ASTR 620: Planetary Processes
Professor Eric Nielsen

Lecture 3: Tides, Hill Sphere, Lagrange Points

Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I’ll let you know in advance if we’ll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Homework 1 will be due this Monday, August 29th, at 11:59:59pm on Canvas

Review of the last class

- Sedna has a semi-major axis of about 500 AU and eccentricity about 0.9: what is the perihelion and aphelion of Sedna?
  - (A) — 50 AU perihelion, 950 AU aphelion
  - (B) — 500 AU for both perihelion and aphelion
  - (C) — 450 AU perihelion, 550 AU aphelion
  - (D) — 5 AU perihelion, 995 AU aphelion
  - (E) — 1000 AU for both perihelion and aphelion

Review of the last class

- Saturn has a 10 AU semi-major axis. Therefore, it has an orbital period of:
  - (A) — 3 years
  - (B) — 10 years
  - (C) — 30 years
  - (D) — 100 years
  - (E) — 300 years

Review of the last class

- Escape velocity is given by:
  - (A) — $v_{esc}(r) = \sqrt{\frac{2GM_\odot}{r}}$
  - (B) — $v_{esc}(r) = \sqrt{\frac{GM_\odot}{r}}$
  - (C) — $v_{esc}(r) = \frac{2GM_\odot}{r}$
  - (D) — $v_{esc}(r) = \frac{GM_\odot}{r}$

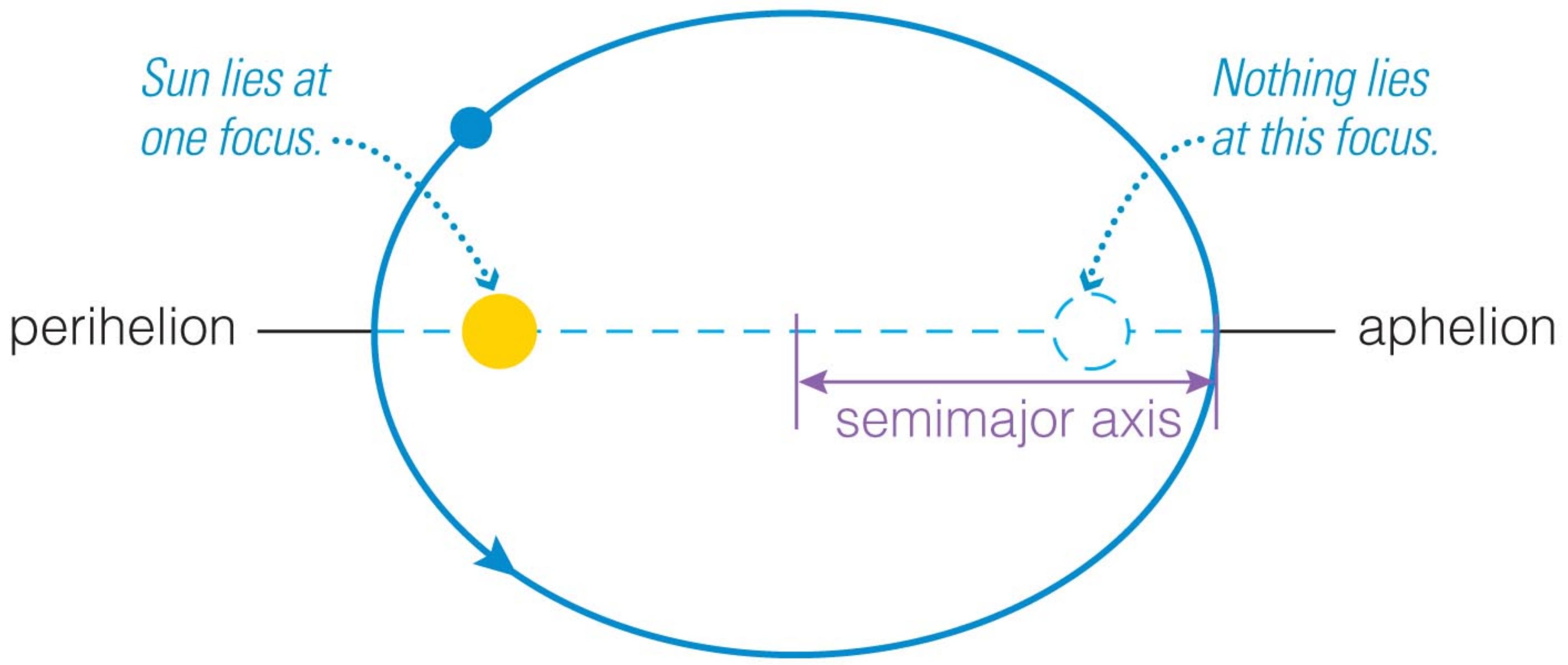
Review of the last class

- Which planet is traveling fastest?
  - (A) — A planet with a 1 AU semi-major axis, eccentricity of 0, when it's 1 AU from the star
  - (B) — A planet with a 10 AU semi-major axis, eccentricity of 0.9, when it's 1 AU from the star
  - (C) — A planet with a 0.53 AU semi-major axis, eccentricity of 0.9, when it's 1 AU from the star
  - (D) — They're all the same distance from the star, so they're all traveling at the same speed

Review of the last class

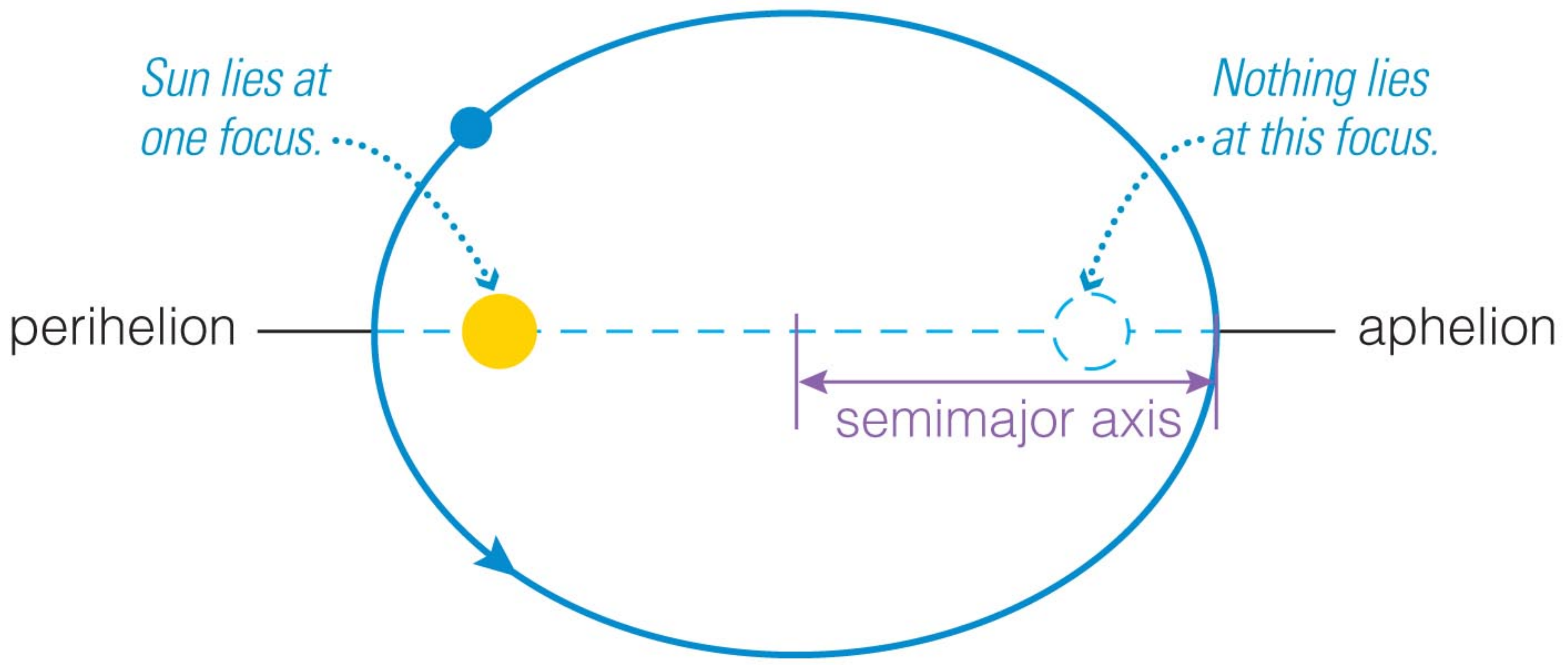
- How would a Hohmann Transfer orbit to Mercury from Earth differ from a Hohmann Transfer orbit to Jupiter from Earth?
  - (A) — It would be the same
  - (B) — At Earth, would have to slow down, then slow down again at Mercury
  - (C) — At Earth, would have to speed up, then slow down at Mercury
  - (D) — At Earth, would have to slow down, then speed up at Mercury
  - (E) — At earth, would have to speed up, then speed up again at Mercury

