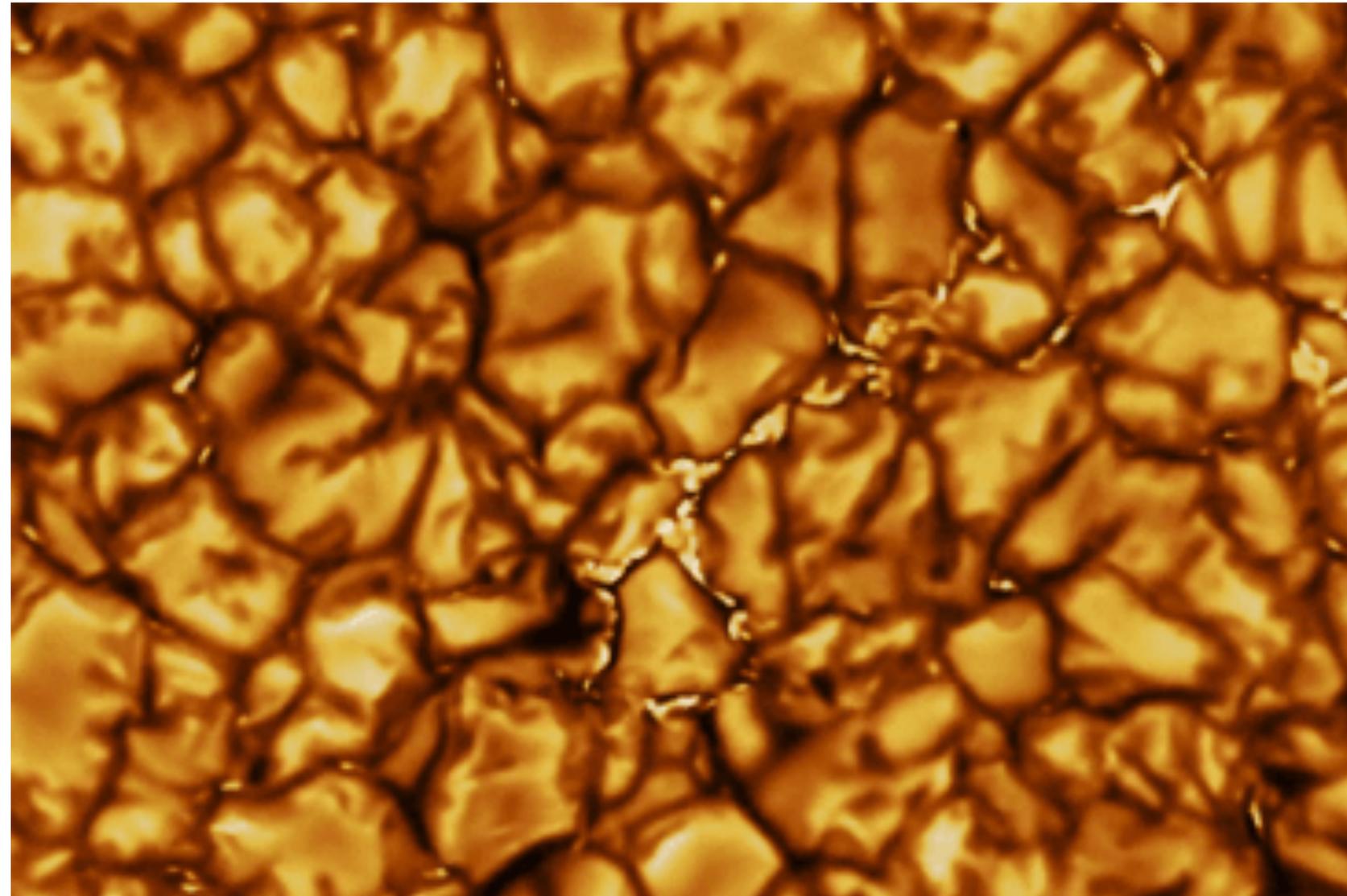
- (2) At this height above Venus's surface, what is the temperature?
- 50 km height, adiabatic lapse rate of 8 K/km, so just need to multiply:
- $\Delta T = 50km * 8 \frac{K}{km} = 400K$
- Given a surface temperature of 700 K, this means a comfortable 300 K temperature.
- (3) What should the name of my resort be? (Puns encouraged)
- The 31-mile-high city

Break

05:00

Convection

- Potential temperature (θ):
 - For an air parcel, the temperature the parcel of air would have if it were expanded (or compressed) adiabatically from its existing temperature+pressure to a pressure of 1 bar
 - (Air at sea level on Earth has a pressure of 1 bar)



Convection

- First law of thermodynamics (and assume adiabatic, so no heat gained or lost):

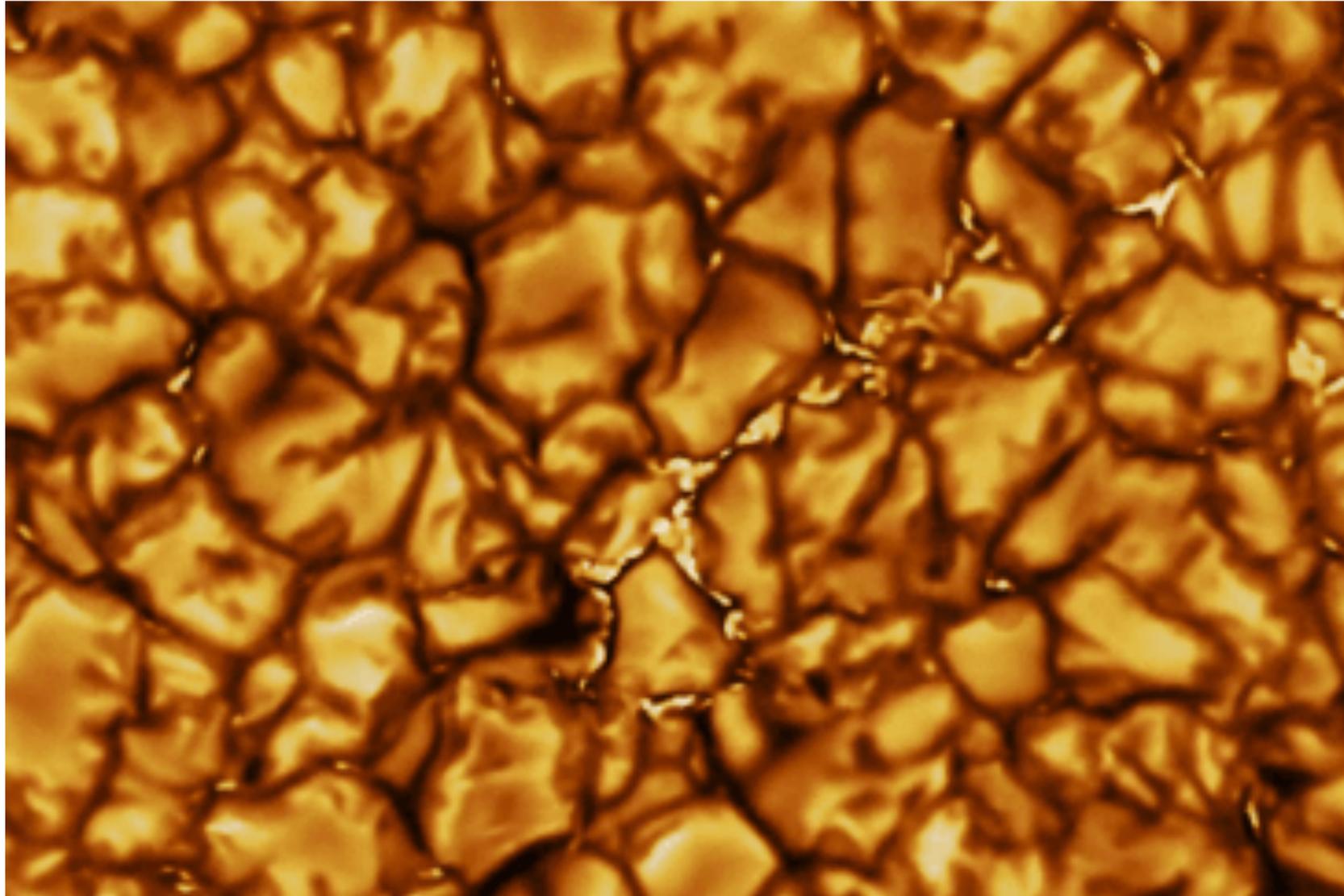
$$dq = C_V dT + PdV = 0$$

- And, from before:

$$C_P dT - \frac{1}{\rho} dP = 0 \qquad \frac{P}{\rho} = RT$$

- We get:

$$\frac{C_P}{R} \frac{dT}{T} = \frac{dP}{P}$$



Convection

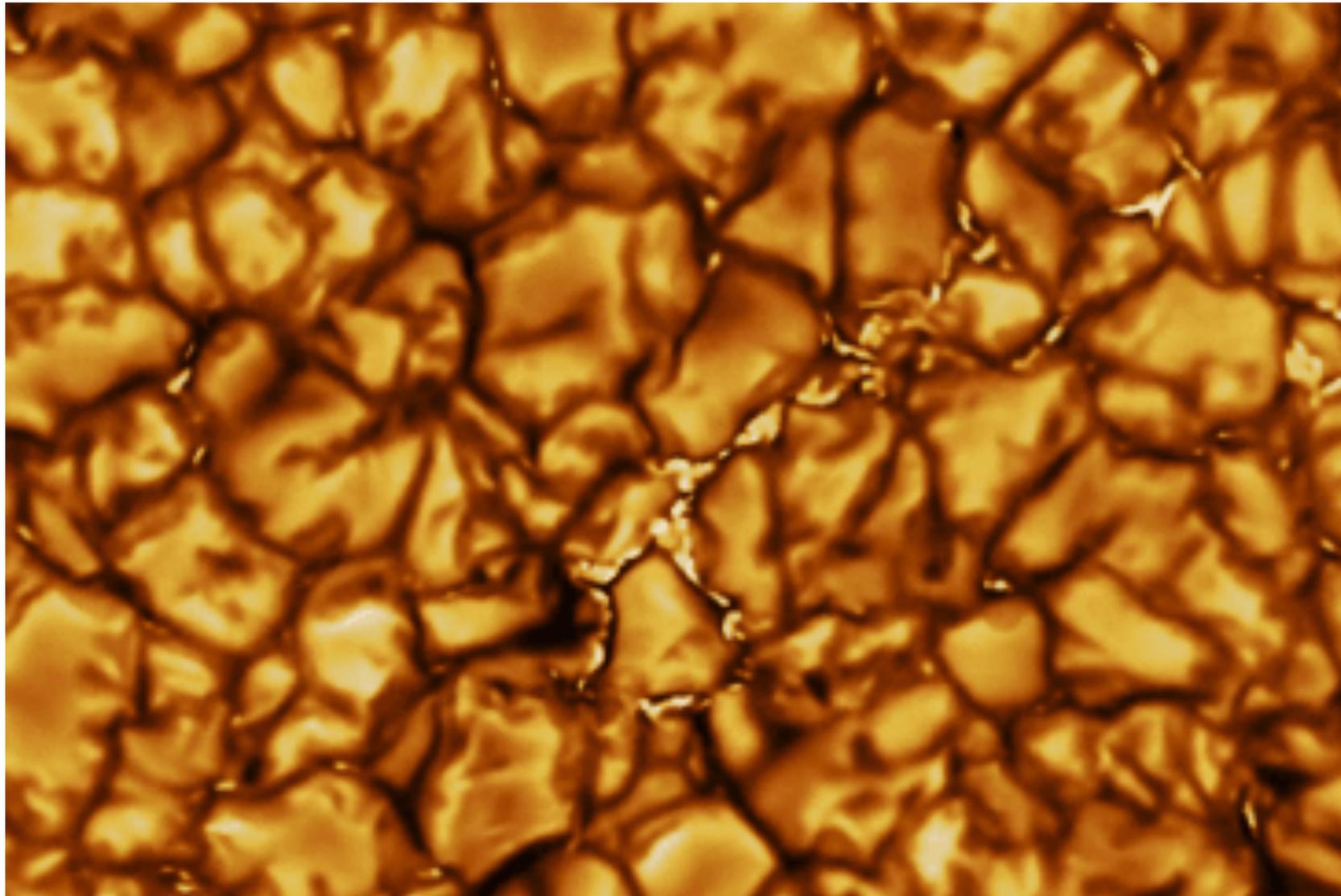
- $\frac{C_P dT}{R T} = \frac{dP}{P}$

- Integrate both sides:

$$\frac{C_P}{R} \int_{T_0}^T \frac{dT}{T} = \int_{P_0}^P \frac{dP}{P}$$

- $\frac{C_P}{R} (\ln(T) - \ln(T_0)) = \ln(P) - \ln(P_0)$

$$\frac{C_P}{R} \ln \left(\frac{T}{T_0} \right) = \ln \left(\frac{P}{P_0} \right)$$



Convection

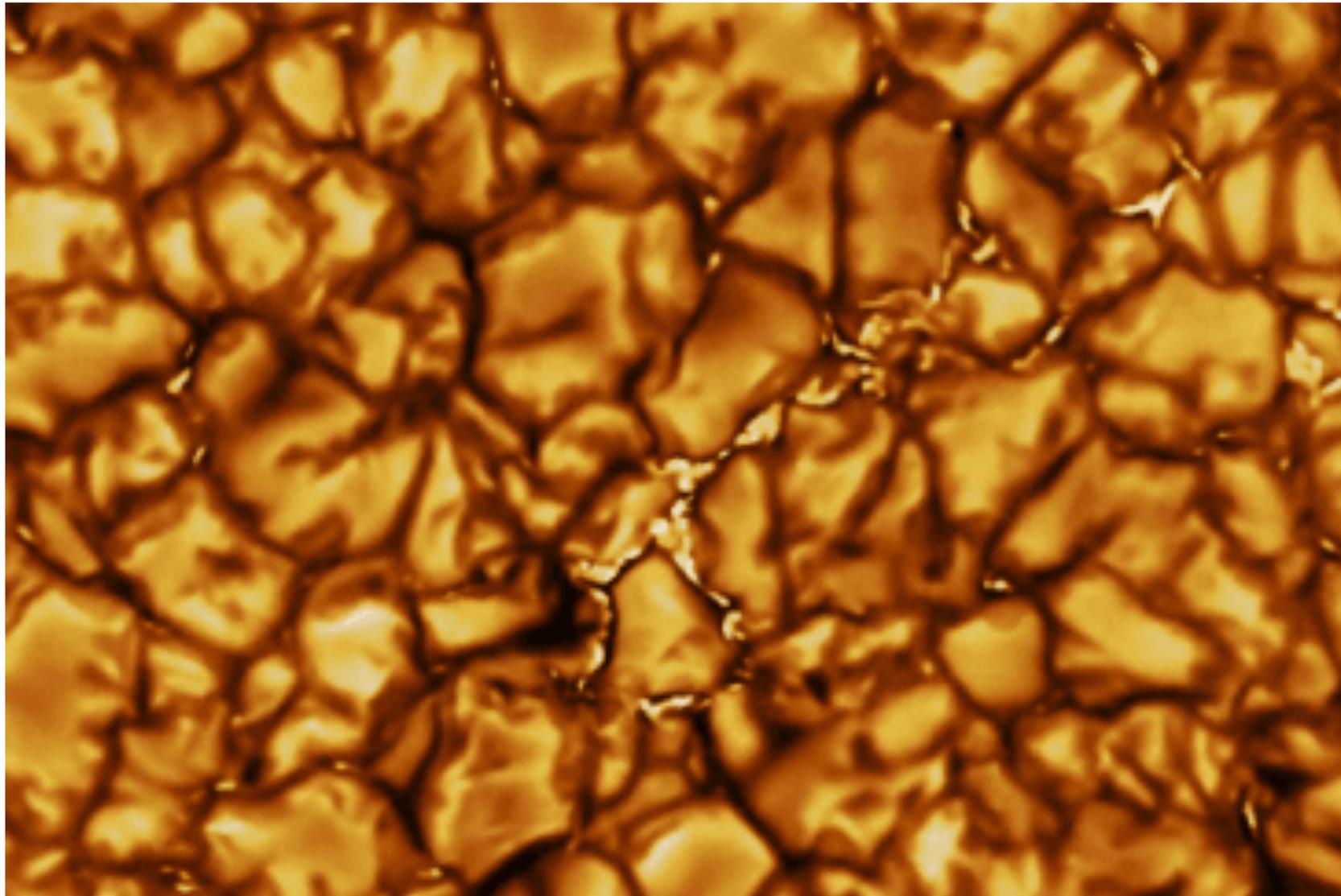
- $$\frac{C_P}{R} \ln \left(\frac{T}{T_0} \right) = \ln \left(\frac{P}{P_0} \right)$$

