ASTR 620: Planetary Processes

Professor Eric Nielsen

Lecture 10: Spectra

Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Midterm in 2 weeks: Wednesday, October 5th (here in class)
- Homework 3 due Wednesday, September 28 at 11:59pm

Review of the last class

- A surface is in equilibrium if:
  - (A) — Every square inch of the surface is covered by a crater
  - (B) — Craters no longer form
  - (C) — Each new crater is accompanied by the destruction of an existing crater
  - (D) — Lava flows wipe out all previous craters

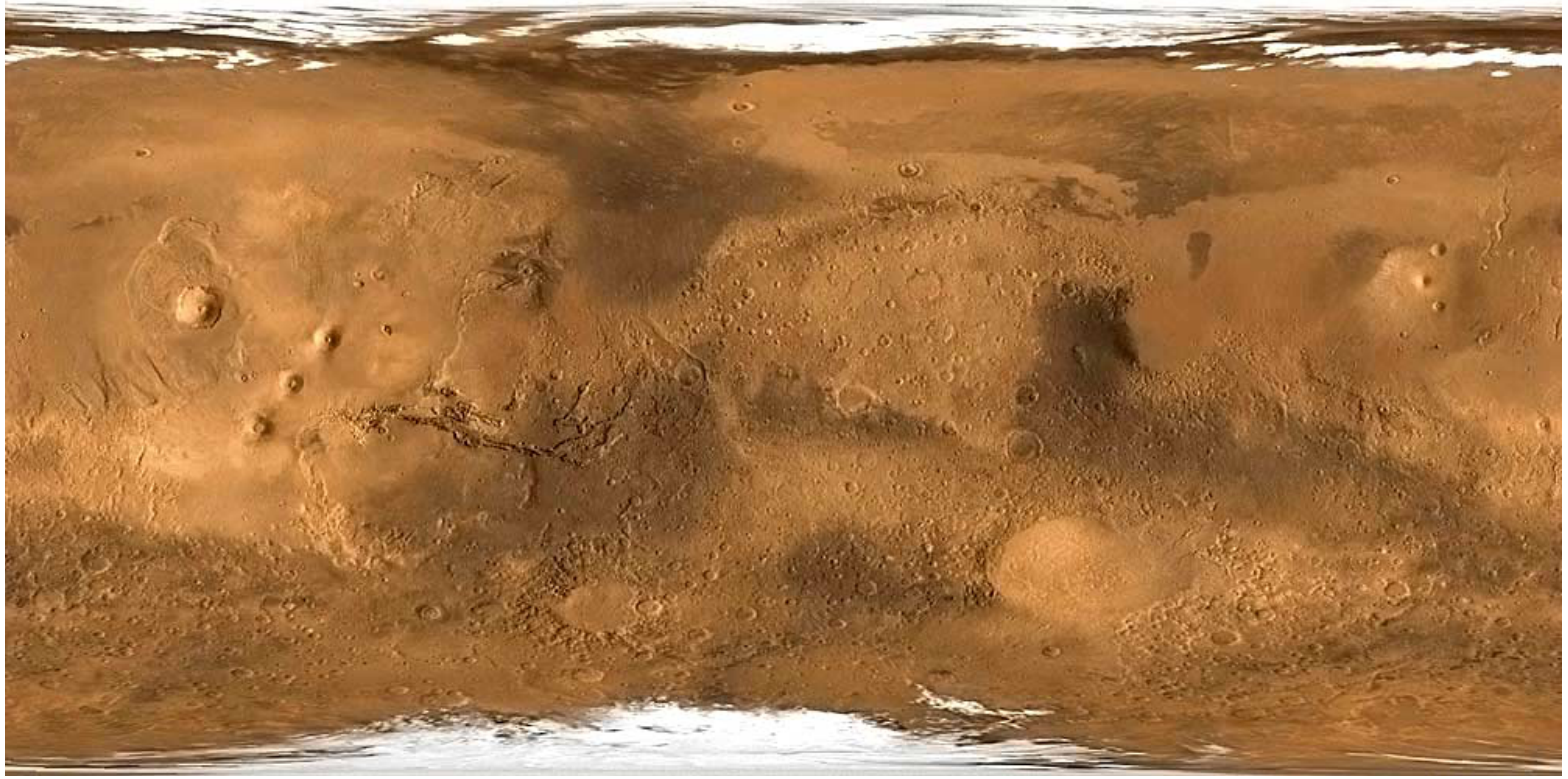
Review of the last class

- Which best describes the distributions of craters and impactors in the Solar System?
  - (A) — There are more large craters than small craters, there are more small impactors than large impactors
  - (B) — There are more small craters than large craters, there are more small impactors than large impactors
  - (C) — There are more large craters than small craters, there are more large impactors than small impactors
  - (D) — There are more large craters than small craters, there are more small impactors than large impactors

Review of the last class

- Which hemisphere of Mars has an older surface?
  - (A) — North
  - (B) — South
  - (C) — They’re equally old

North



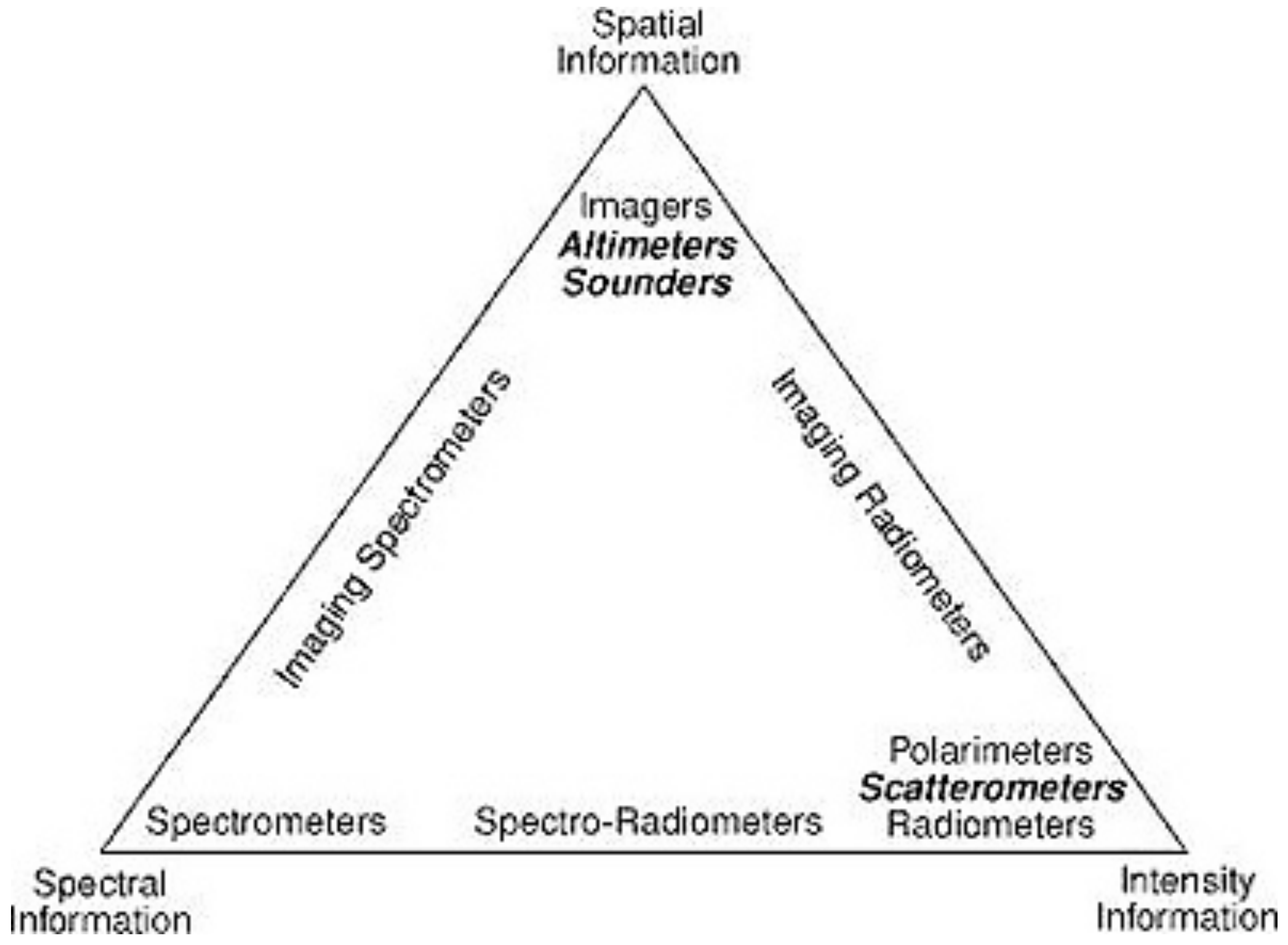
South

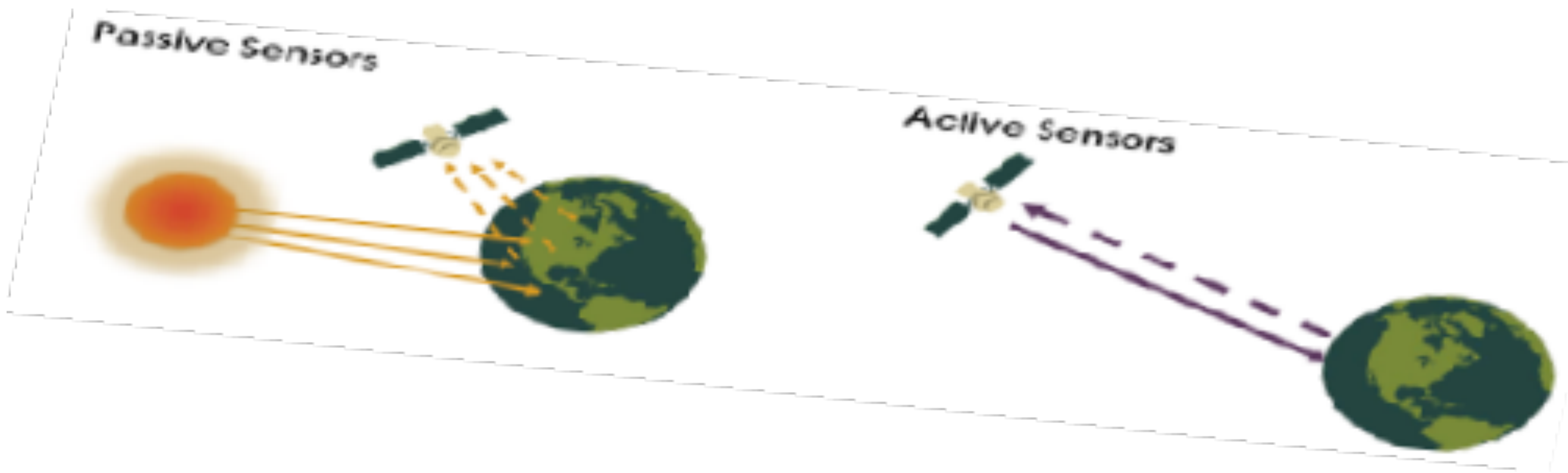
Review of the last class

- Small meteors (10 cm) that enter Earth's atmosphere at high speeds:
  - (A) — Bounce back into space
  - (B) — Form very small craters on the ground
  - (C) — Completely evaporate before they hit the ground
  - (D) — Break up, with the fragments falling at terminal velocity