Orbital elements



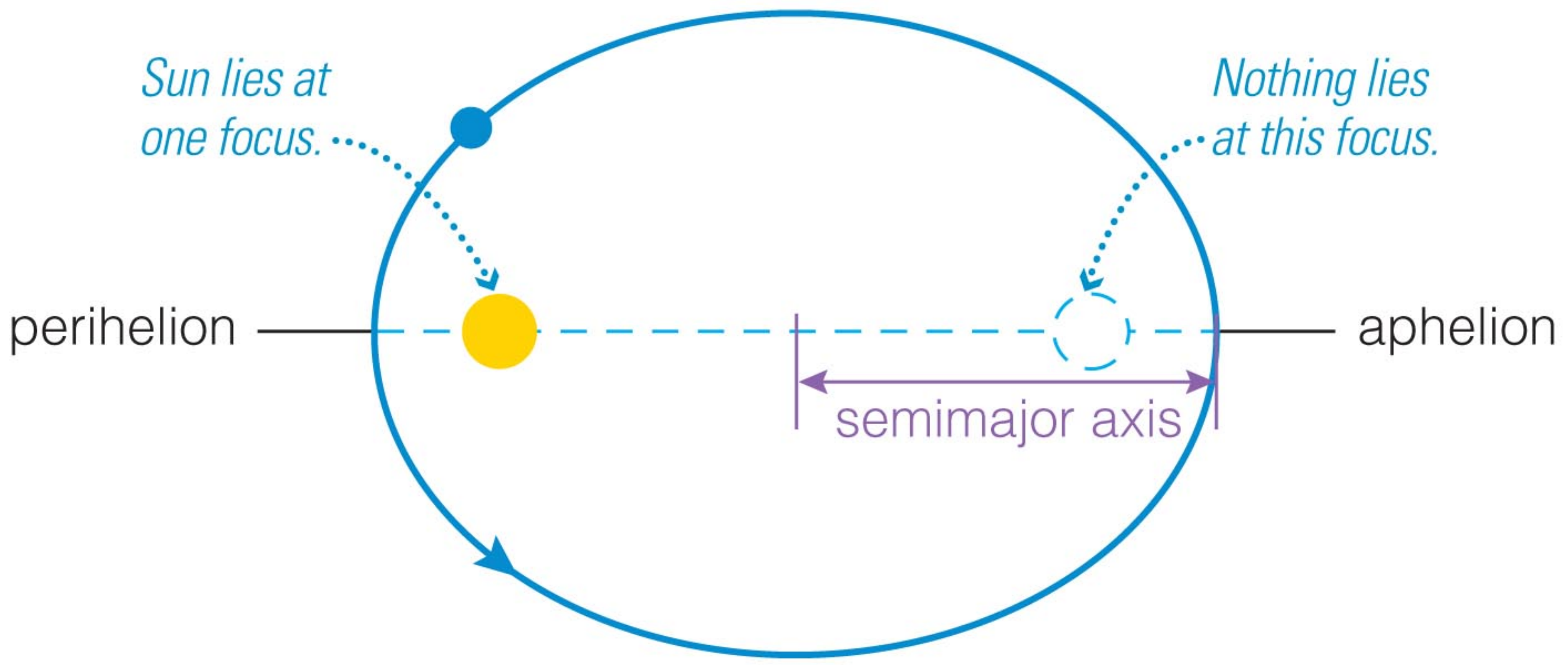
Orbital elements

semi-major axis (a)



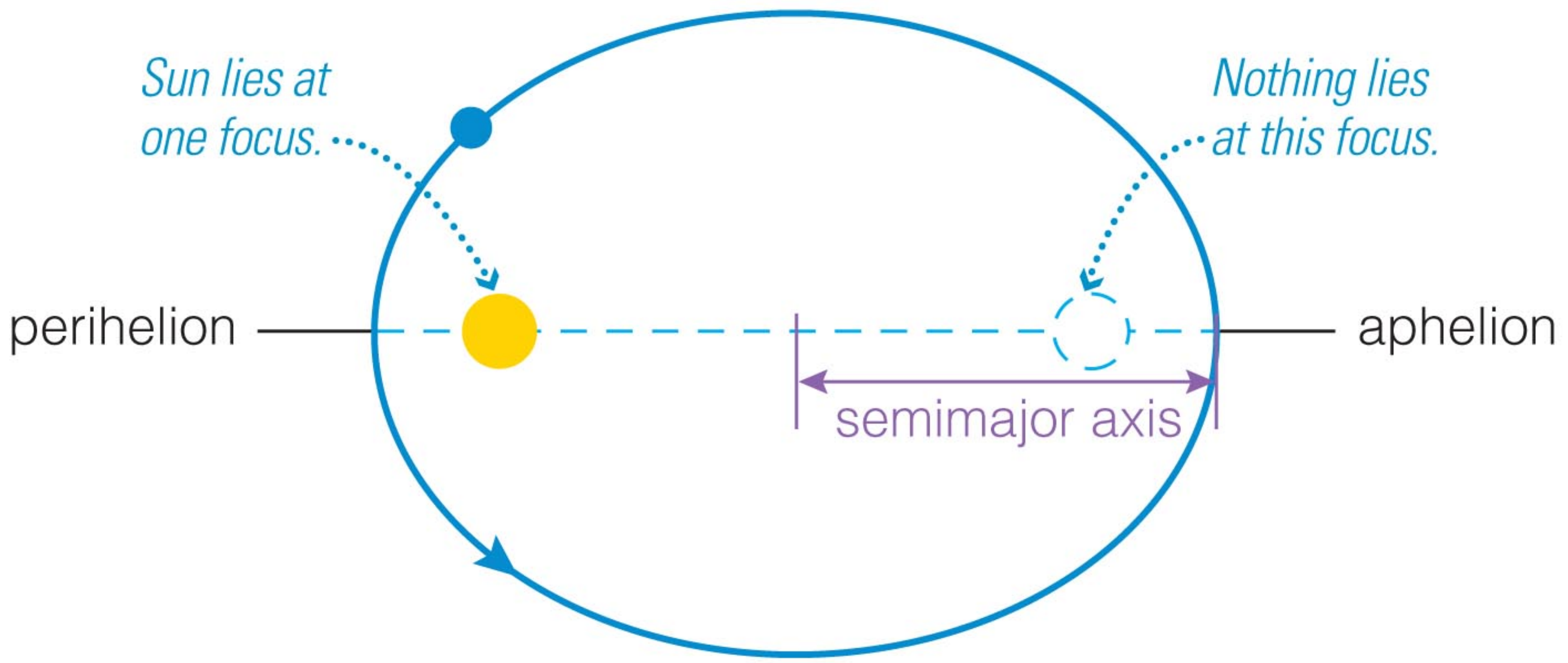
Orbital elements

semi-major axis (a)
eccentricity (e)