- $$T = T_0 \left(\frac{P}{P_0} \right)^{\frac{R}{C_P}}$$

- Or, if we can write in terms of the potential temperature θ (temperature a parcel of gas would have if moved adiabatically from its current temperature/pressure to a pressure of 1 bar):

- $$T_0 = \theta \quad P_0 = 1 \text{ bar}$$

- $$\theta = T \left(\frac{P_0}{P} \right)^{\frac{R}{C_P}}$$

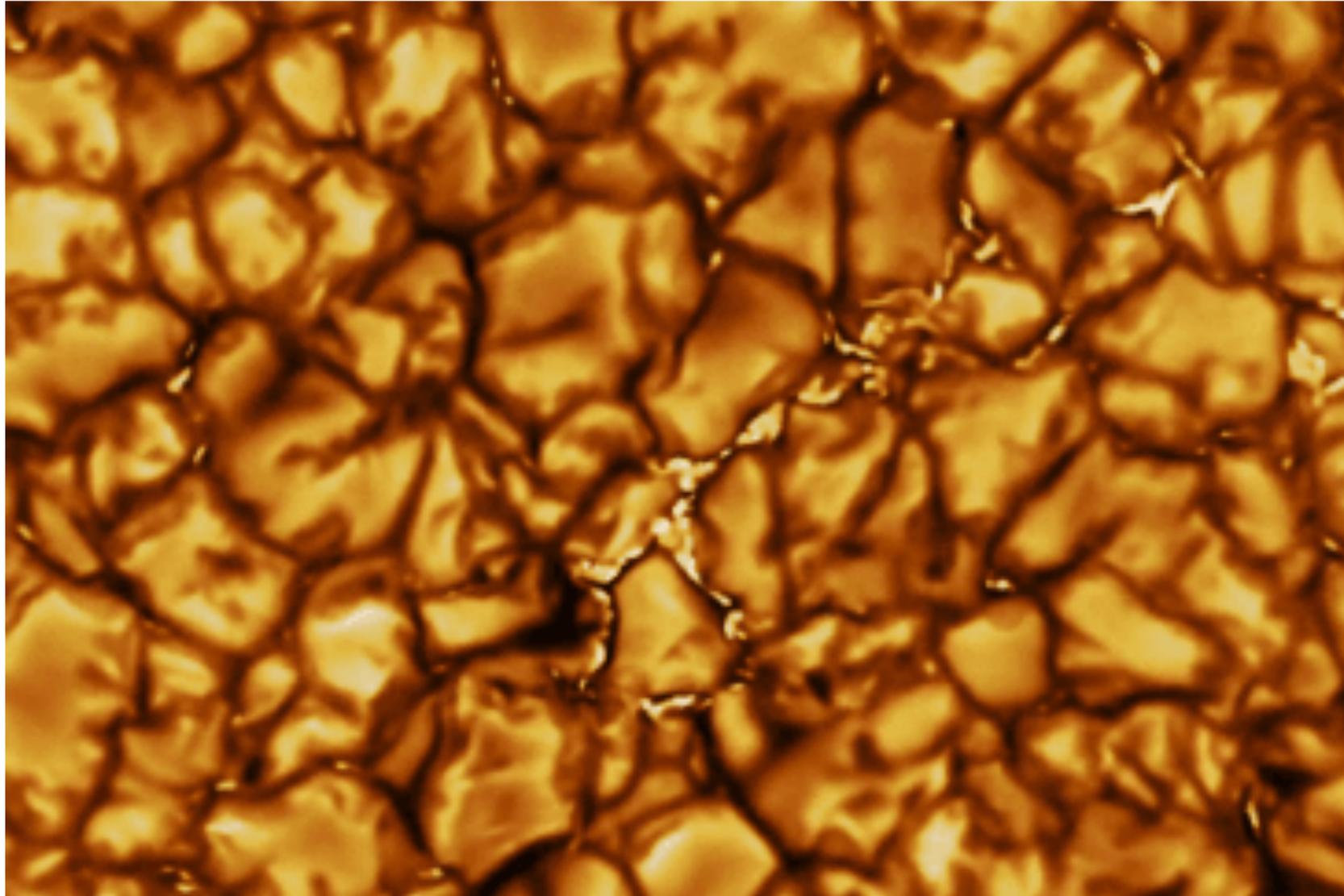


Convection

- Poisson's Equation:

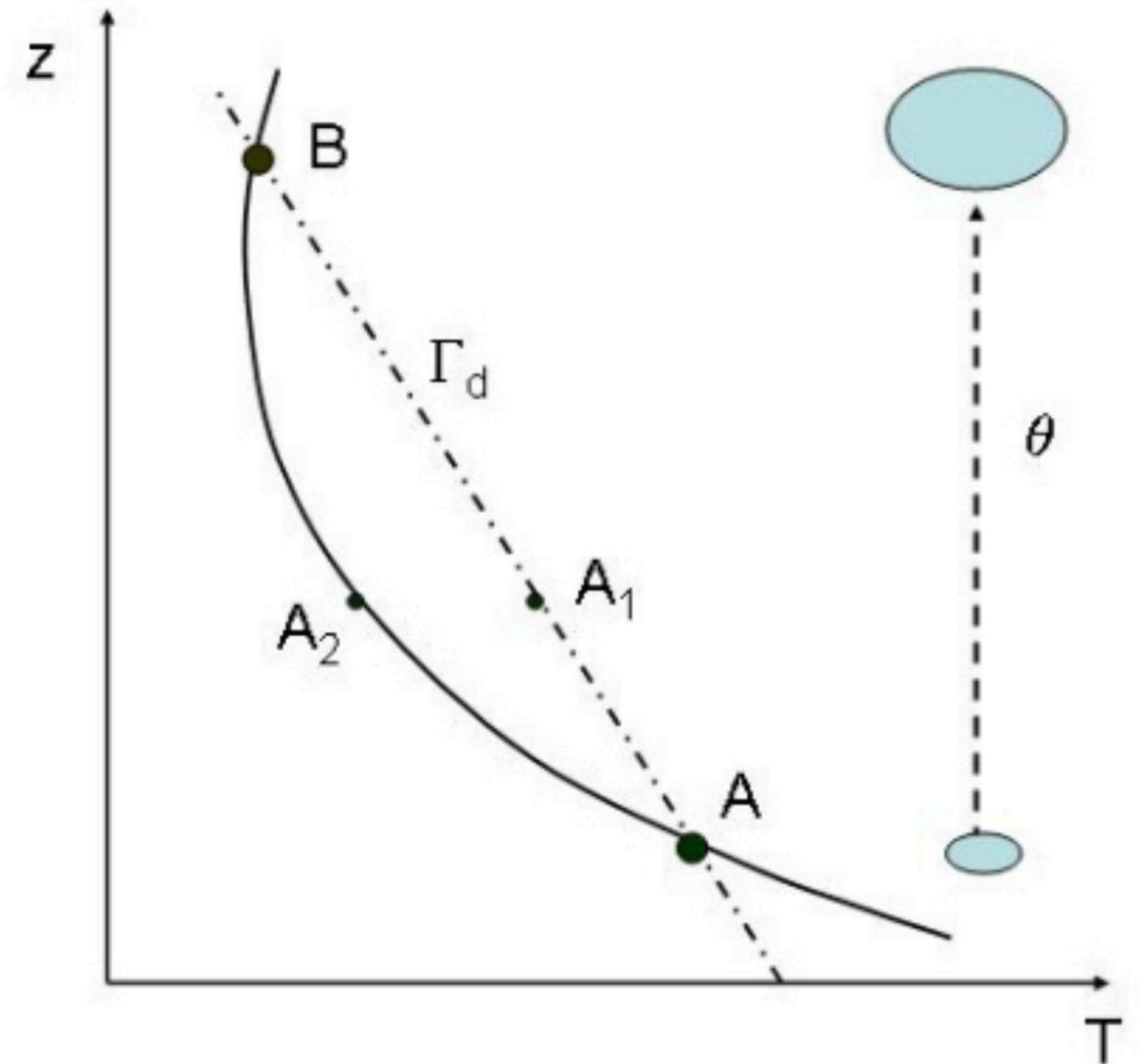
$$\theta = T \left(\frac{P_0}{P} \right)^{\frac{R}{C_P}}$$

- θ is a conserved quantity for an air parcel that moves through an atmosphere adiabatically