Remote Sensing

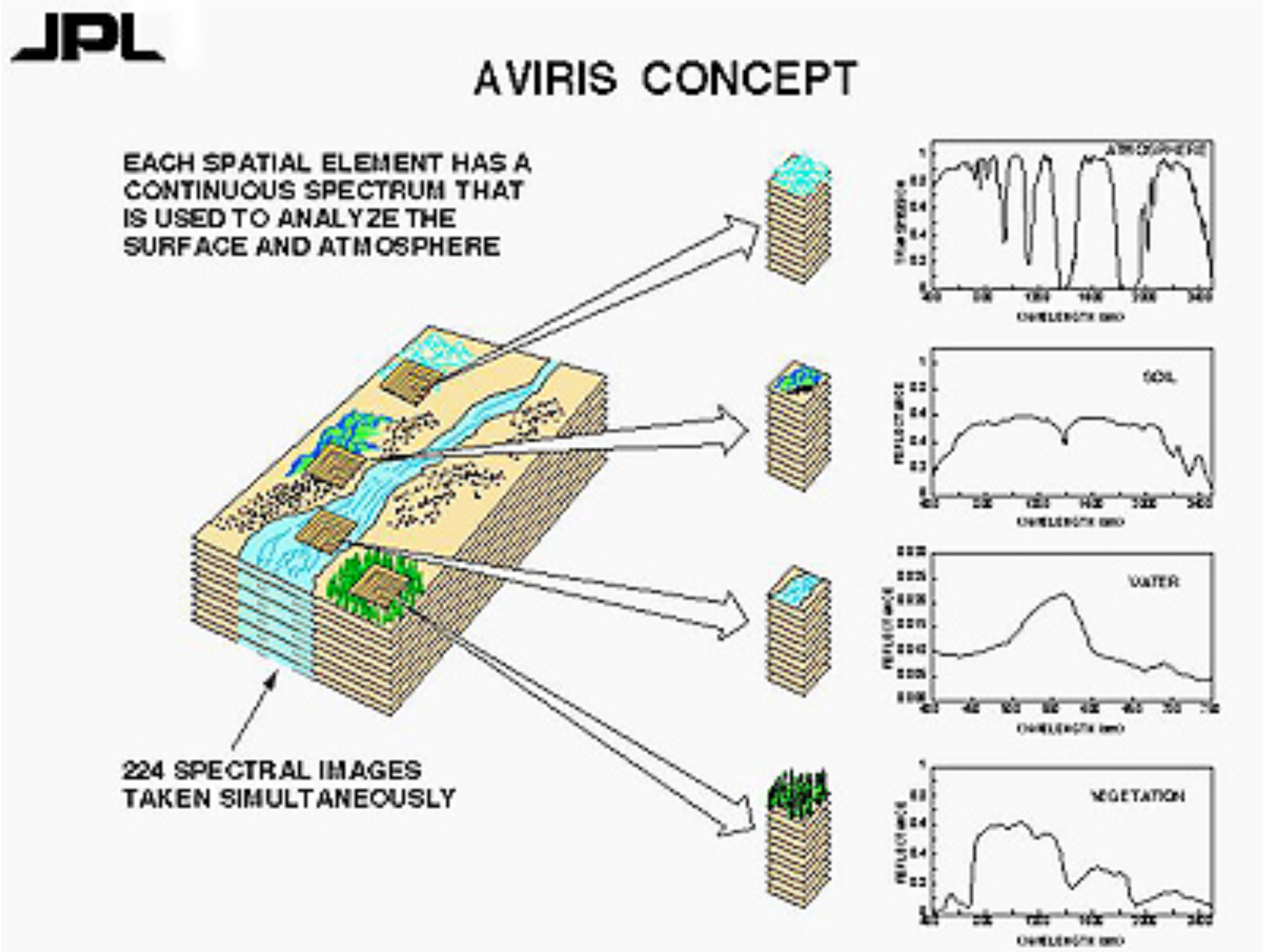
- In Planetary Science (and astronomy) we collect information on distant objects through light
- Photometry (intensity): total brightness
- Spectra: light broken up by wavelength
- Spatially resolved images: light broken up by angular position



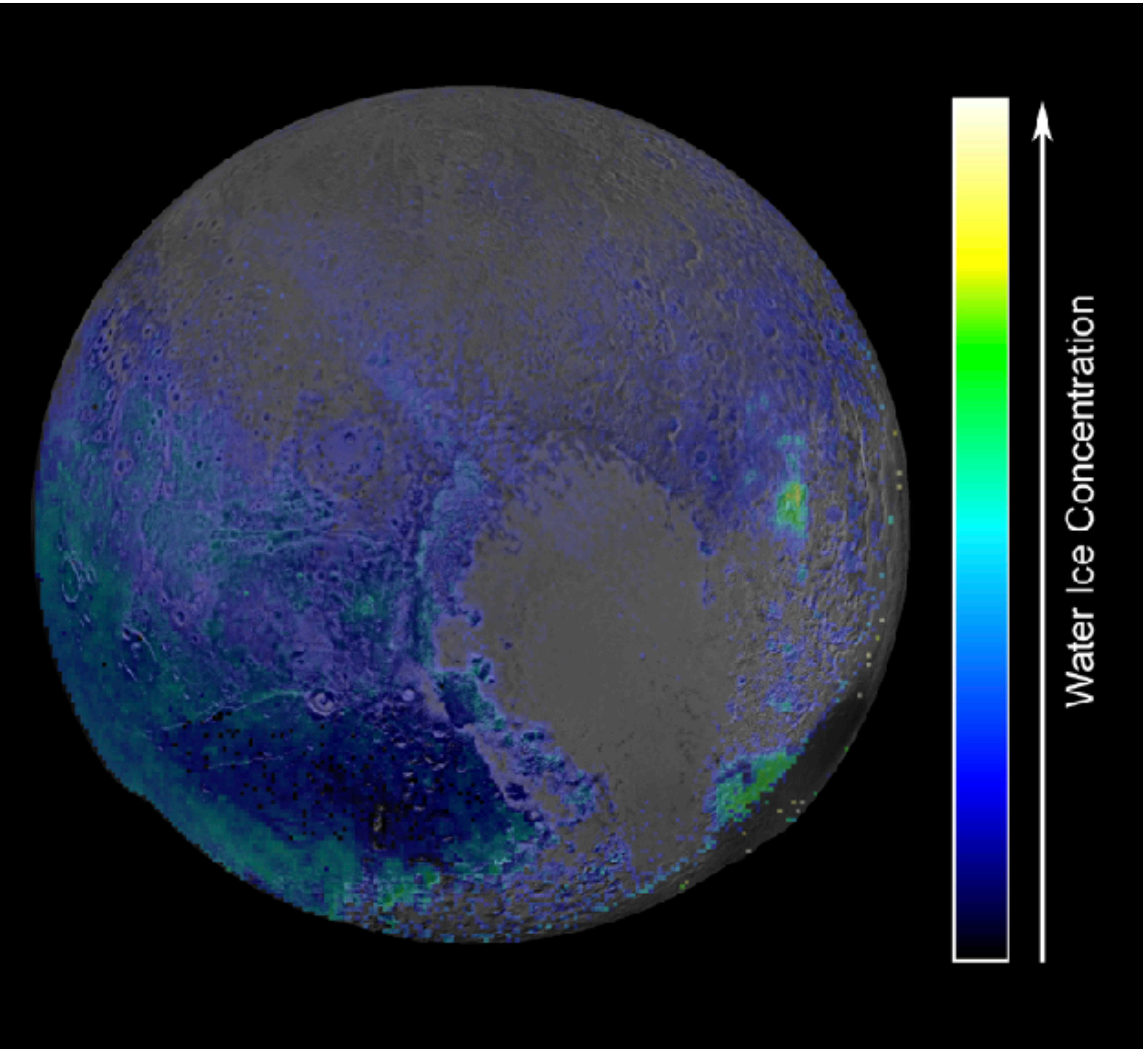


Hyperspectral Imaging (Planetary Science)