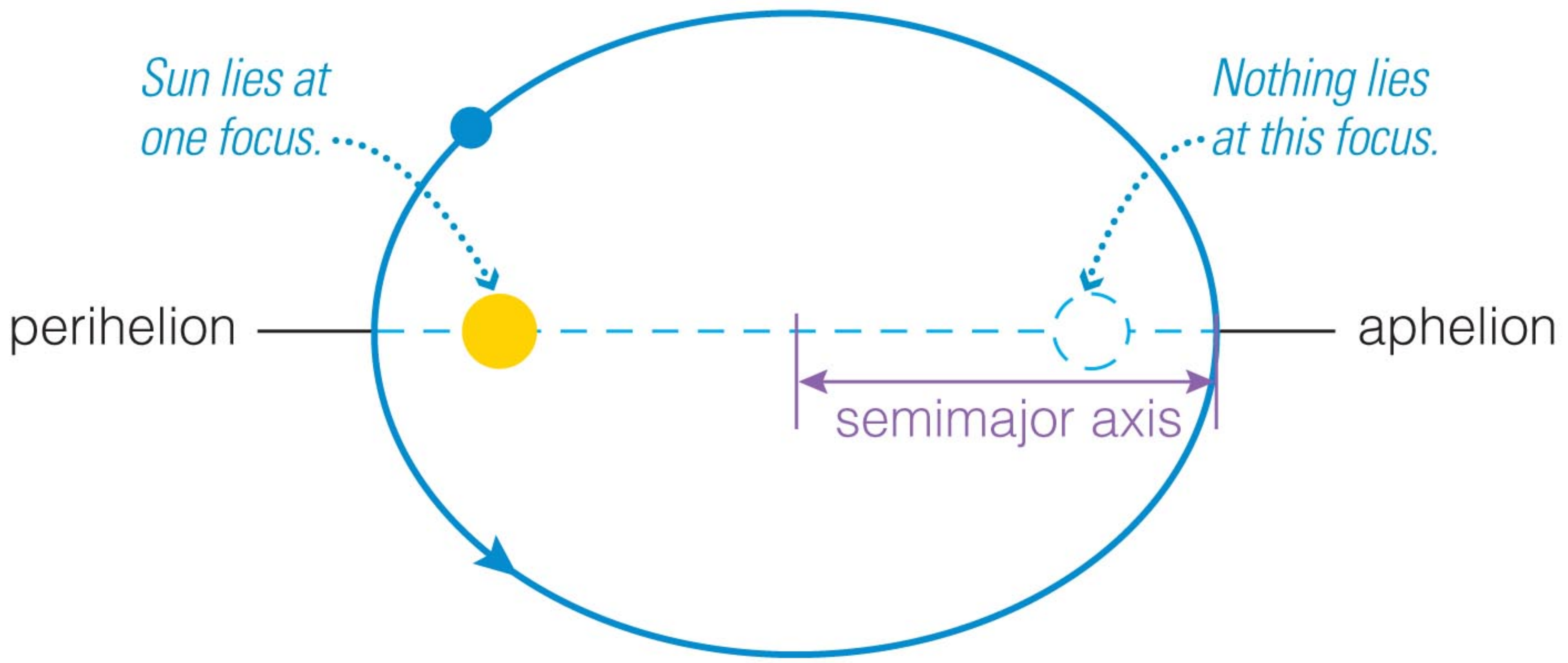
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)



Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)



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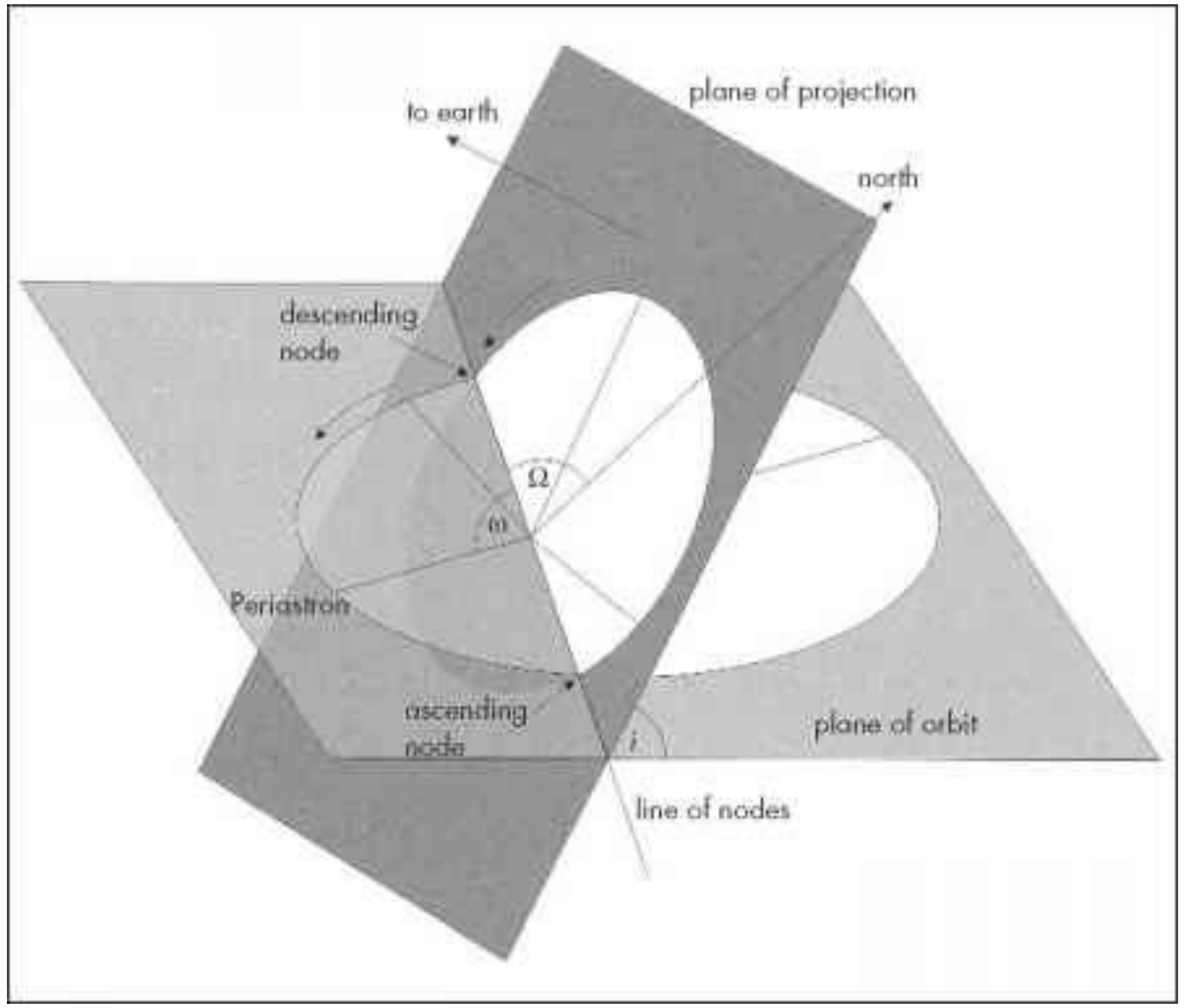
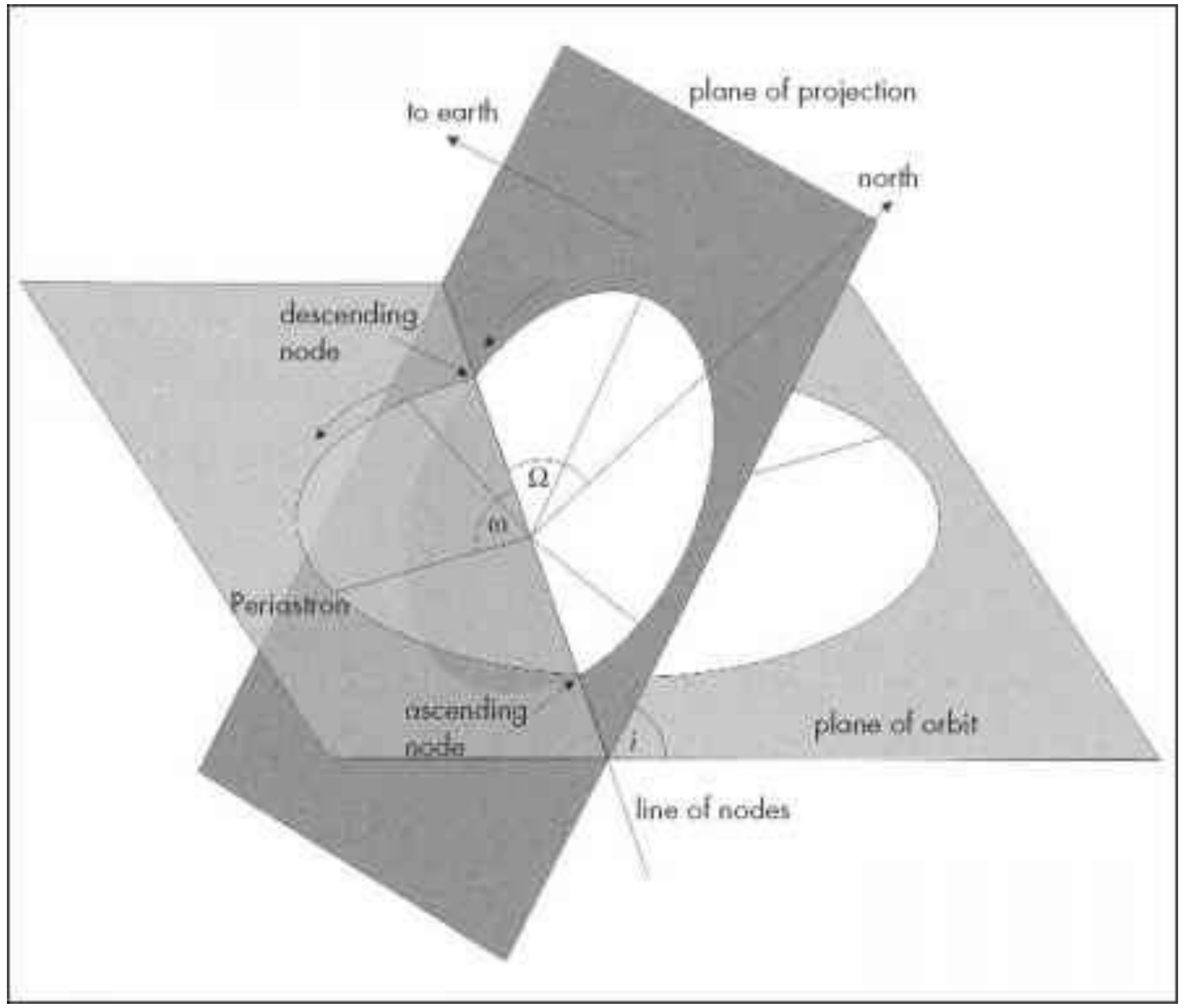