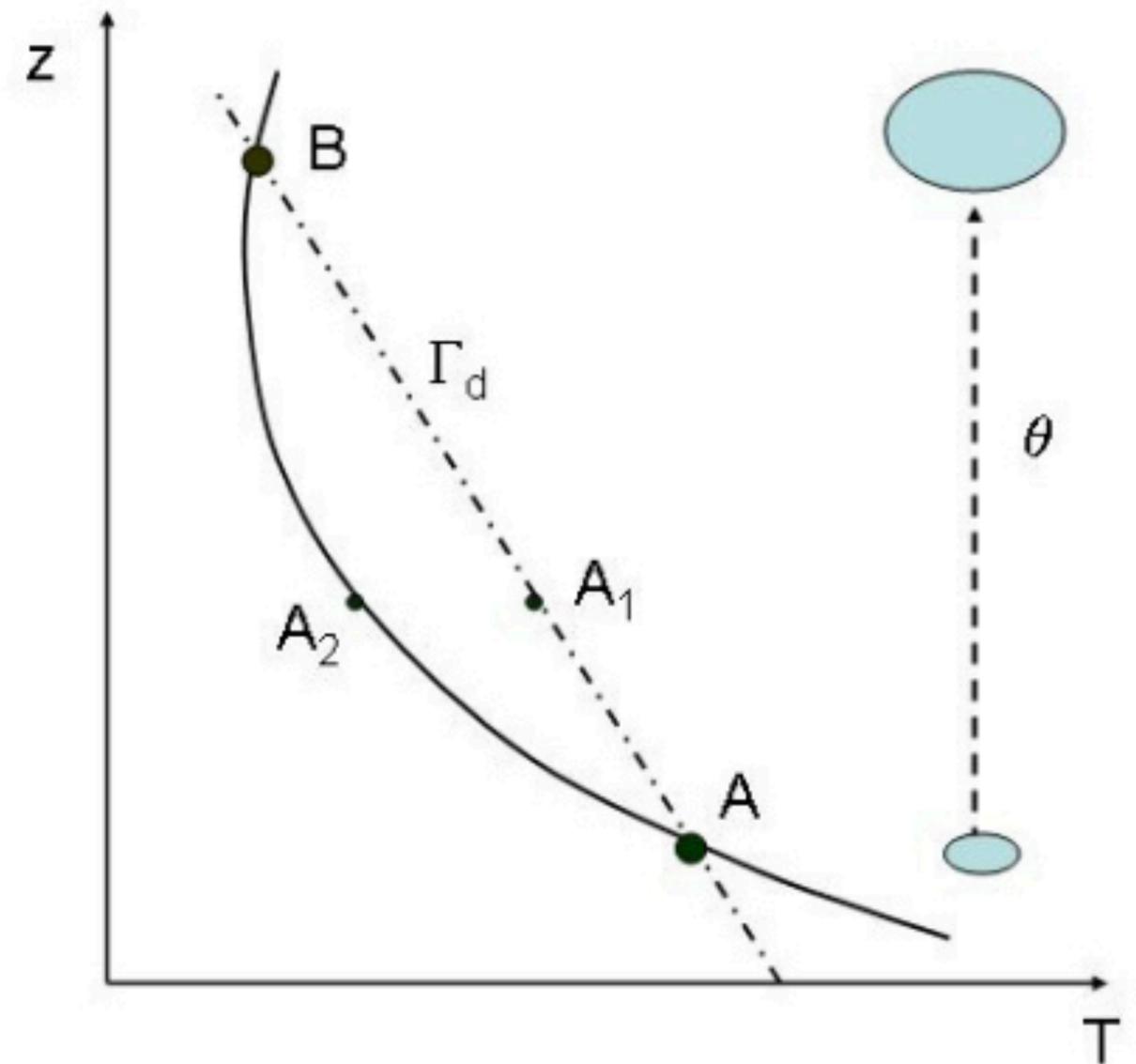
Vertical Stability

- When we lift a parcel of gas from z to $z + \Delta z$, what happens to it?
- We need to compare the actual lapse rate (“environmental lapse rate,” how temperature changes with height) to adiabatic lapse rate to determine fate of the parcel
- Specific temperature (not the actual temperature) remains constant (dashed line)



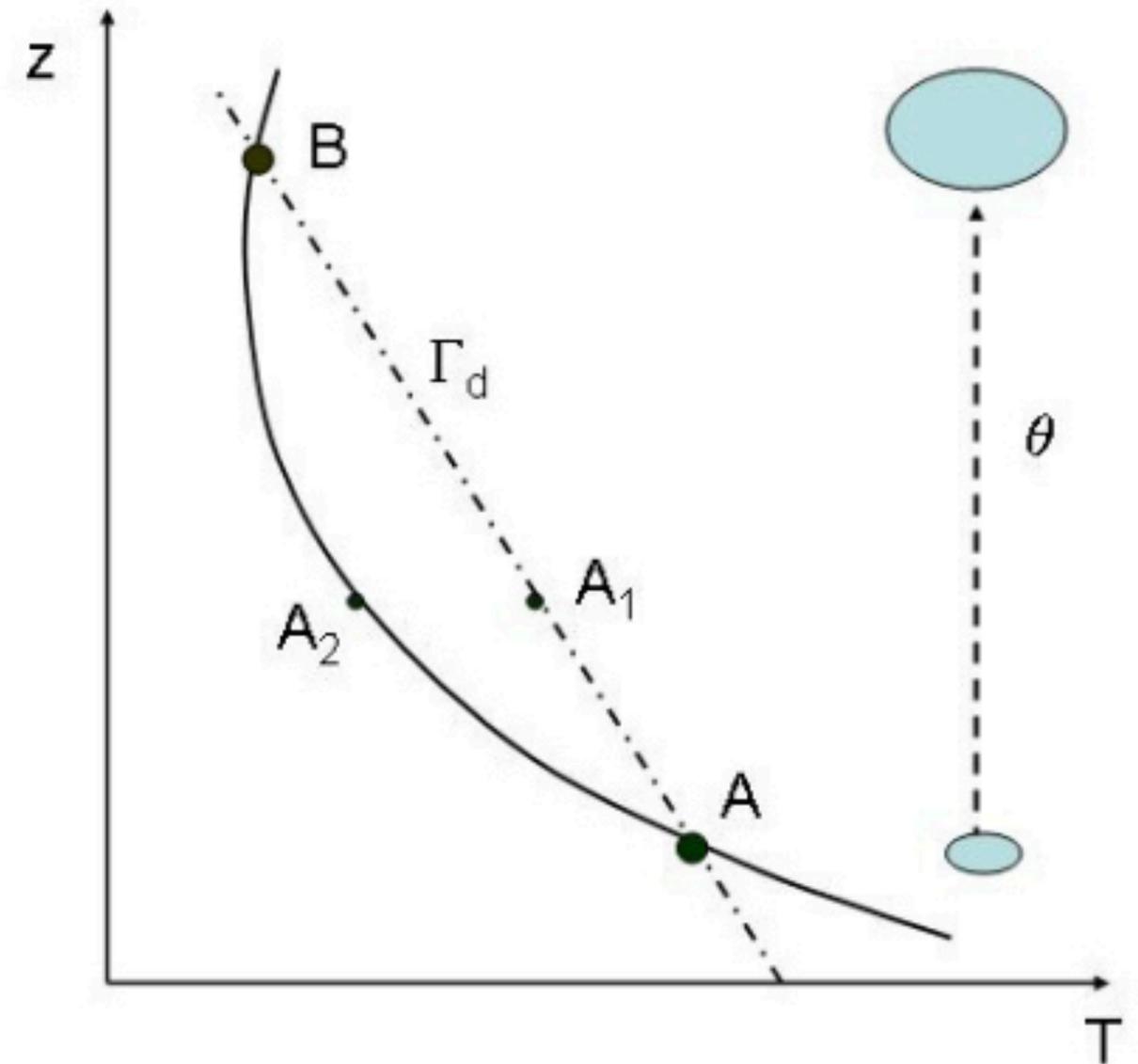
Vertical Stability

- dot-dashed line is adiabatic lapse rate
- solid line is environmental lapse rate
- A parcel initially at A will rise adiabatically
- At A1, parcel is hotter than surroundings at that height (A2), and will continue to rise
- A parcel at point B is stable (will not rise)