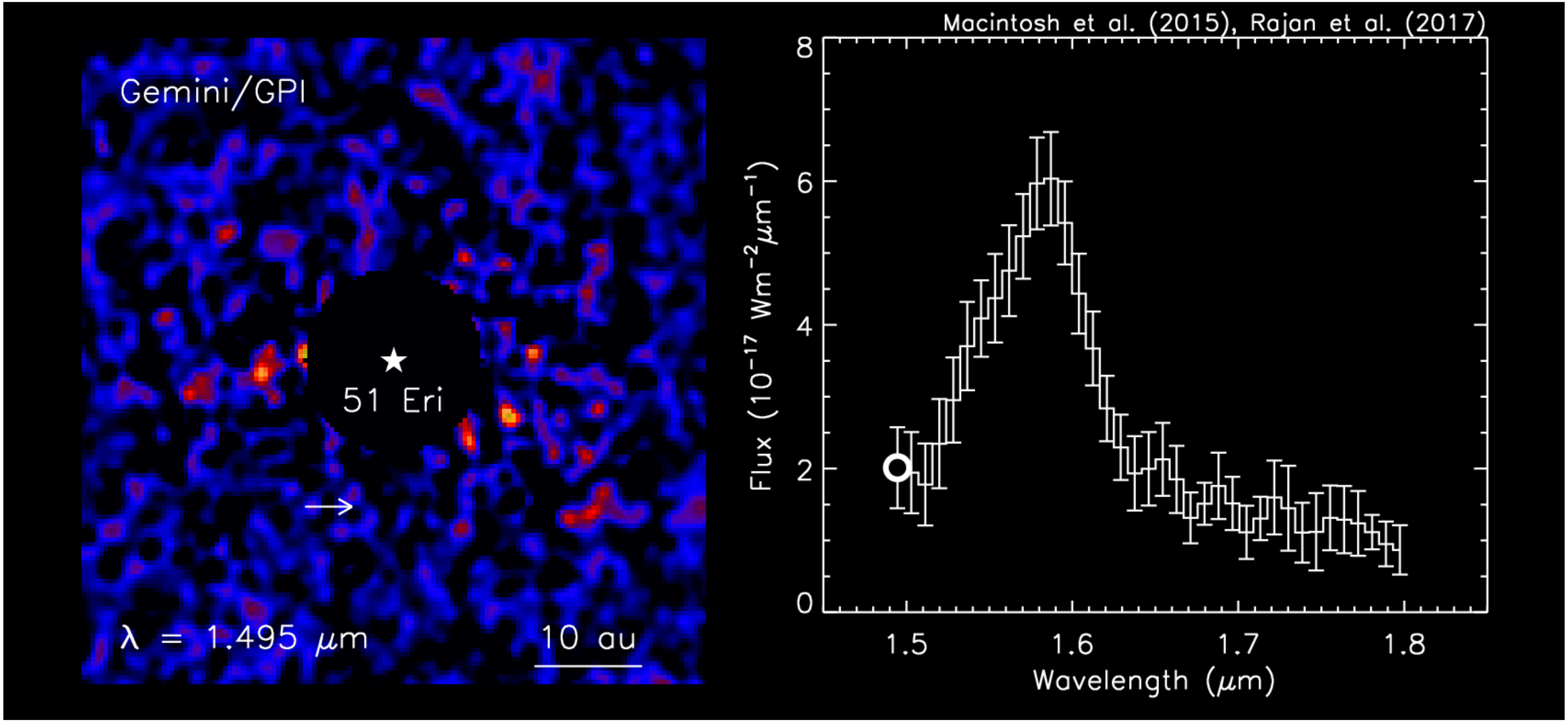
- Combination of imaging and spectroscopy:
    - Spatially resolved in the x (east-west, say) and y (north-south, say) directions
    - Spectrally resolved in the wavelength direction
    - Data “cube” is x vs y vs lambda



Hyperspectral Imaging (Planetary Science)

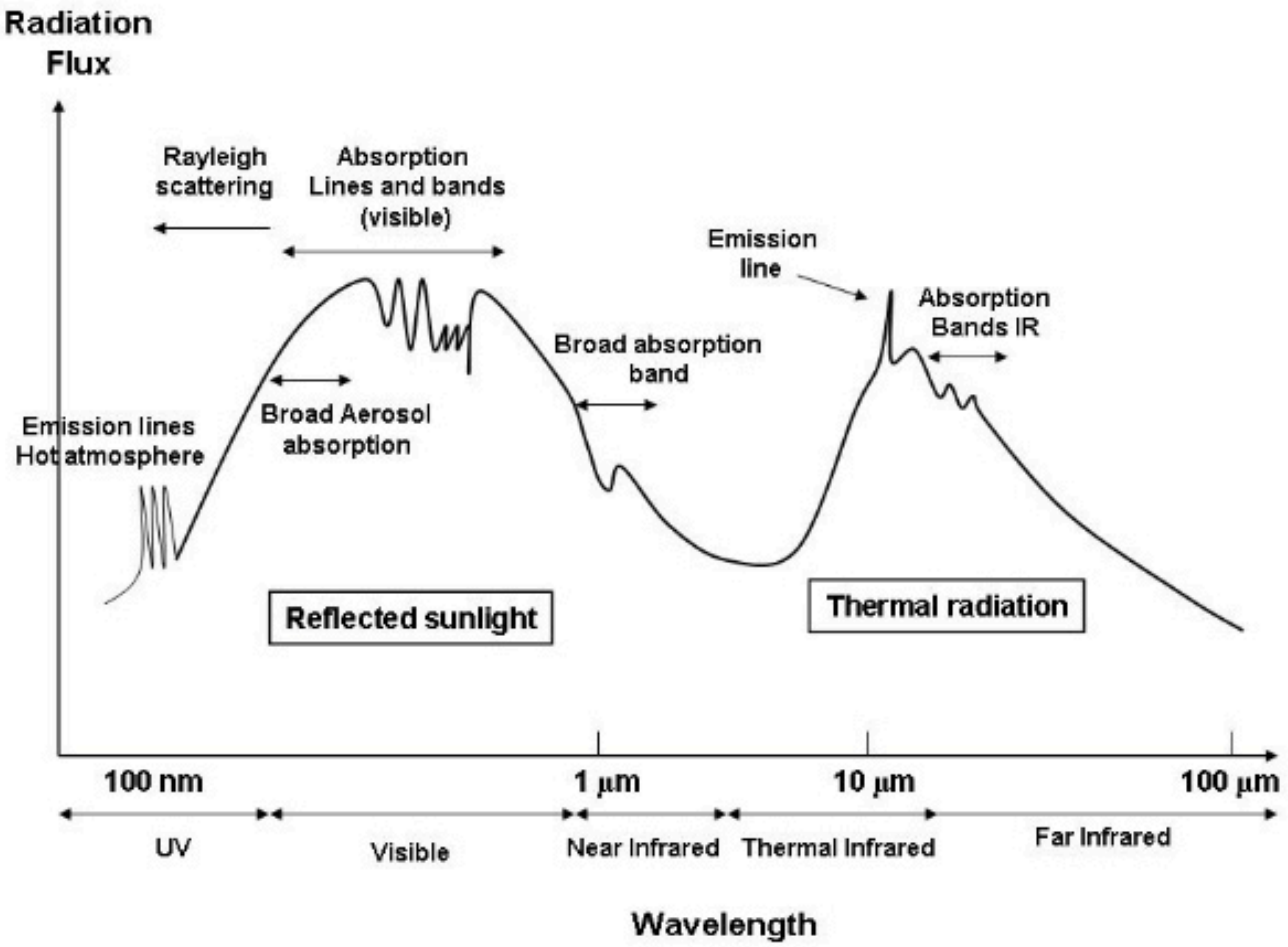


Integral Field Spectroscopy (Astronomy)



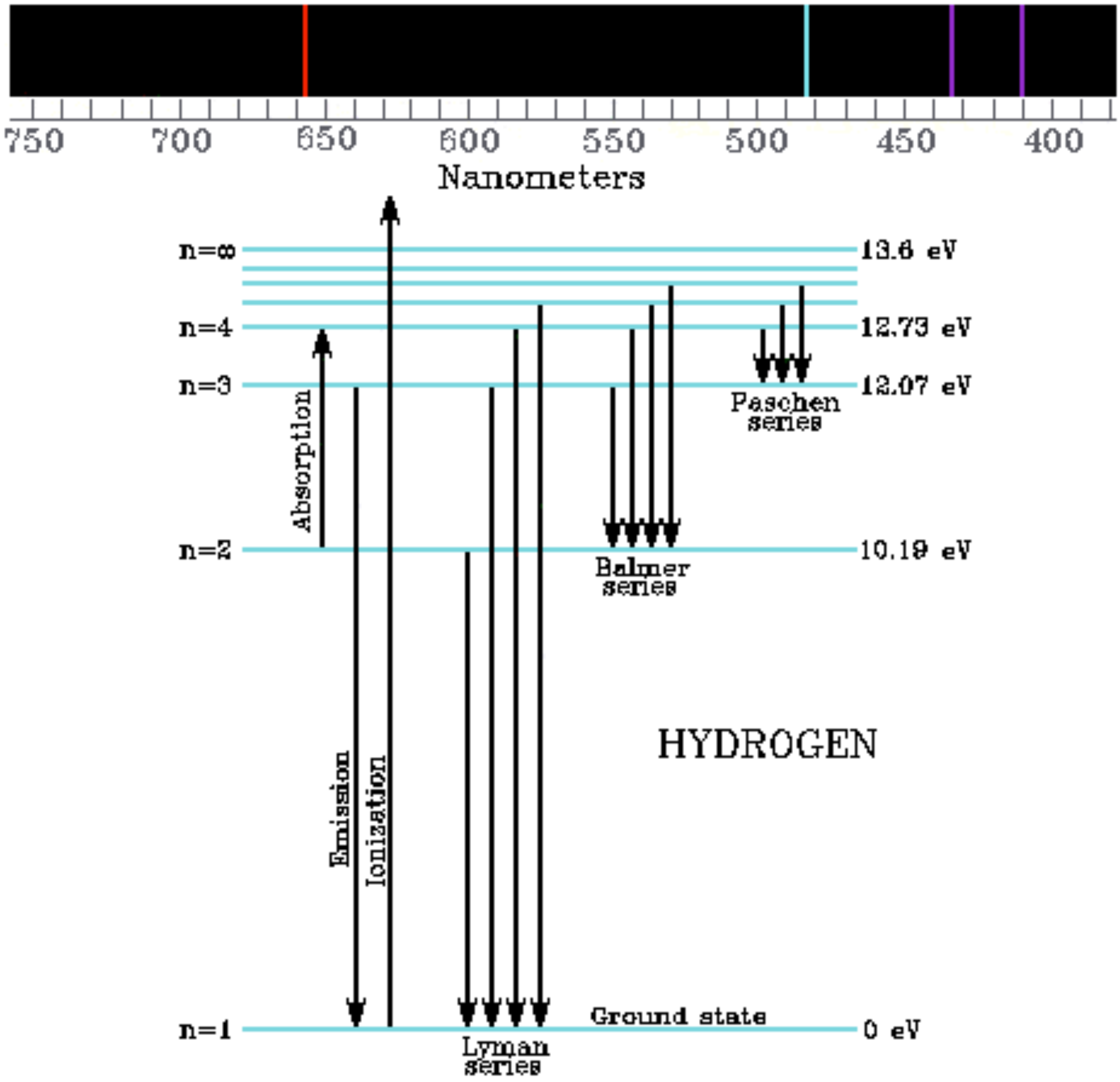
Planetary Spectra

- Most planetary spectra have two components:
  - Reflected sunlight (starlight) spectrum, usually in the visible
    - Depends on angle between sun, planet, and observer
  - Thermal emission, mainly in the infrared, approximately a blackbody given the object's temperature. Internal heat comes from:
    - Reprocessed sunlight
    - Internal heat sources (e.g. radioactive decay)
    - Residual heat from formation