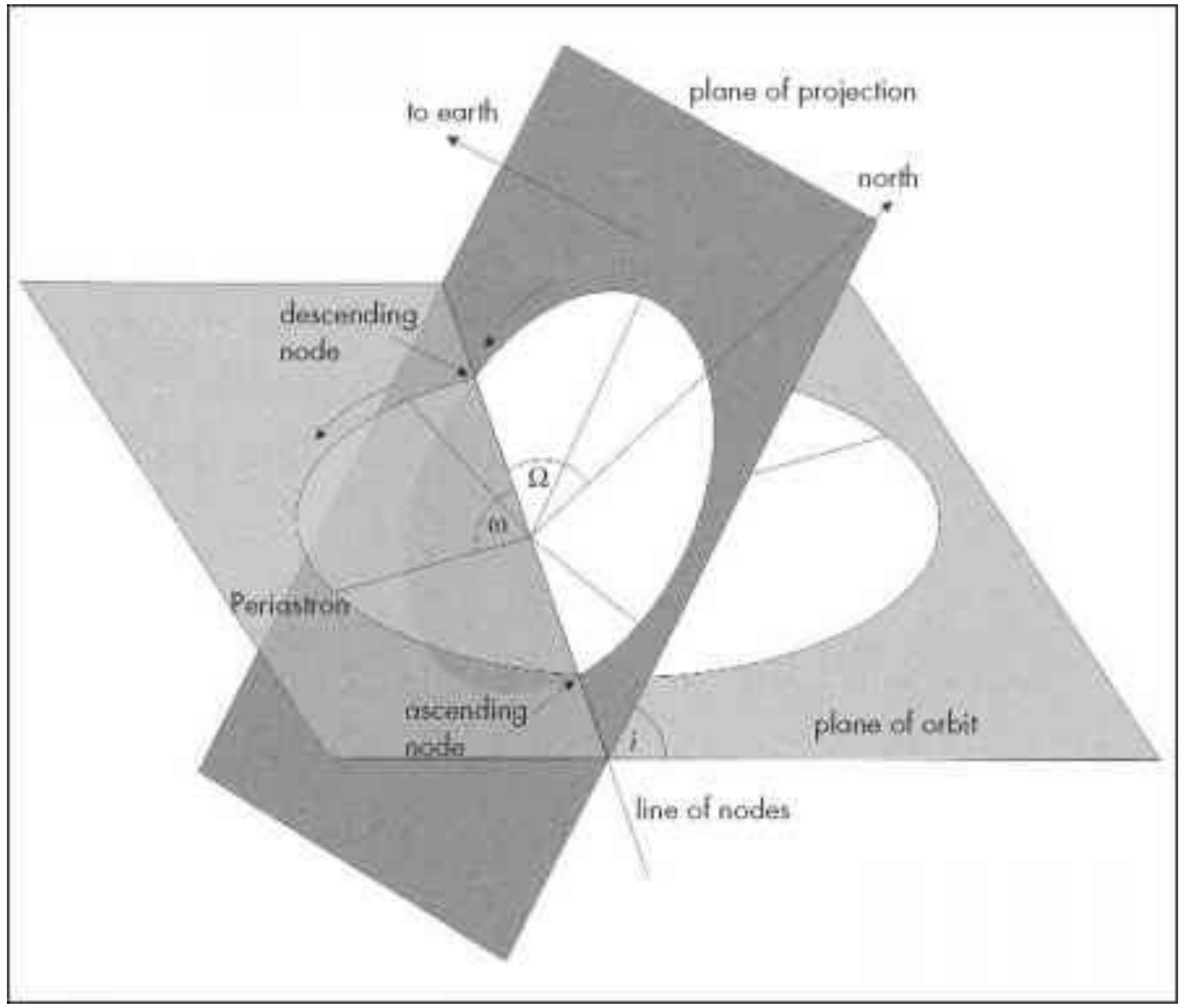
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)