Vertical Stability

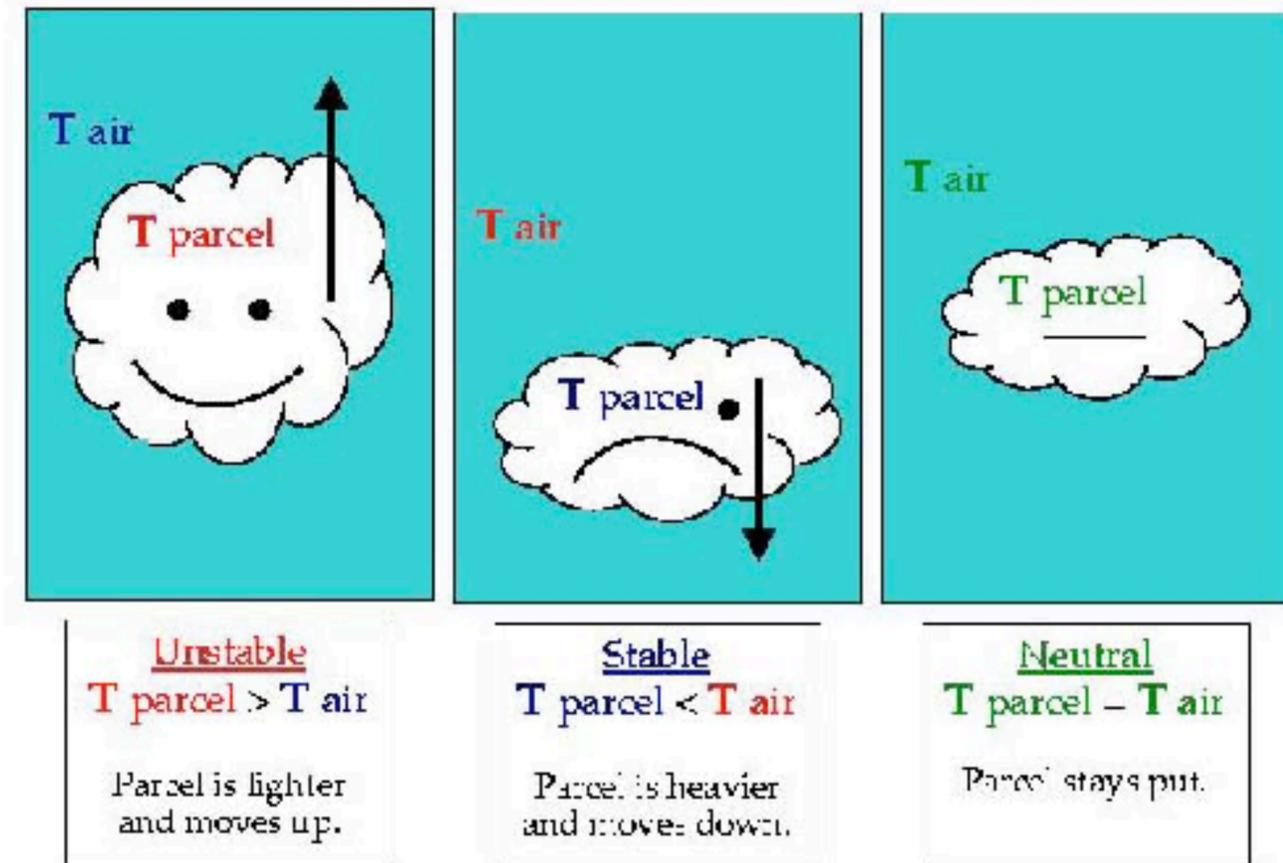
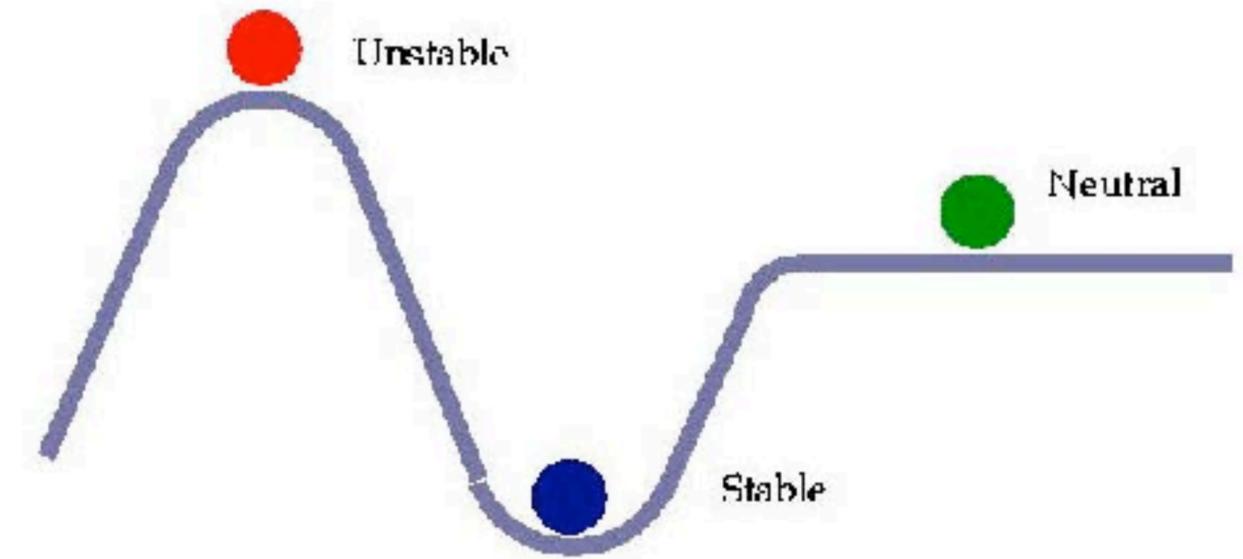
- At A, slope of temperature profile is less than the slope of the adiabatic lapse rate:
 - Decrease of temperature with altitude is greater than adiabatic
 - “Superadiabatic” — atmosphere is unstable to vertical displacement, parcel rises
- At B slope inverts, atmosphere becomes stable against vertical displacement



Vertical Stability

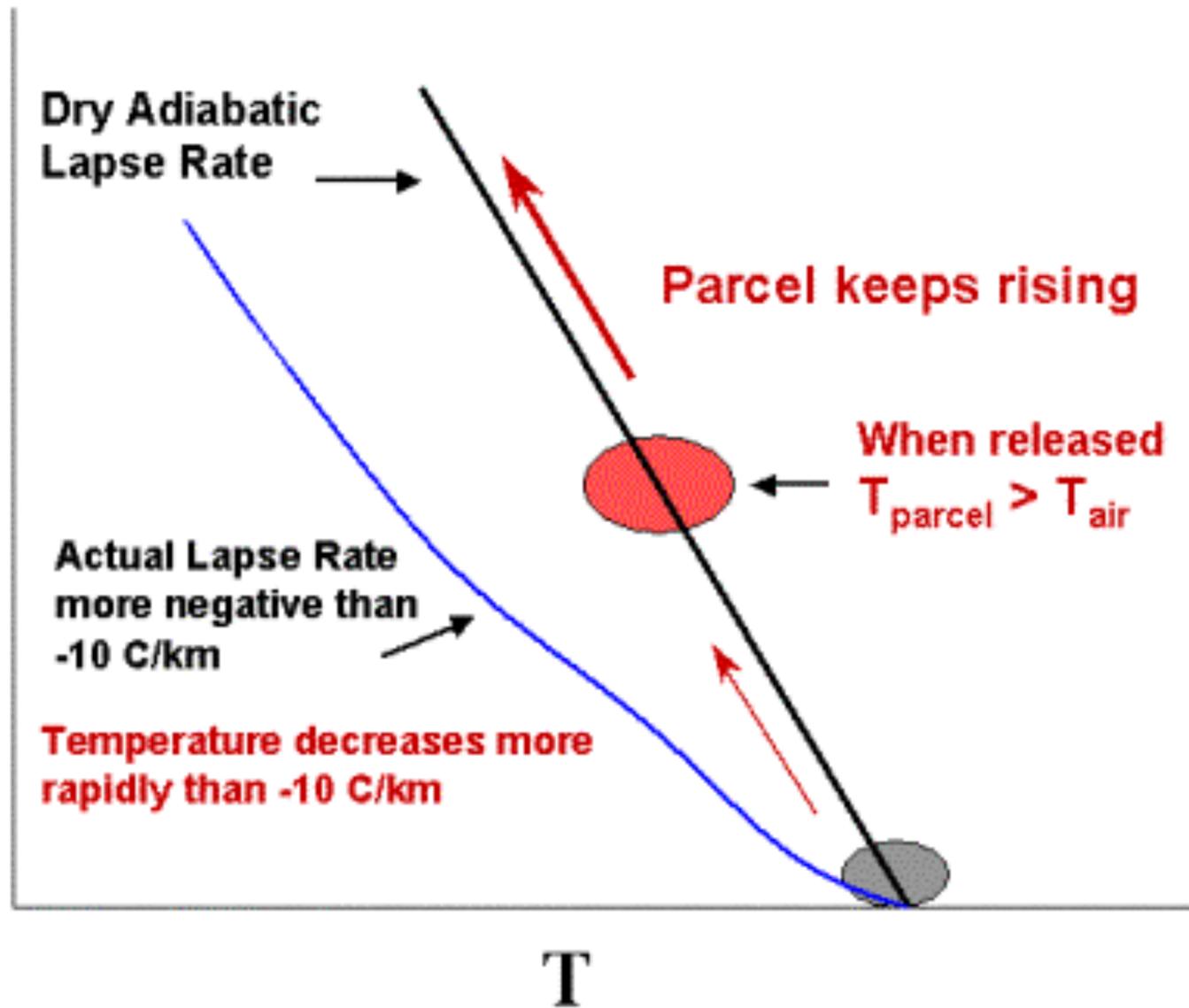
- What happens to a parcel that's moved up a small amount:
 - Unstable atmosphere: parcel continues to move upward
 - Stable atmosphere: parcel sinks back down to where it started
 - Neutral atmosphere: parcel remains where it was moved to