Absorption and Emission Spectra

- Atoms absorb (or emit) a photon when an electron moves between two energy levels
- Ground state (n=1) is the “closest” to the nucleus, and the lowest energy state for an electron
- Moving up to a higher energy level absorbs a photon
- Moving down to a higher energy level emits a photon
- The energy spacing between higher energy levels (n=large to n=large + 1) is smaller than the spacing between lower energy levels

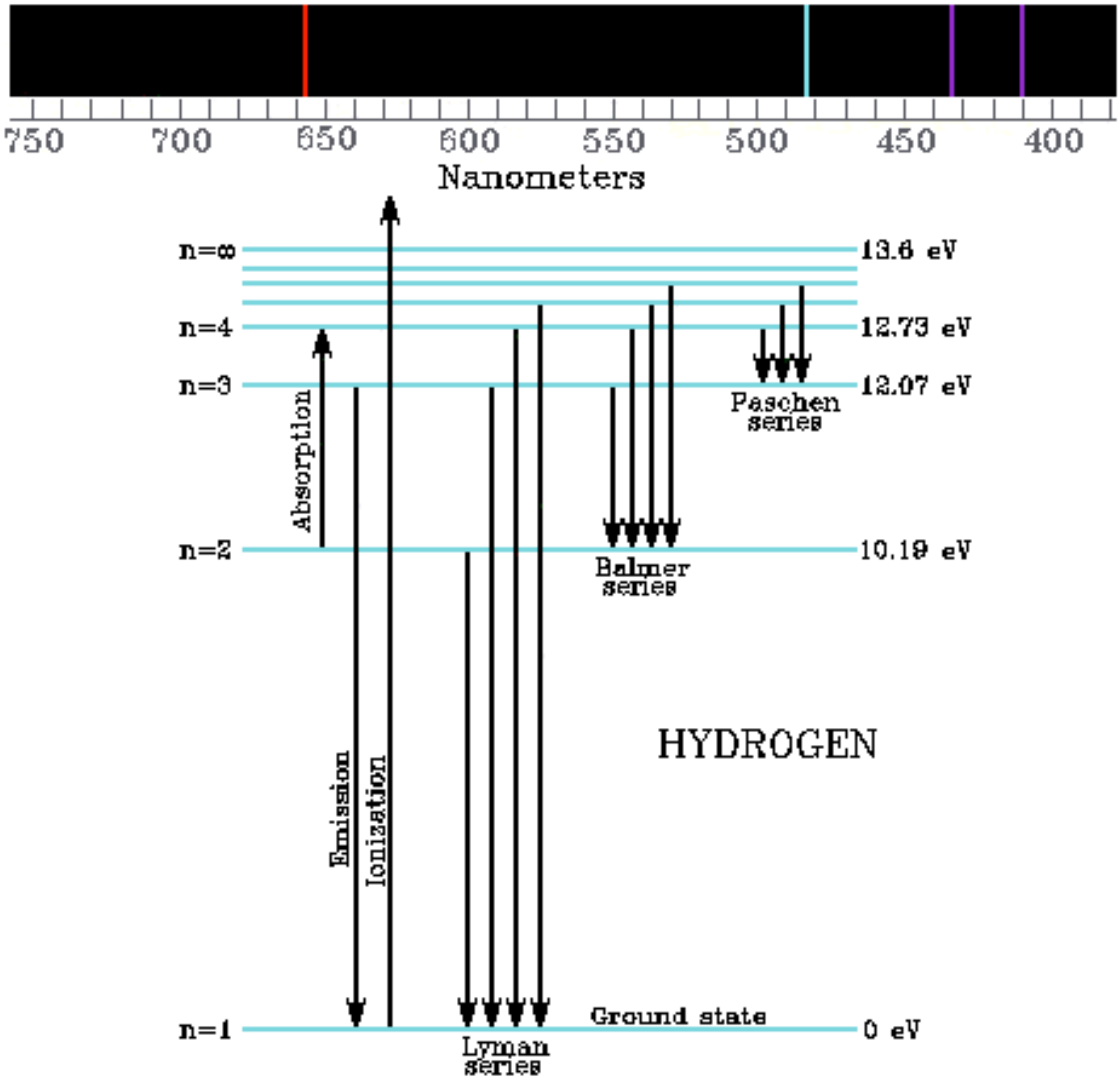


Emission

- Requires atom starts in an excited (not ground) state
- Can spontaneously move to a lower state and emit a photon
- Intensity of emission spectrum from level j to level i is given by:

$$N_j h\nu_{ji} A_{ji}$$

- N_j: Number of atoms in state j per unit volume
- $h\nu_{ji}$: energy difference between the two levels
- A_{ji}: Einstein A coefficient for spontaneous emissions (number of transitions per second of atom from excited state j to state i)

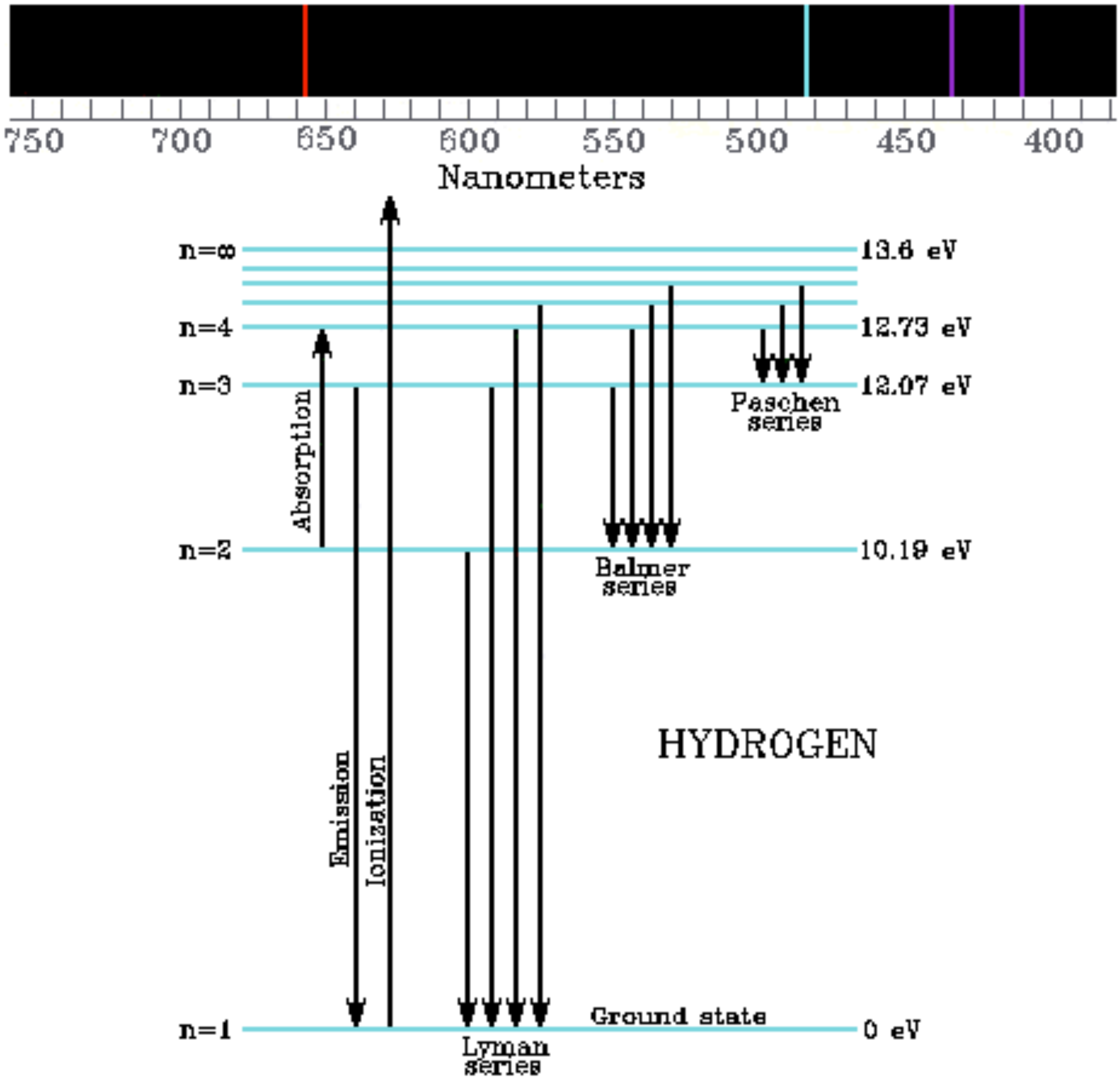


Absorption

- Intensity of absorbed light resulting from electrons initially in state i move up to level j:

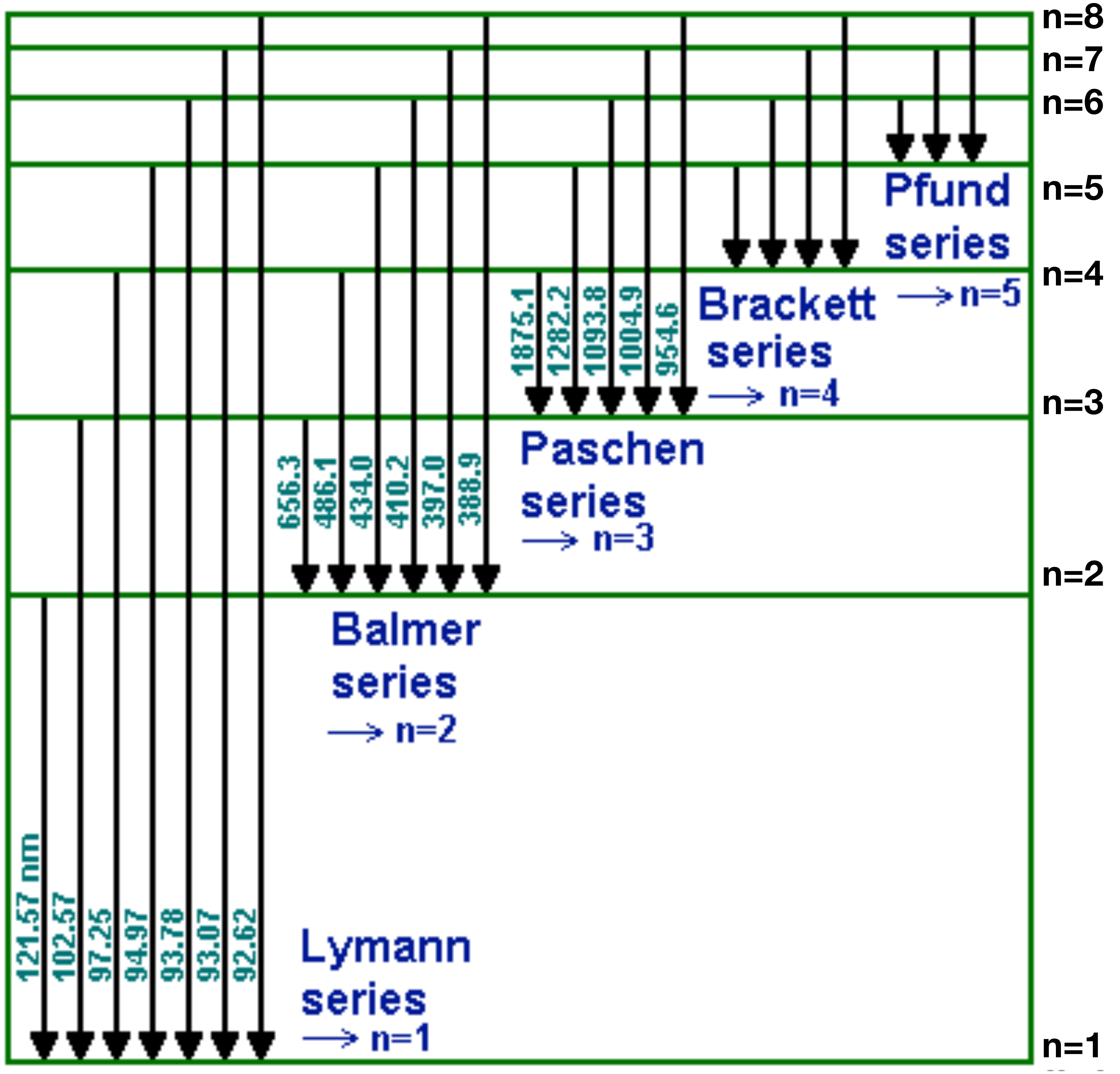
  $N_i h\nu_{ij} B_{ij} \rho_\nu$

- N_i: Number of atoms in state i per unit volume
- $h\nu_{ij}$: energy difference between the two levels
- B_{ij}: Einstein B coefficient for absorption
- ρ_ν: density of radiation with frequency ν



Hydrogen

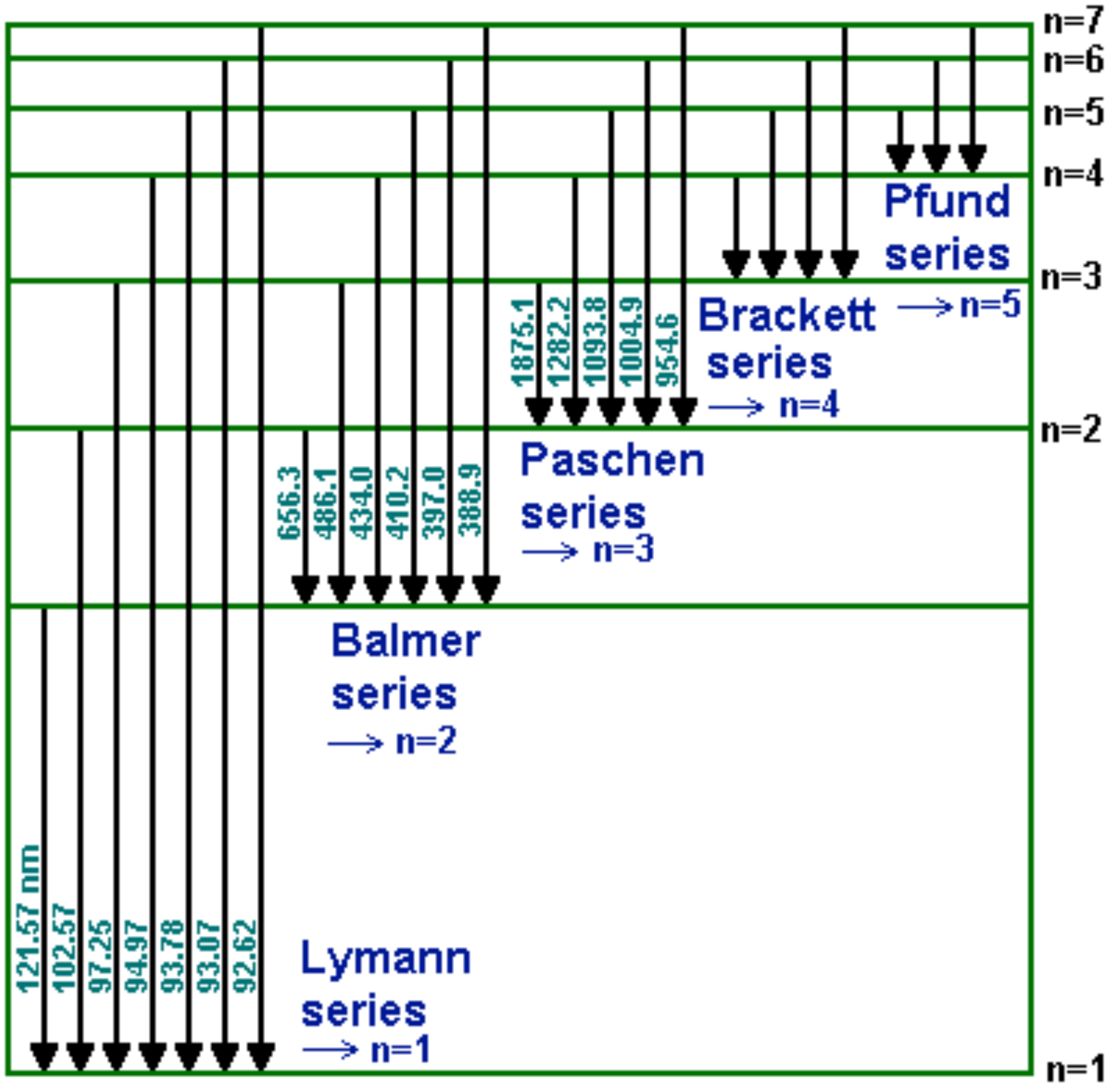
- The Schrodinger equation can be solved for hydrogen and other single-electron atoms (and not much else)
- The energy for each energy level (n) is given by:
- $$E_n = -\frac{m_e Z^2 e^4}{8h^2\epsilon_0^2 c}\frac{1}{n^2} = -R\frac{Z^2}{n^2}$$
- m_e: electron mass
- Z: Number of protons
- e: electron charge
- h: Planck constant
- ϵ_0: permitivity of free space
- c: speed of light
- R: Rydberg constant



Hydrogen

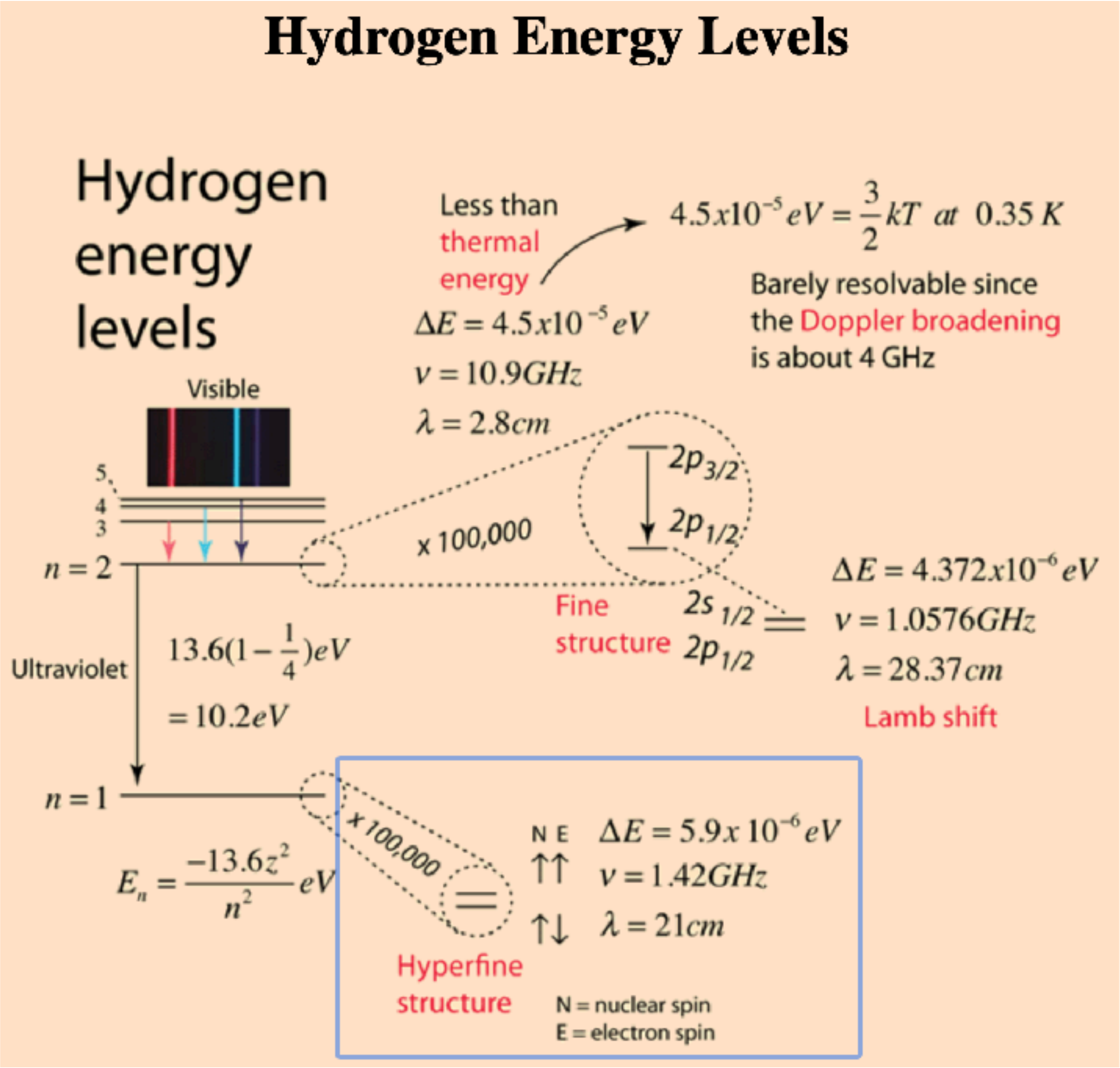
- Hydrogen spectrum comes from changes in n due to electrons jumping between different energy levels in the atom:

$$\frac{1}{\lambda} = \frac{1}{hc}(E_{n=1} - E_{n=2}) = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$



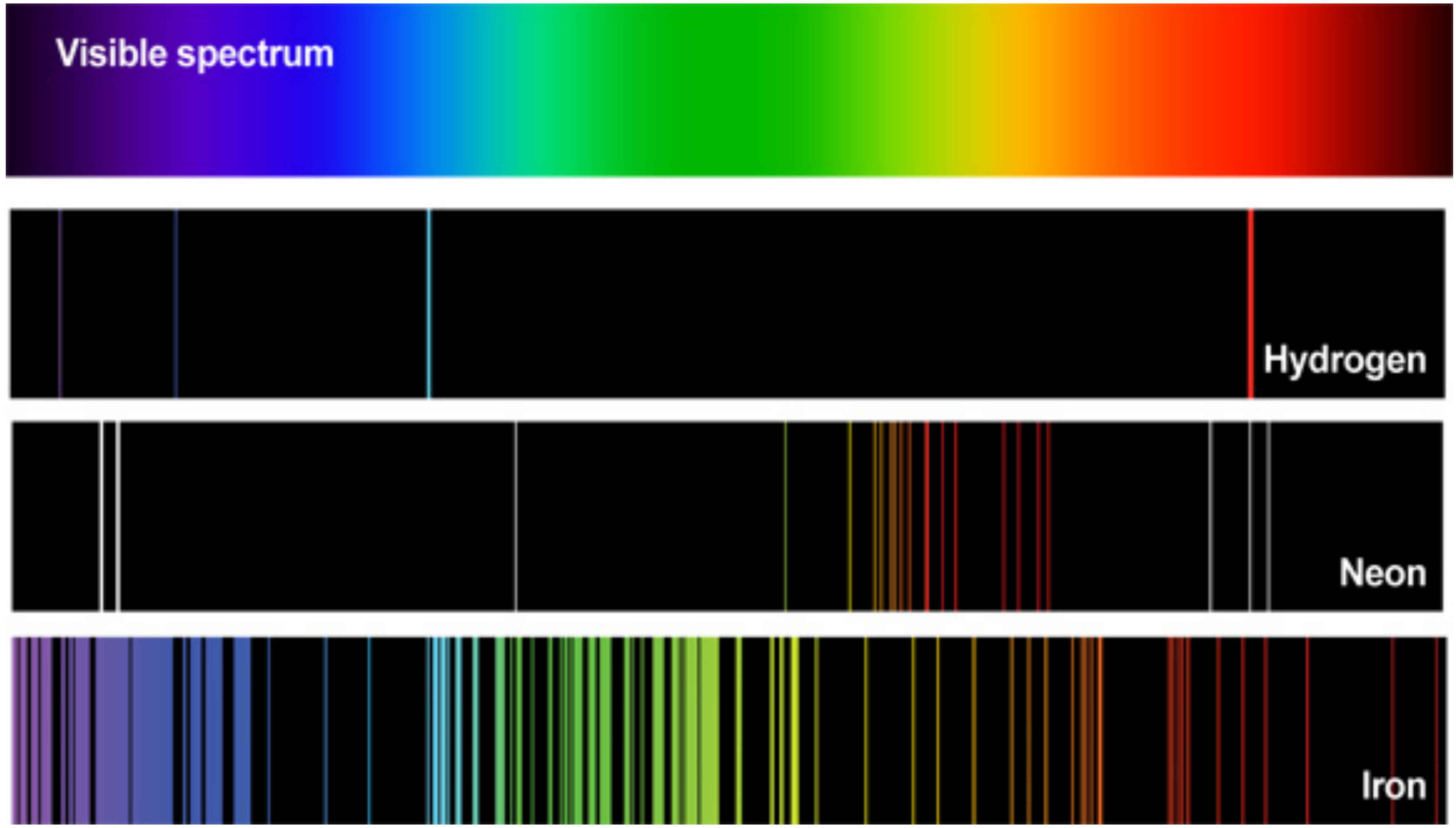
Hydrogen

- Fine structure: interaction between electron spin and electron's orbital angular momentum
- Hyperfine structure: interaction between electron spin and nuclear spin



Atomic Spectra

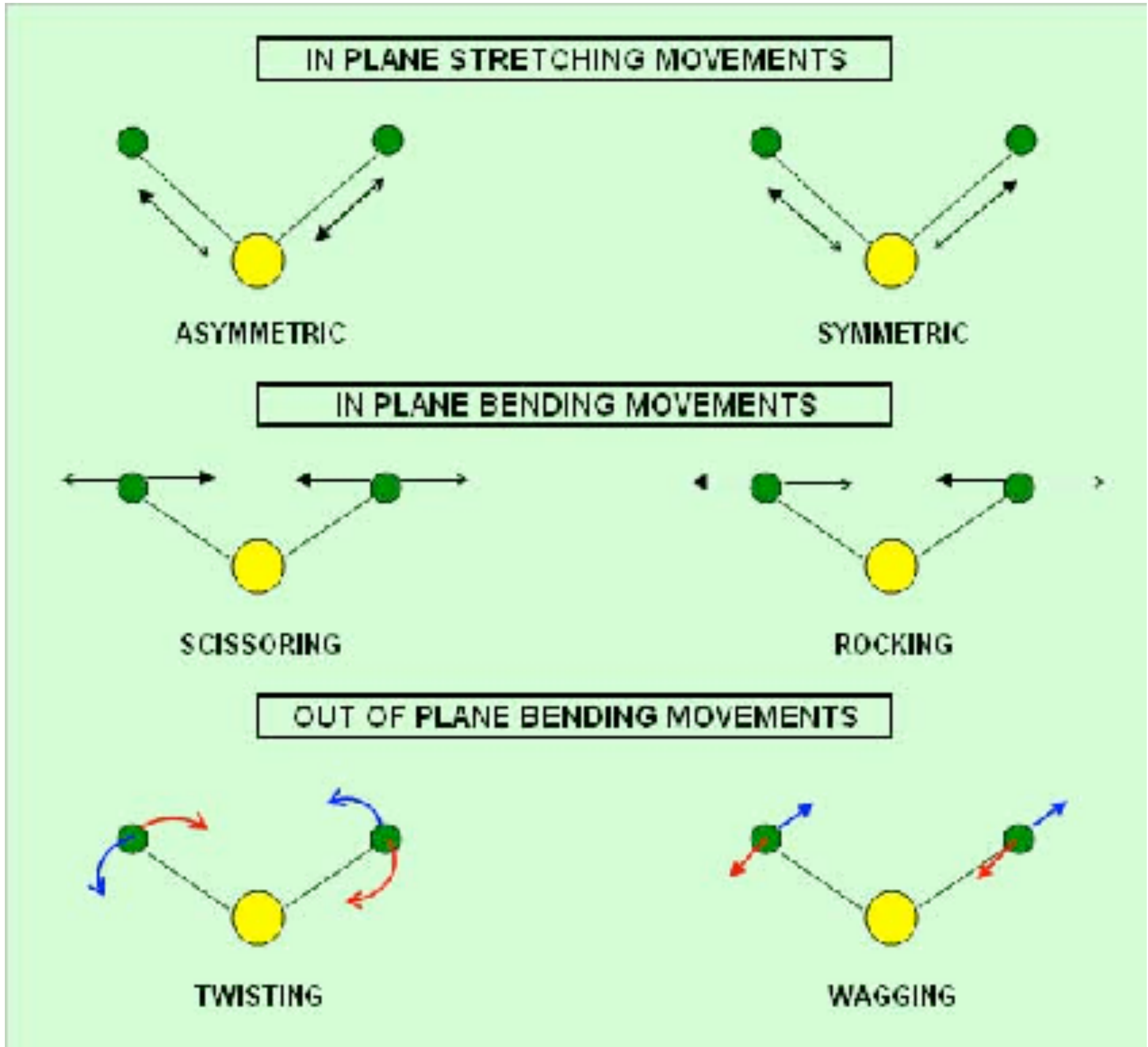
- Atomic lines dominate stellar spectra (except M stars, where you start to get molecules)
- These lines result from electron transitions between energy levels
    - Electron energy levels (and angular momentum) are quantized
- Highest energy transitions: electron energy levels
- Next-highest energy transitions: vibrational
- Smallest energy transitions: rotational