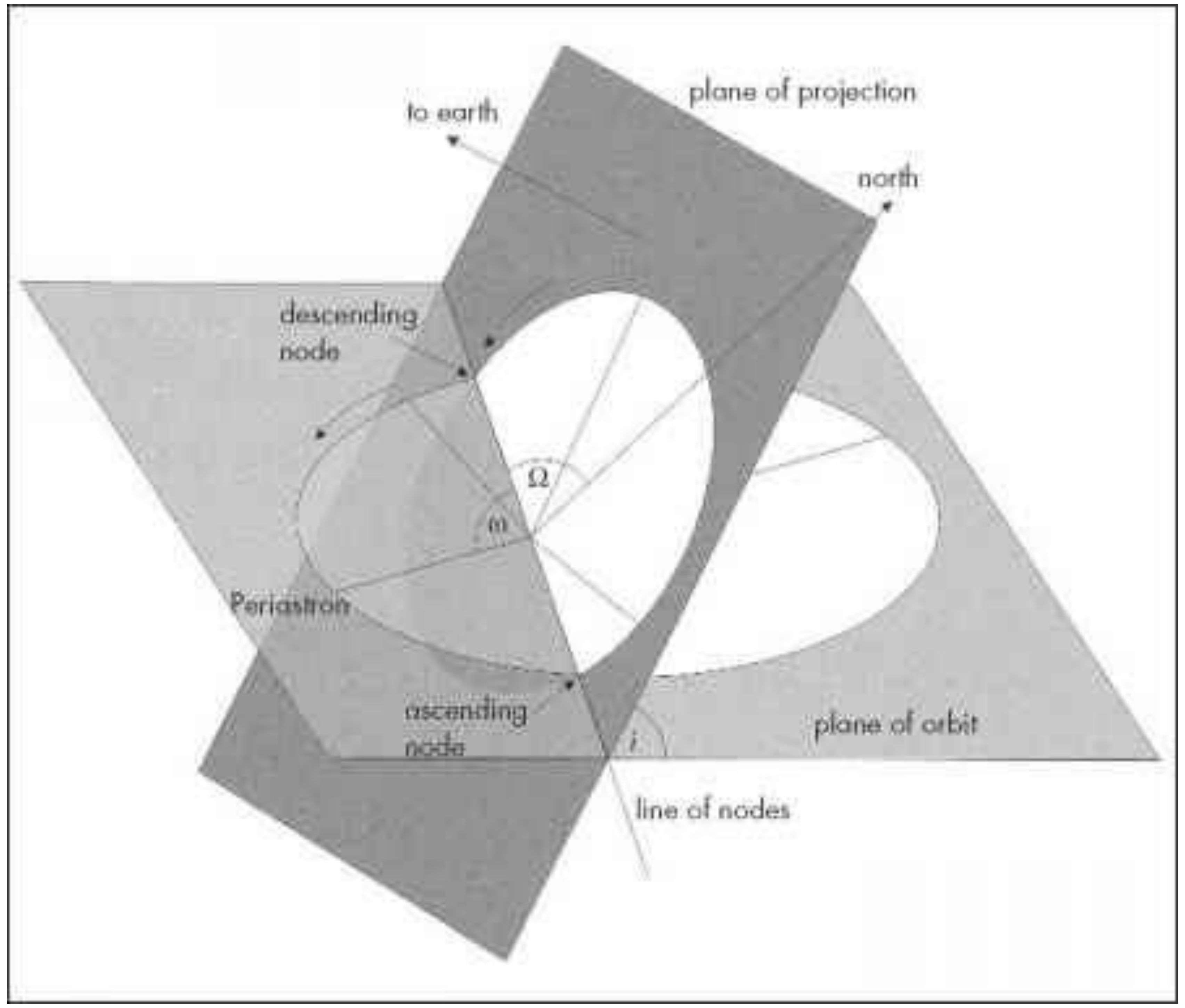
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)
Position Angle of Nodes (Ω)



Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)
Position Angle of Nodes (Ω)
Argument of Periastron (ω)



Anomalies

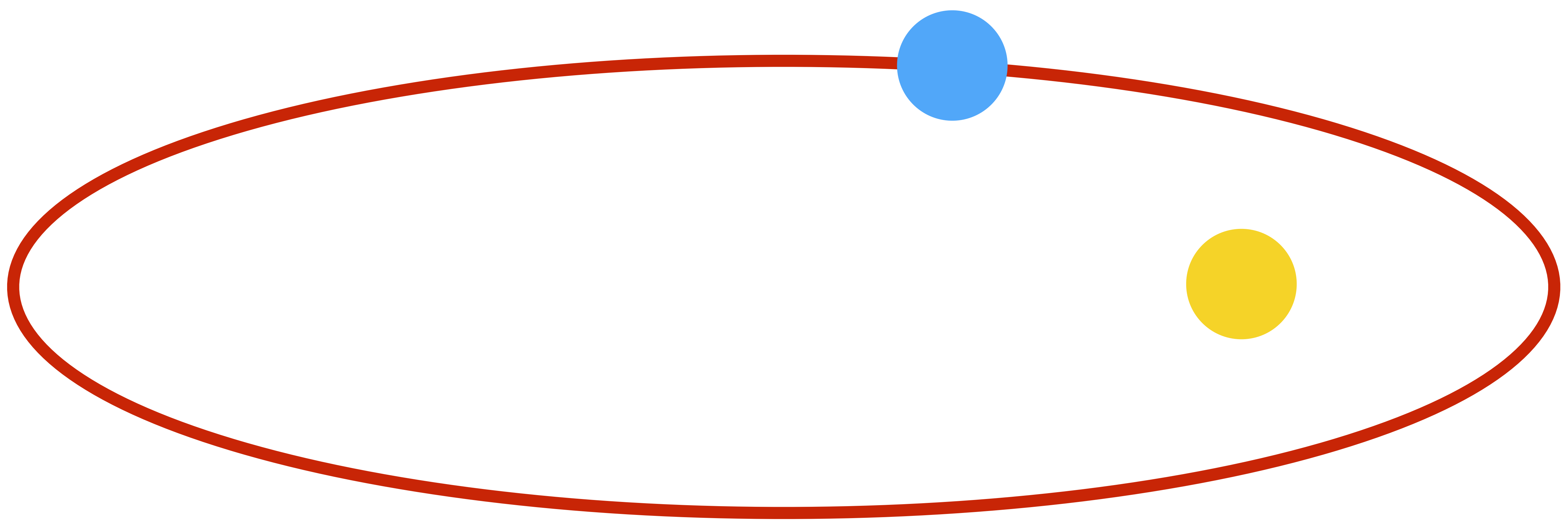
Anomalies

Mean Anomaly (*M*, 0-2π) — Fraction of a period since last periastron

Anomalies

Mean Anomaly (M, 0-2π) — Fraction of a period since last periastron

Eccentric Anomaly (E, 0-2π) — Angle between periastron, center of ellipse, and (kind of) planet



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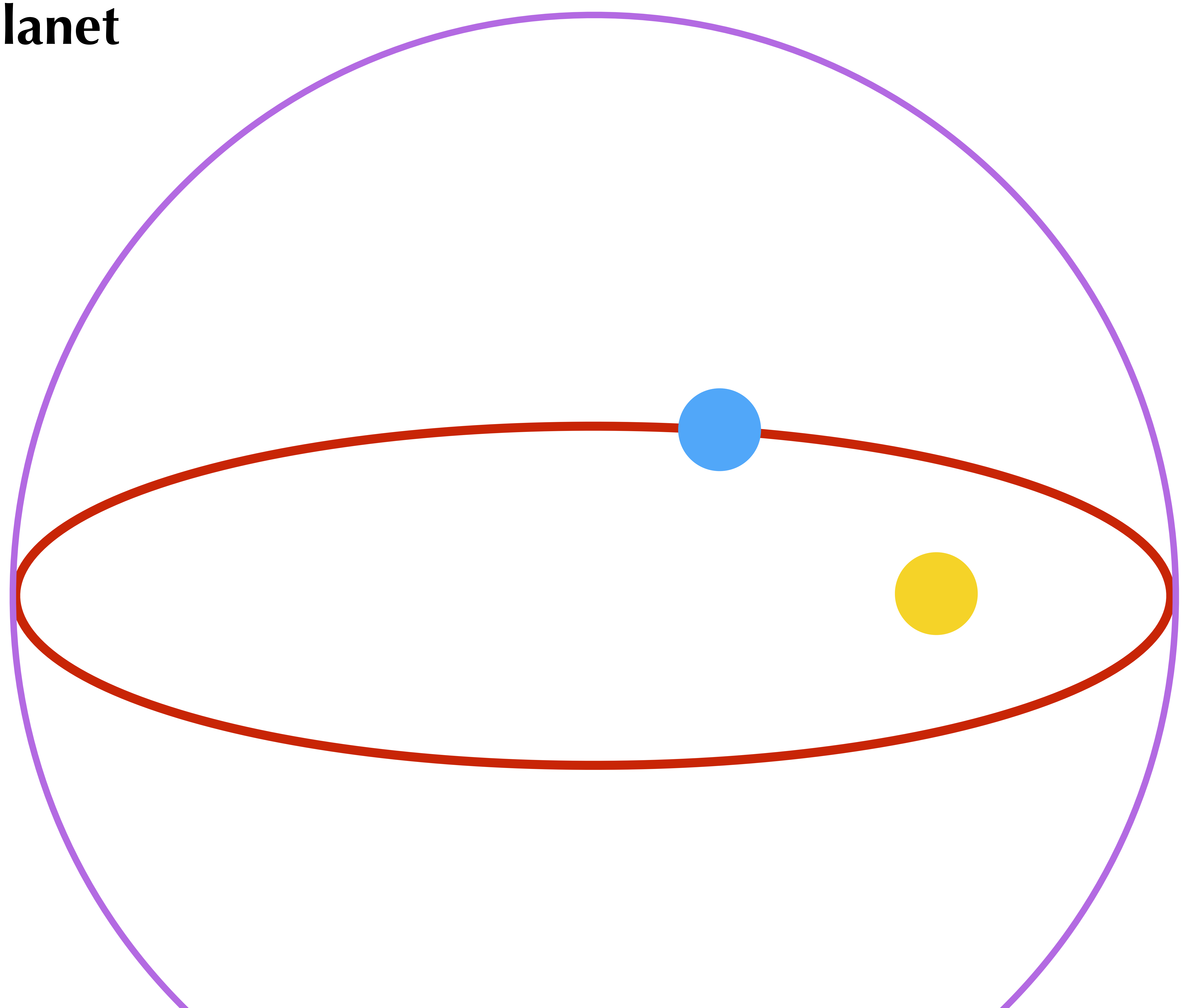