ATMOSPHERIC STABILITY

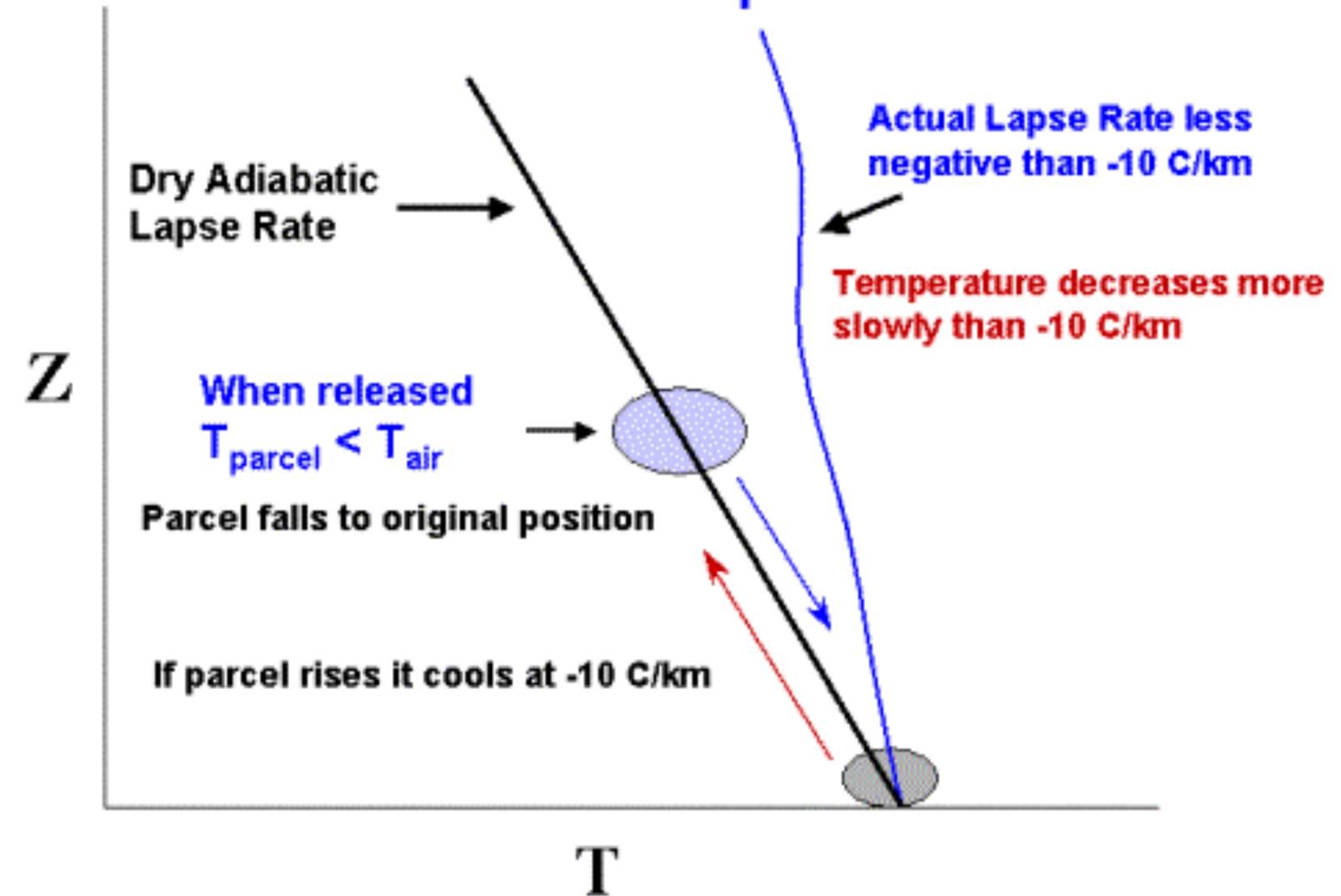


Vertical Stability

Unstable Atmosphere

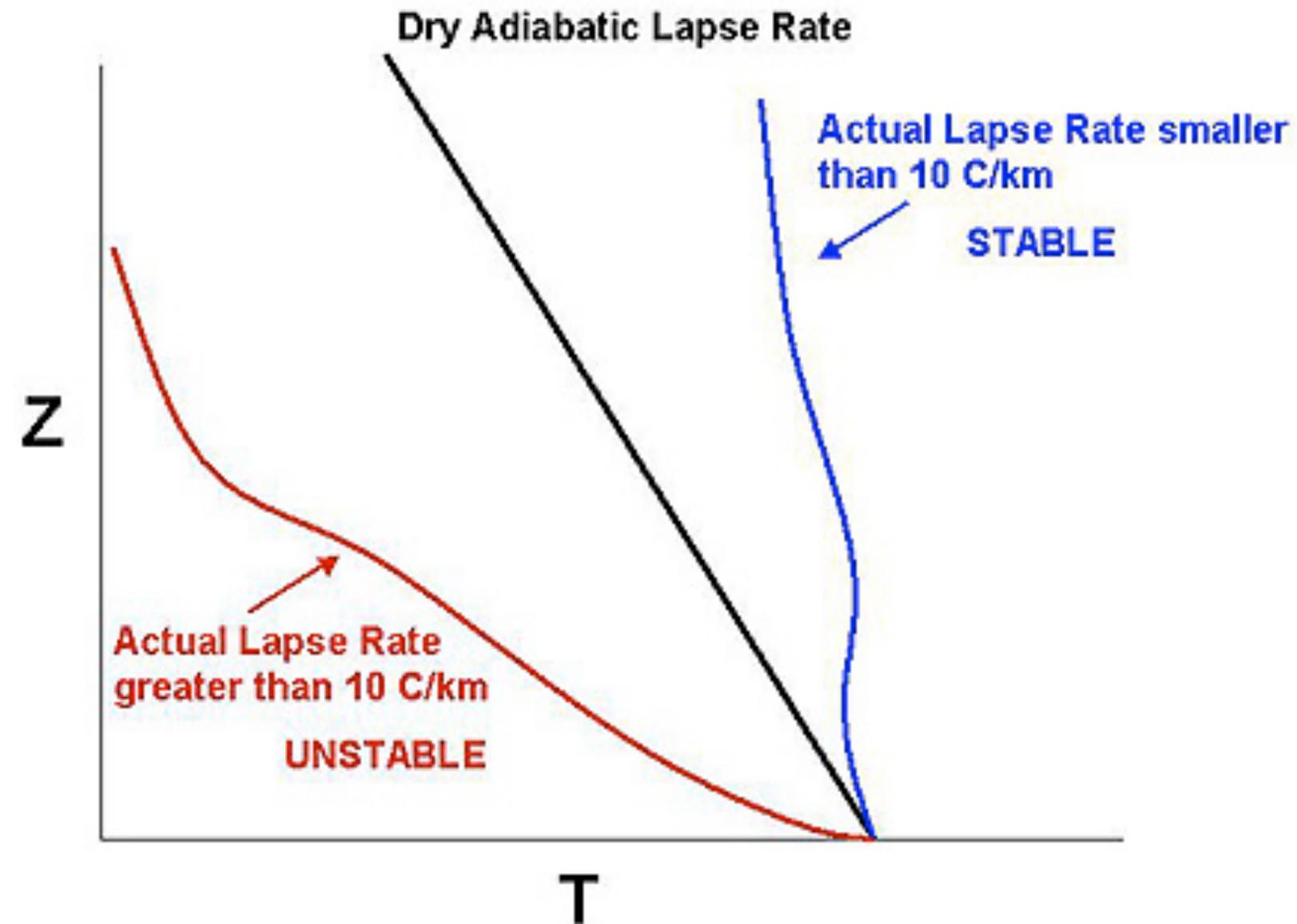


Stable Atmosphere



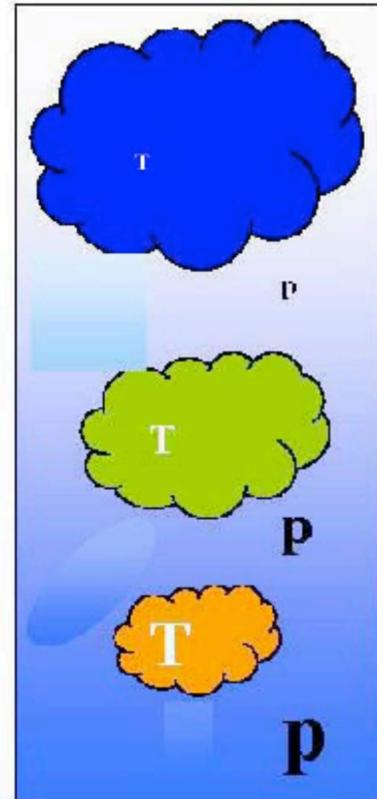
Vertical Stability

- Stable air:
 - slow change in temperature with height
 - suppresses convection
- Unstable air:
 - Rapid change in temperature with height
 - enhances convection
 - caused by surface becoming much warmer than air



Vertical Stability

ATMOSPHERIC STABILITY (continued)



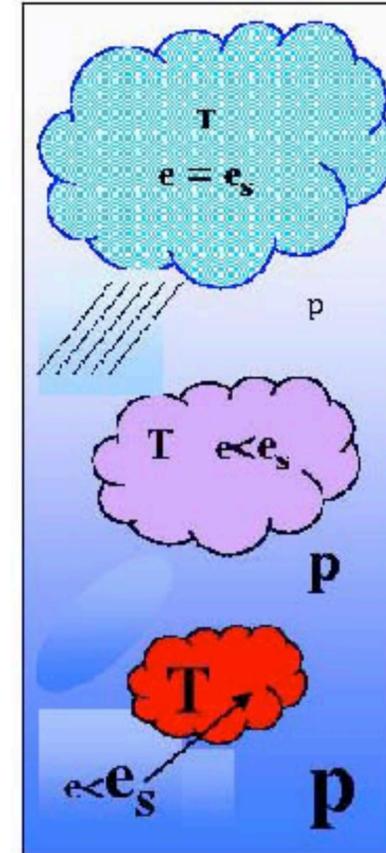
Rising air parcel encounters decrease in surrounding pressure and expands.

Expansion is adiabatic and implies a decrease in parcel temperature.

The dry adiabatic lapse rate:

$$\Gamma_d = 9.8^\circ\text{C/kilometer}$$