Break

Molecular Spectra

- The nuclei of molecules can move with respect to each other as well:
  - The whole molecule can rotate
  - The individual nuclei can vibrate (the bonds between them can stretch/contract and bend)
- These motions are also quantized (just like electron energy levels)
- Photons (which carry energy) are emitted or absorbed when a molecule moves from one rotation/vibration state to another



Rotation of Molecules

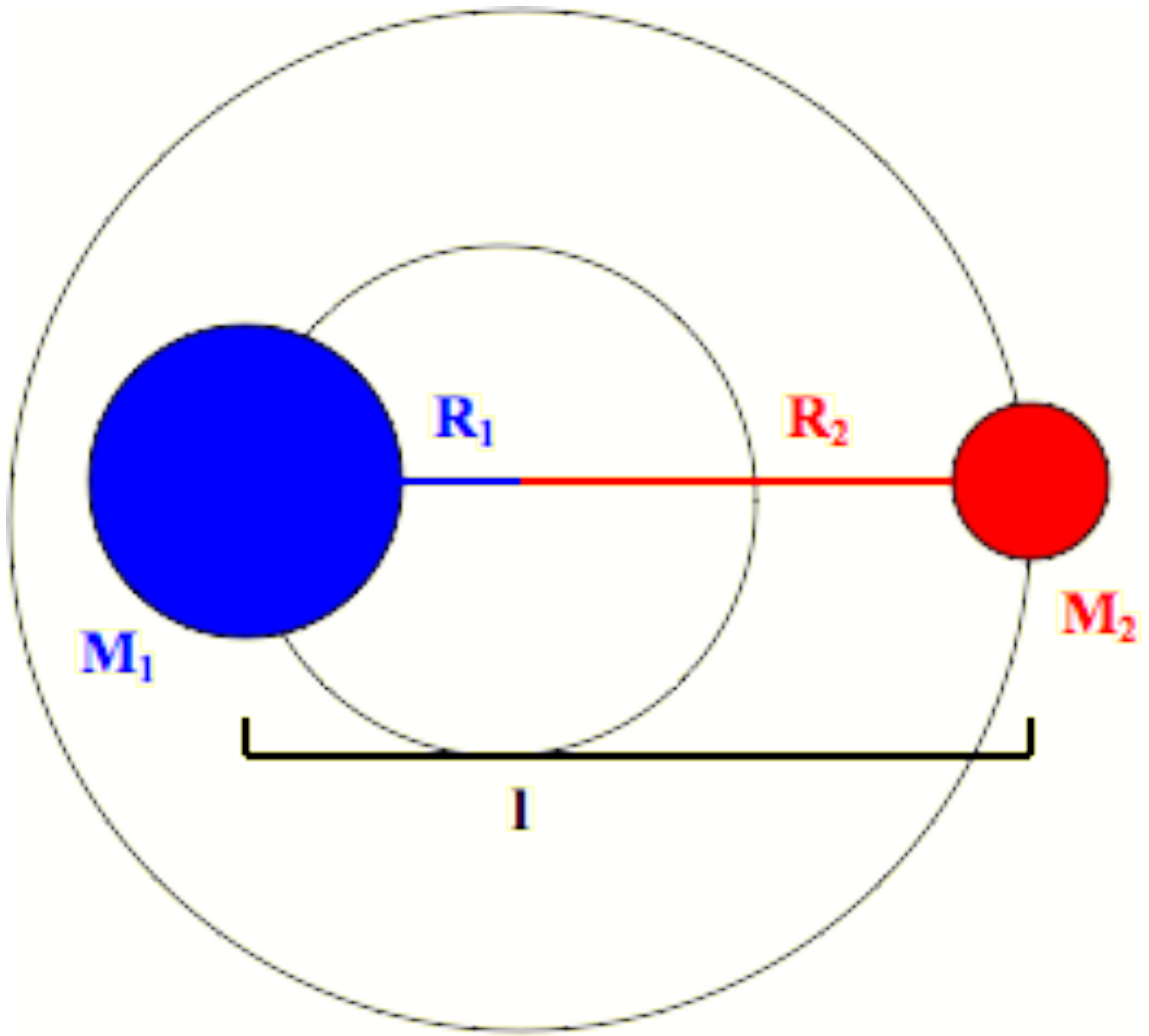
- Consider two (different) atoms in a molecule (CO, say)
- Momentum of inertia for rotation about the center of mass:

$$I = M_1 R_1^2 + M_2 R_2^2$$

- Solving the Schrodinger equation, rotational energy levels are given by:

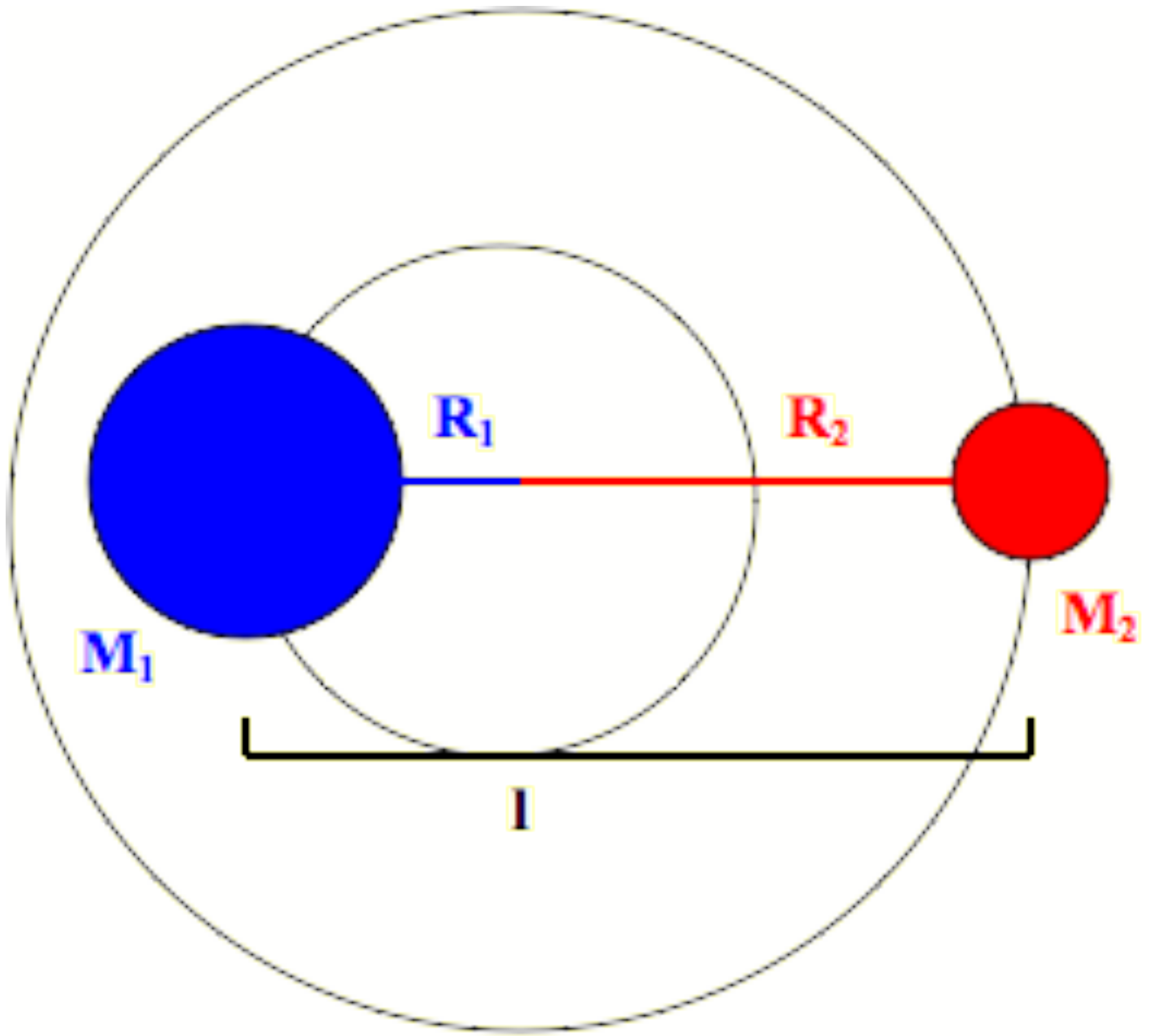
$$E_r = \frac{h}{8\pi^2 c I} J(J+1) = BJ(J+1)$$

- h: Planck's constant
- J: rotational quantum number (an integer)
- B: rotational constant
- c: speed of light



Rotation of Molecules

- $$E_r = \frac{h}{8\pi^2 cI} J(J+1) = BJ(J+1)$$
- Molecular transitions written as upper-lower (1-0, 17-16)
- If J=0, then $E_r = 0$ (no rotational zero-point energy)
- A molecule will only show a pure rotation spectrum if it has a permanent dipole moment
- Rotational constant B has units of inverse-length, wavenumber