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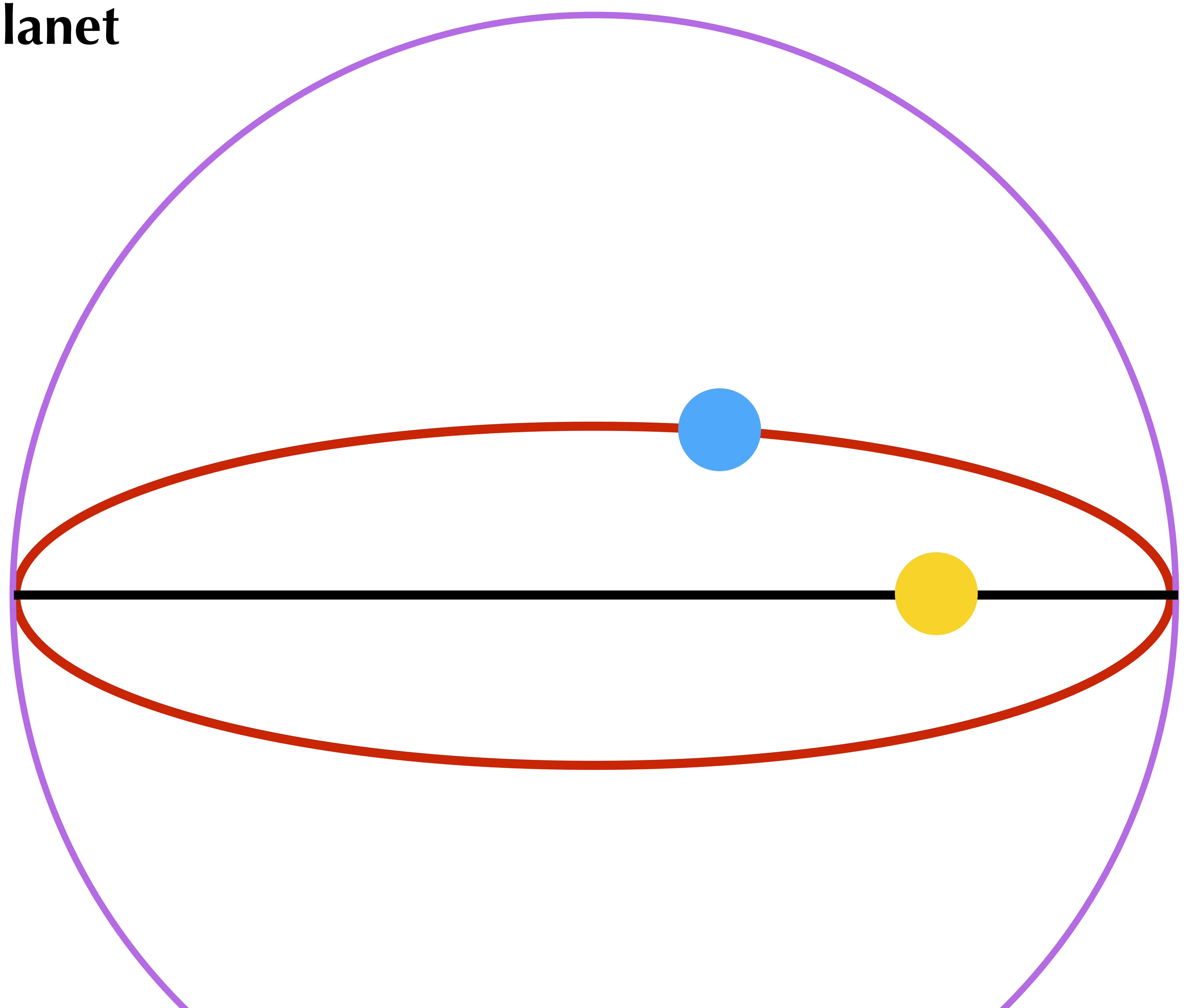

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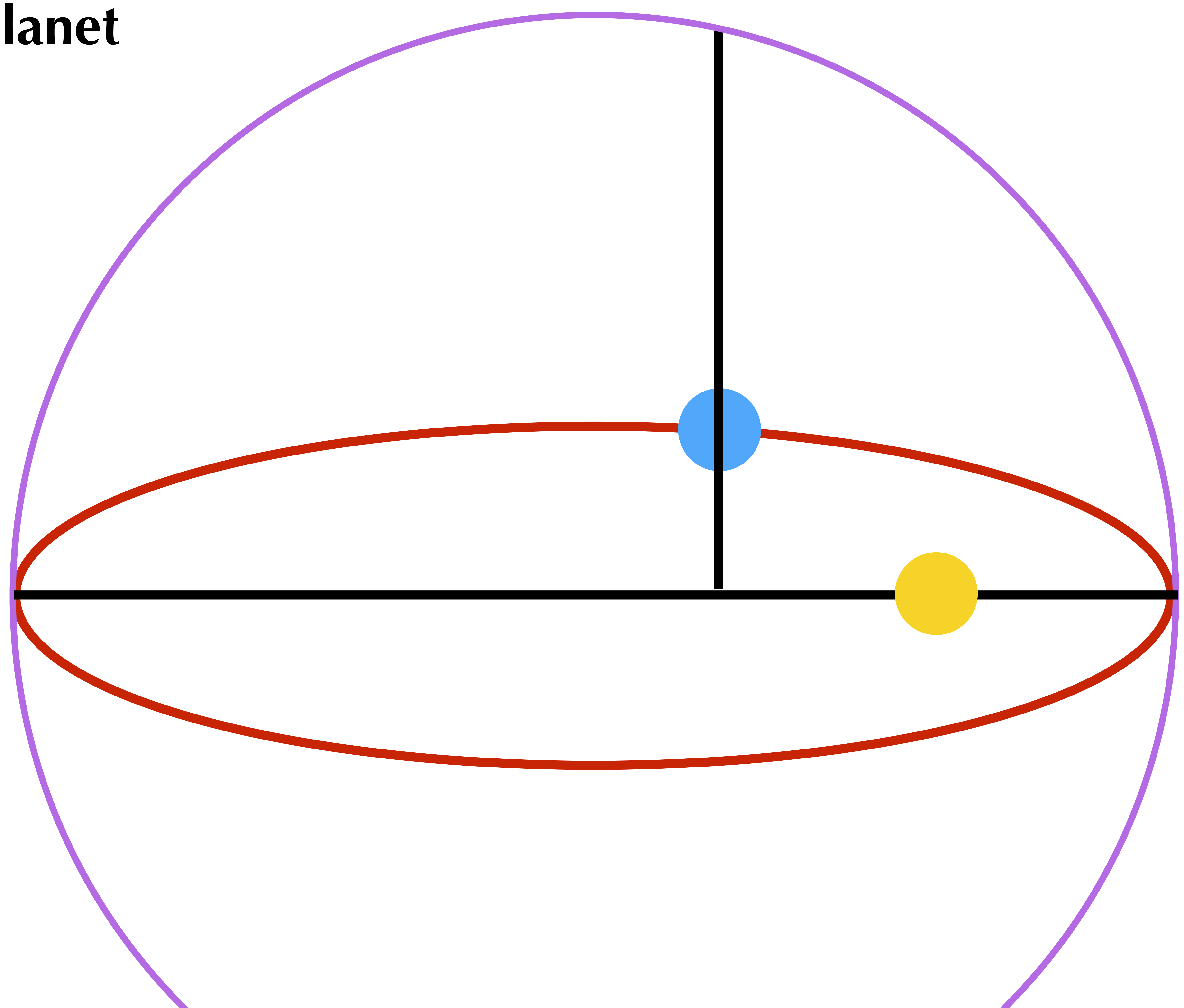