STABILITY OF MOIST AIR



Rising air parcel includes water vapor.

Adiabatic cooling reduces its ability to contain water.

Vapor condenses and releases latent heat.

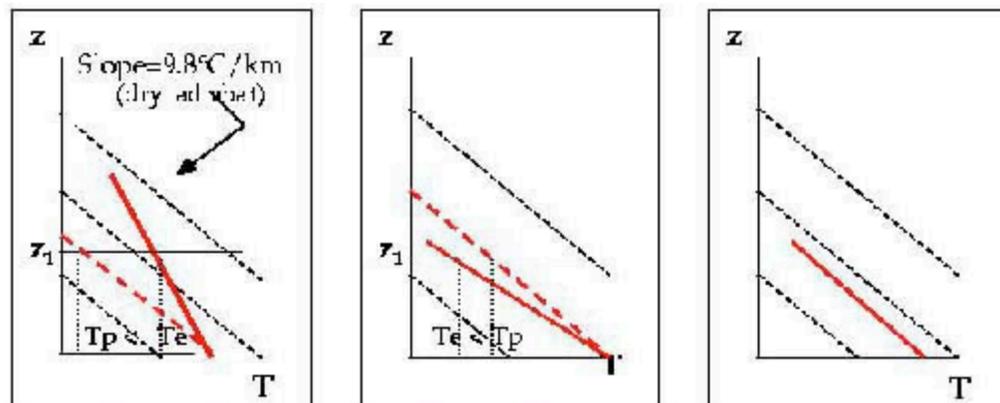
Cooling rate of air is reduced.

Moist adiabatic lapse rate:

$$\Gamma_{\text{moist}} = 6.5^\circ\text{C/kilometer}$$

Stability of Dry Air

In the diagrams below solid red line is the environmental lapse rate, and dashed red is the dry adiabat for a parcel rising from the surface in that environment. T_p and T_e are the parcel and environment temperature at level Z_1 .



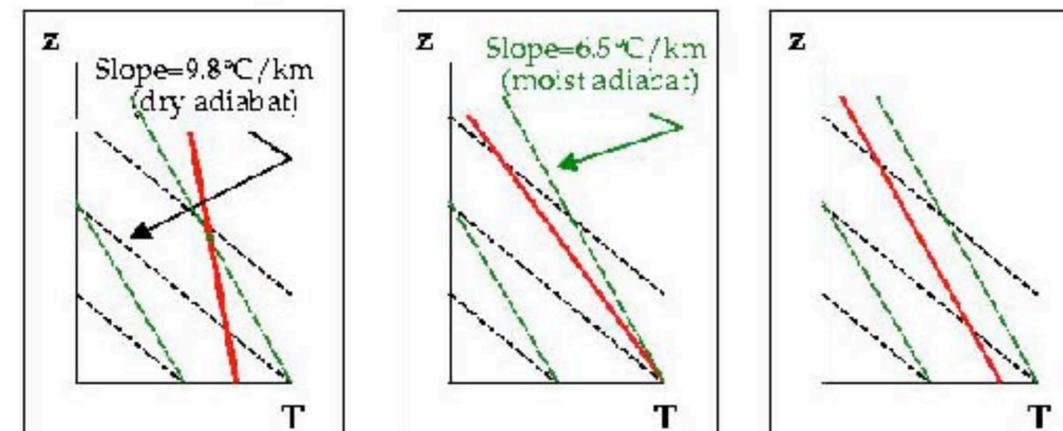
STABLE

UNSTABLE

NEUTRAL

Stability of Saturated Air

In the diagrams below solid red line is the environmental lapse rate.