Response Card Question

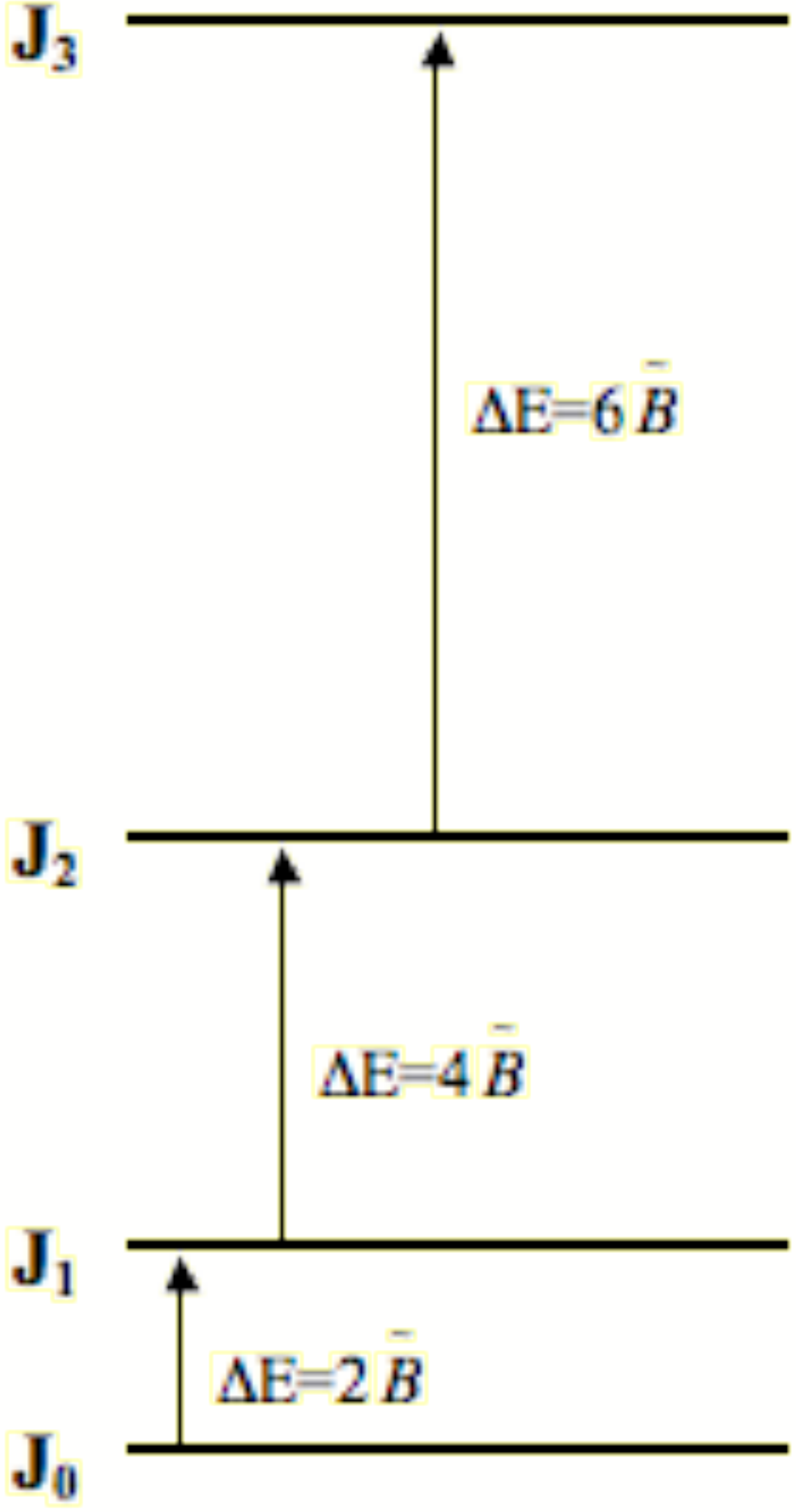
- A certain energy transition has a wavenumber of $10^6 cm^{-1}$. What is the wavelength of the light emitted or absorbed from this transition?
  - (A) — $3x10^{16} cm$
  - (B) — $10^6 cm$
  - (C) — $1 cm$
  - (D) — $10^{-6} cm$
  - (E) — $3x10^{-16} cm$

Rotation of Molecules

- $$E_r = \frac{h}{8\pi^2 cI} J(J+1) = BJ(J+1)$$
- Transitions are allowed when $\Delta J = 1$
- What's the energy of the transition from state J+1 to state J?
- $\Delta E = E(J+1) - E(J) = B(J+1)(J+2) - BJ(J+1)$

  $\Delta E = B(J^2 + 3J + 2 - J^2 - J) = B(2J+2)$

  $\Delta E = 2B(J+1)$
- So we expect lines at 2B, 4B, 6B, 8B, …
- Rotational lines are equally spaced in energy (and so frequency)



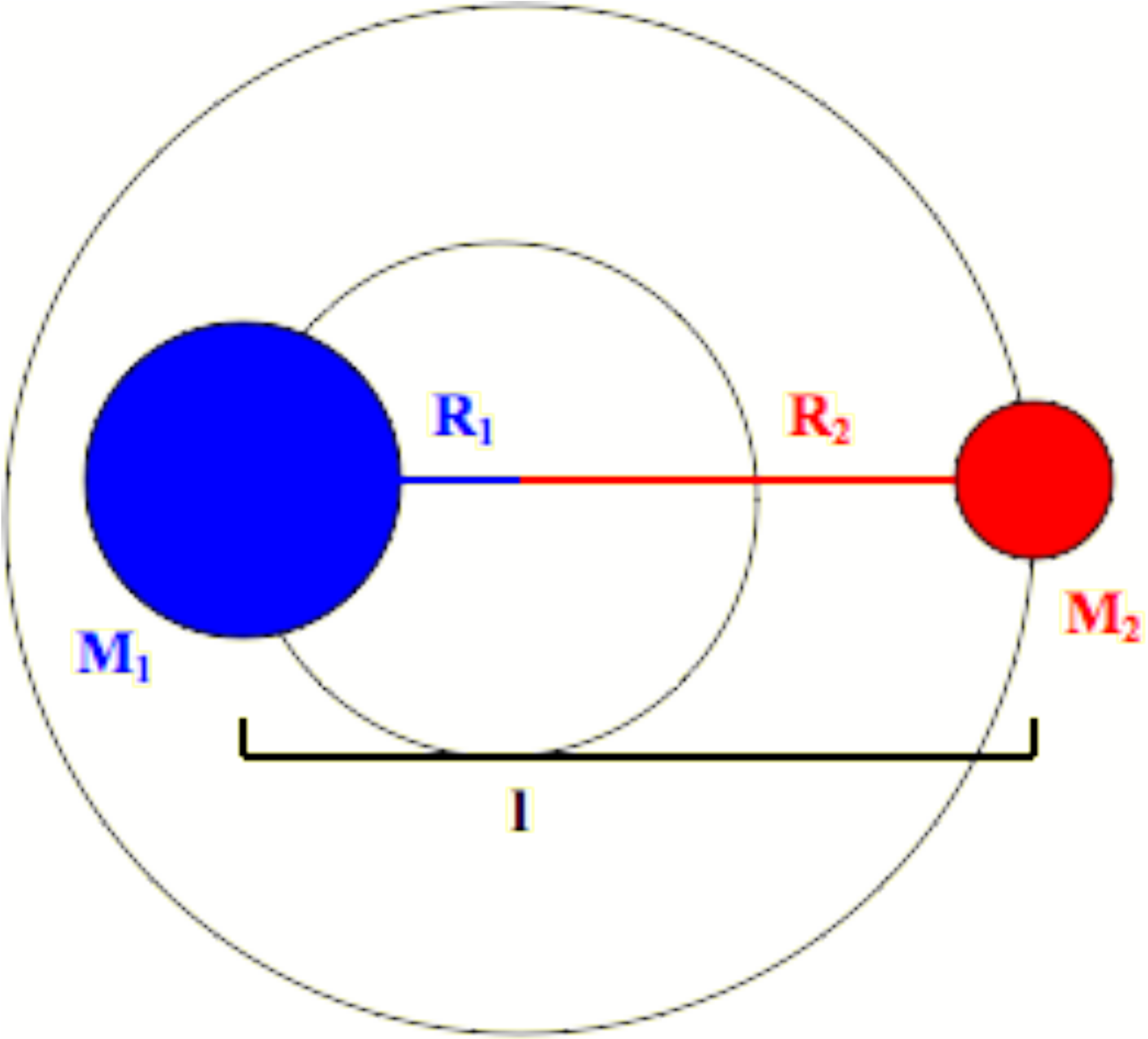
Order of magnitude

- $I = M_1 R_1^2 + M_2 R_2^2$ $\quad m_p = 1.7x10^{-27} kg$

  $E_r = \frac{h}{8\pi^2 c I} J(J+1) = BJ(J+1)$ $\quad h = 6.7x10^{-34} Js$

  $\Delta E = 2B(J+1)$

- Consider the molecule NO (nitrogen+oxygen)
- (1) (order-of-magnitude) what is the bond length R ($R_1 + R_2$) between the two atoms? (hint: in a molecule, you can think of it as the two electron clouds touching each other)
- (2) What is the moment of inertia (I) for NO? (hint: order of magnitude, so assume $R_1 \approx R_2 \approx R$)
- (3) What is B for NO?
- (4) What type of light would be emitted/absorbed for NO rotational transitions? (e.g., visible, X-ray, …)



Order of magnitude

- $I = M_1 R_1^2 + M_2 R_2^2$

  $$E_r = \frac{h}{8\pi^2 cI} J(J+1) = BJ(J+1)$$

  $$\Delta E = 2B(J+1)$$

- Consider the molecule NO (nitrogen+oxygen)
- (1) (order-of-magnitude) what is the bond length R (R1+R2) between the two atoms? (hint: in a molecule, you can think of it as the two electron clouds touching each other)
- The typical size of an atom is about half an angstrom (round up to 1), 1 angstrom is 10^{-10} m
- (2) What is the moment of inertia (I) for NO?
- Nitrogen has an atomic number of 7, Oxygen 8, assume the atomic weight is double that, 14 nucleons for N, 16 for O, or 30 total.

Order of magnitude

- Since we assume the bond length is about the same for both:
- $I = (M_1 + M_2)R^2 = 30m_pR^2$
- Mass of the proton is 10^{-27} kg, R is 1 angstrom, $10^{-10}m$
- $I = 30(10^{-27}kg)(10^{-10}m)^2 = 3x10^{-46}m^2kg$
- (3) What is B for NO?
- $B = \frac{h}{8\pi^2 cI} = \frac{(7x10^{-34}m^2kg/s)}{8\pi^2(3x10^8 m/s)(3x10^{-46}m^2kg)} = \frac{10^{-34}}{100x10^{-38}}m^{-1} = \frac{1}{10^{-2}}m^{-1} = 100m^{-1}$
- (4) Let's convert this to a wavelength by inverting:
- $Wavelength = \frac{1}{Wavenumber} = \frac{1}{100m^{-1}} = 0.01m = 1cm$ Microwave!

For next time

- Reading: de Pater & Lissaeuer Chaper 4, section 4.1, 4.2 (intro only)
- Homework 3 due Wednesday, September 28 at 11:59pm