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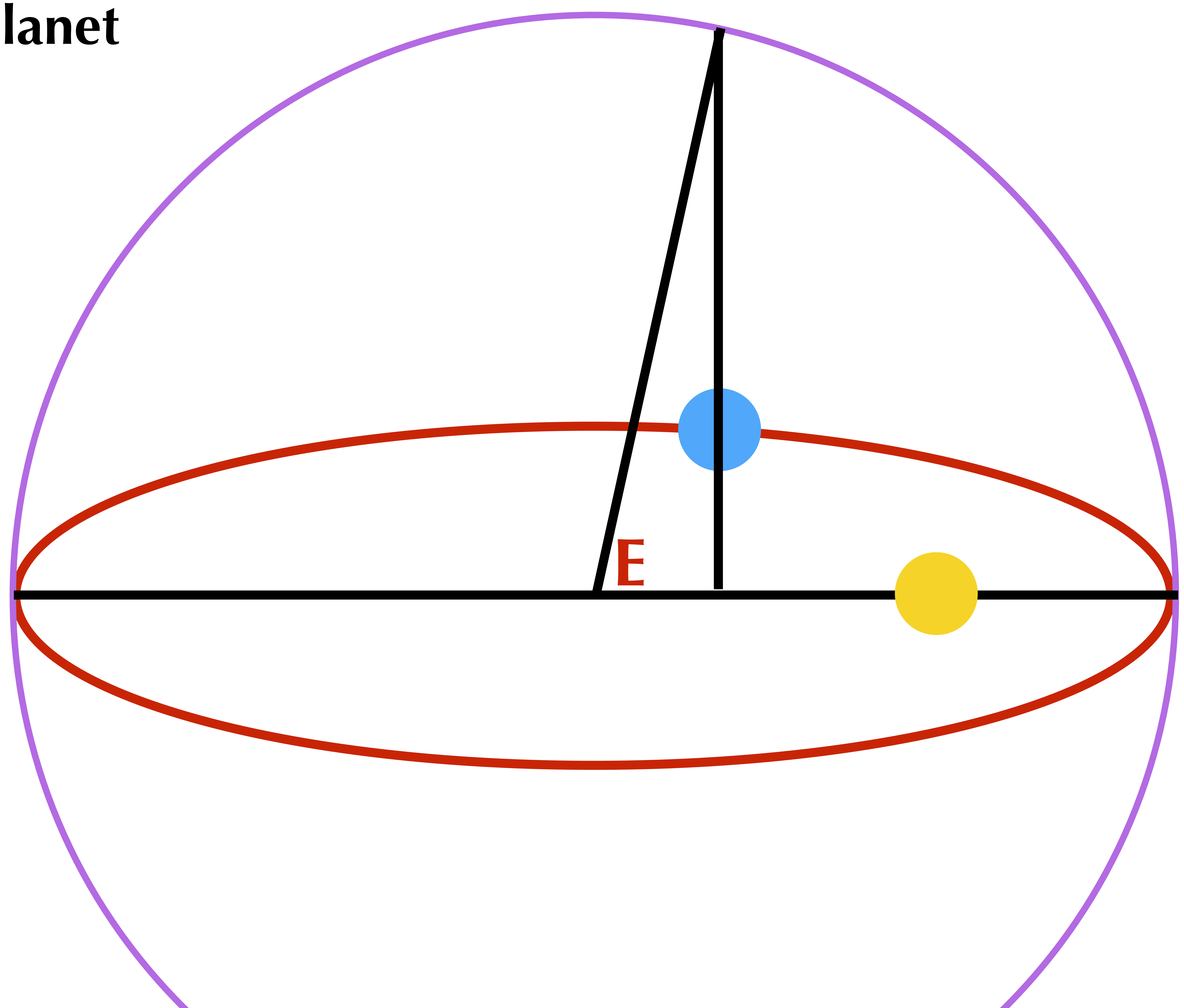

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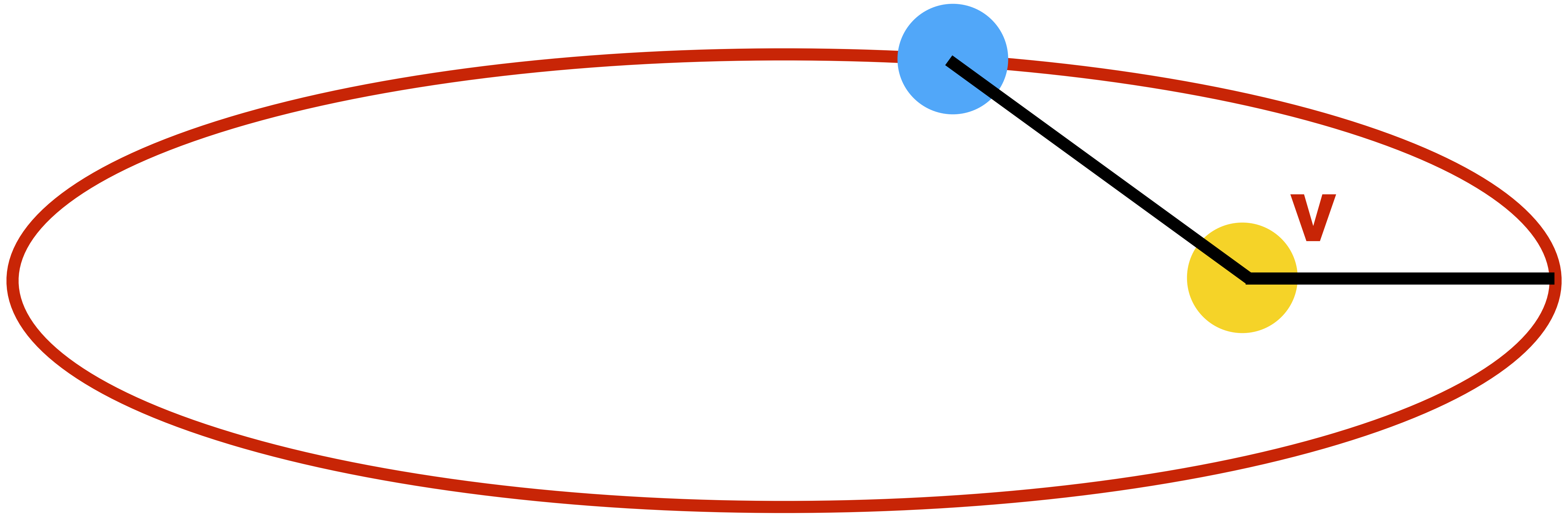
Mean Anomaly (M, 0-2π) — Fraction of a period since last periastron

Eccentric Anomaly (E, 0-2π) — Angle between periastron, center of ellipse, and (kind of) planet