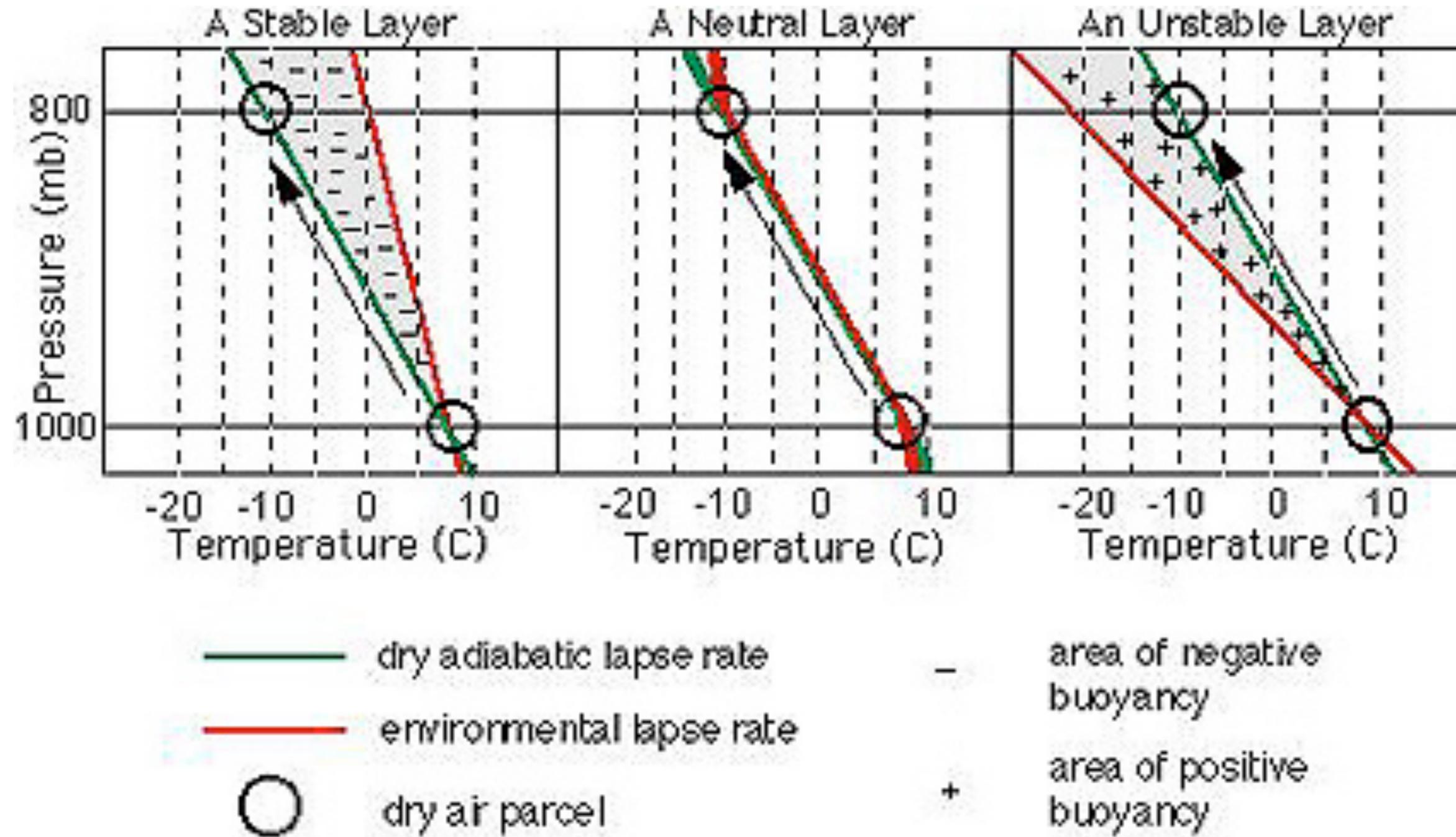


STABLE

UNSTABLE

NEUTRAL

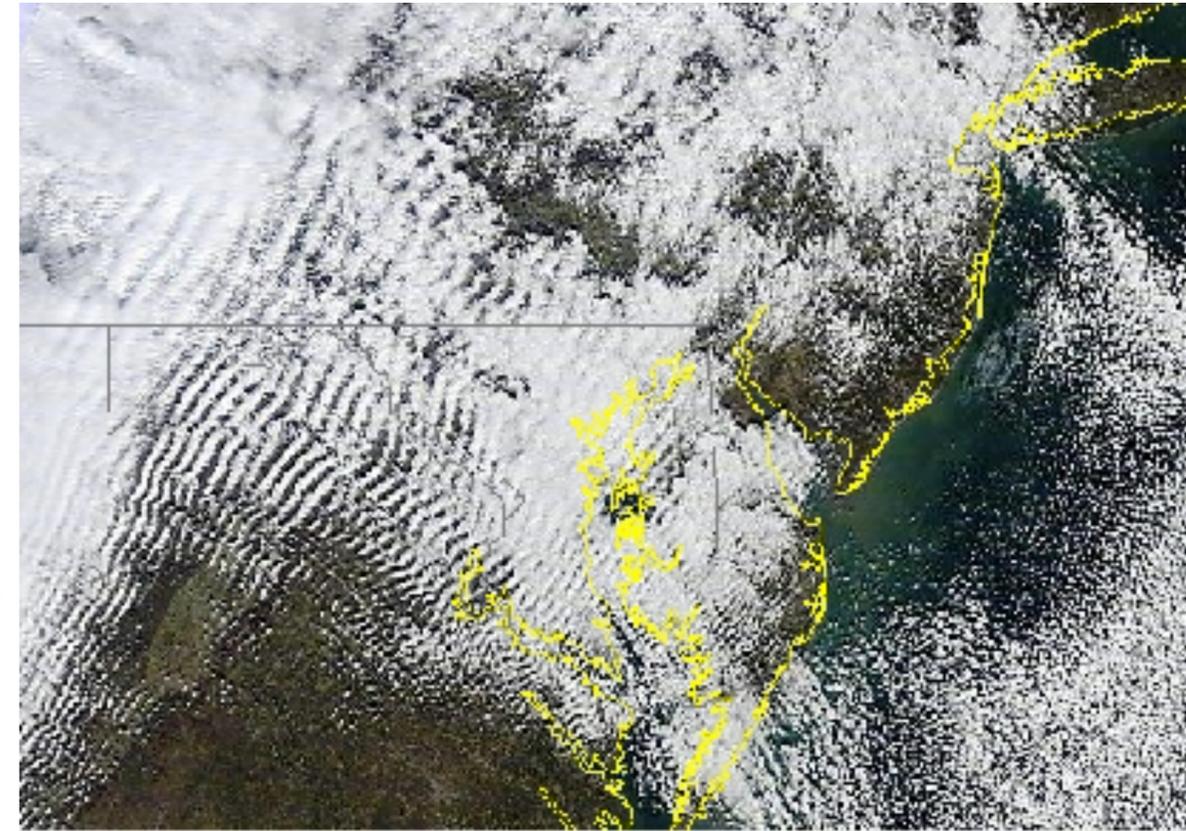
Vertical Stability



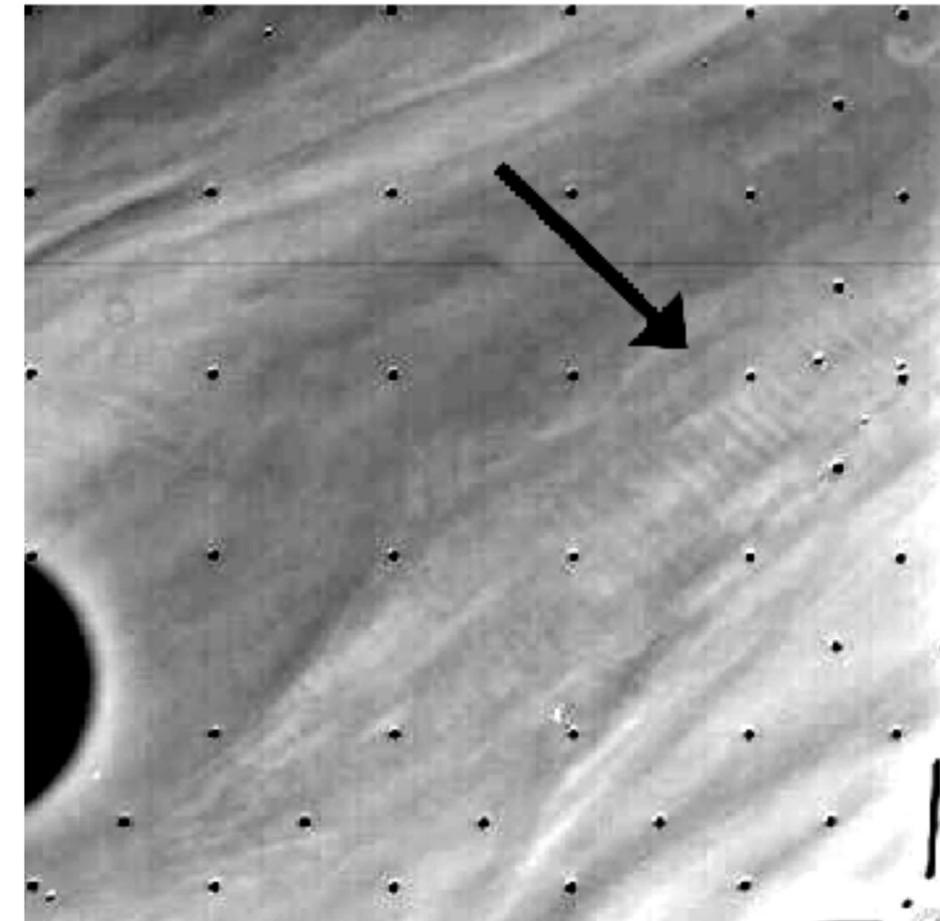
Gravity Waves

- Air parcels (in stable regions) undergo buoyancy oscillations in association with gravity waves
 - Maybe excited by flow over mountainous terrain or by an intense local disturbance

Earth, over the Appalachians



Jupiter, from Voyager



For next time

- Midterm Wednesday in class