True Anomaly (ν, 0-2π) — Angle between periastron, star, and planet



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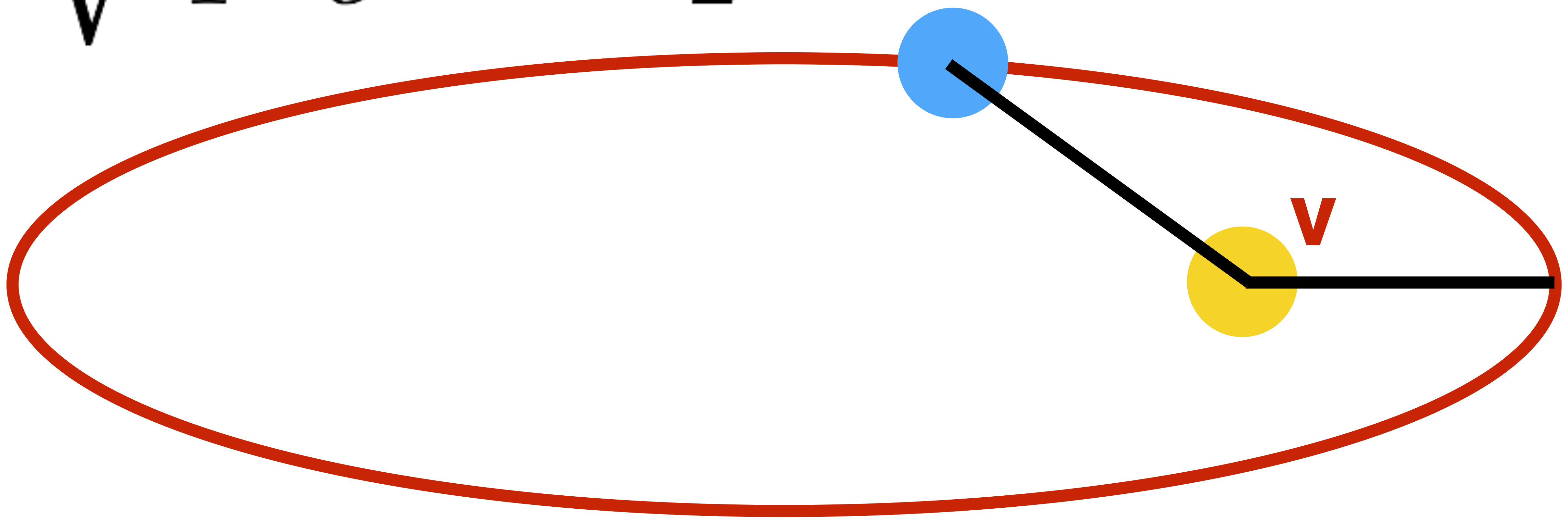
$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\,\tan\frac{E}{2}$$



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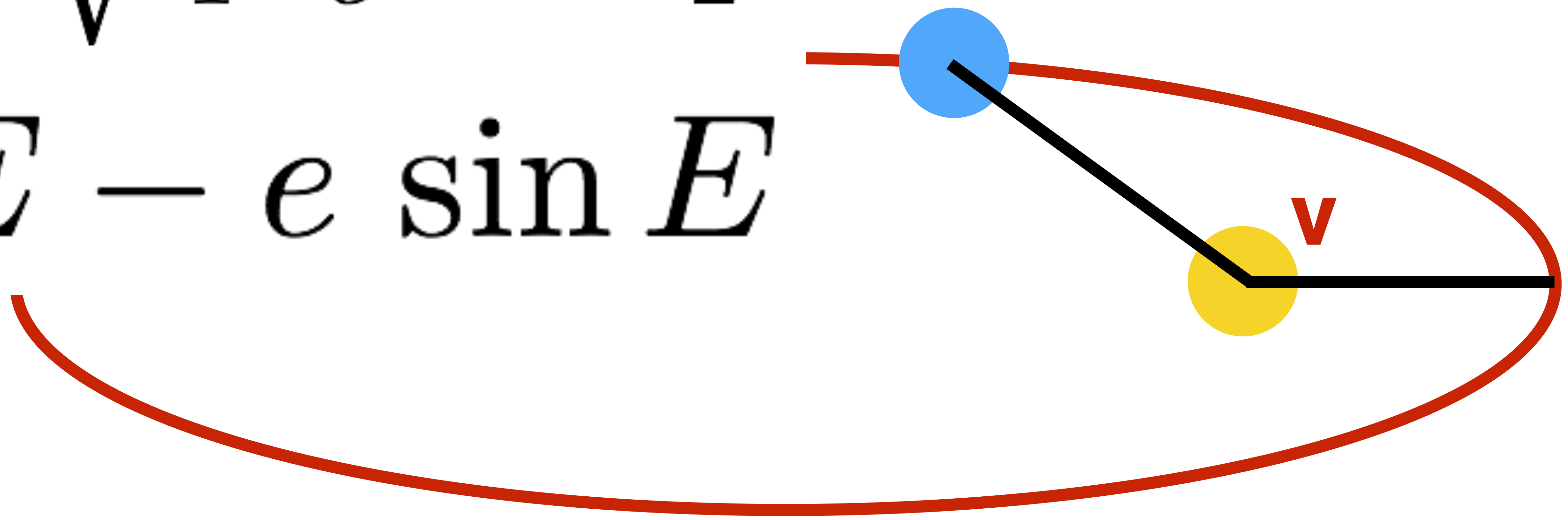
$$M = E - e\sin E$$



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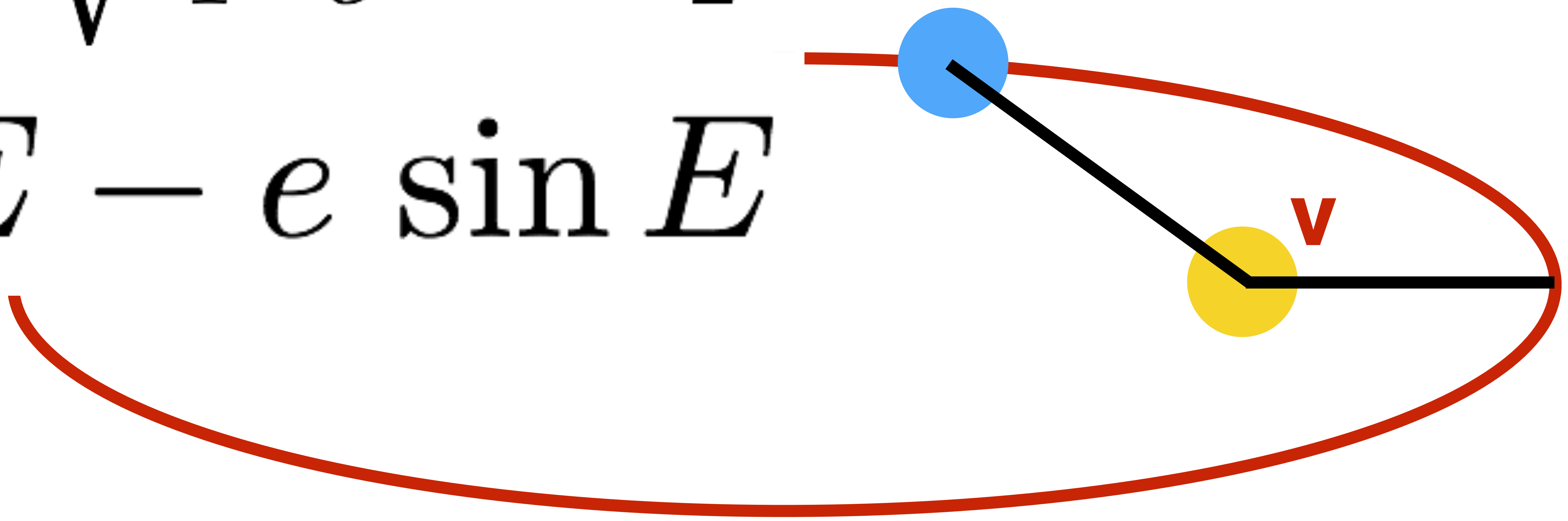


$$\Delta RA = \frac{a\,(1-e^2)}{1+e\,\cos(\nu)}\,(\cos\nu\cos i\cos\Omega\sin\omega\ +\ \cos\nu\sin\Omega\cos\omega\ +\ \sin\nu\cos i\cos\Omega\cos\omega\ -\ \sin\nu\sin\Omega\sin\omega)$$

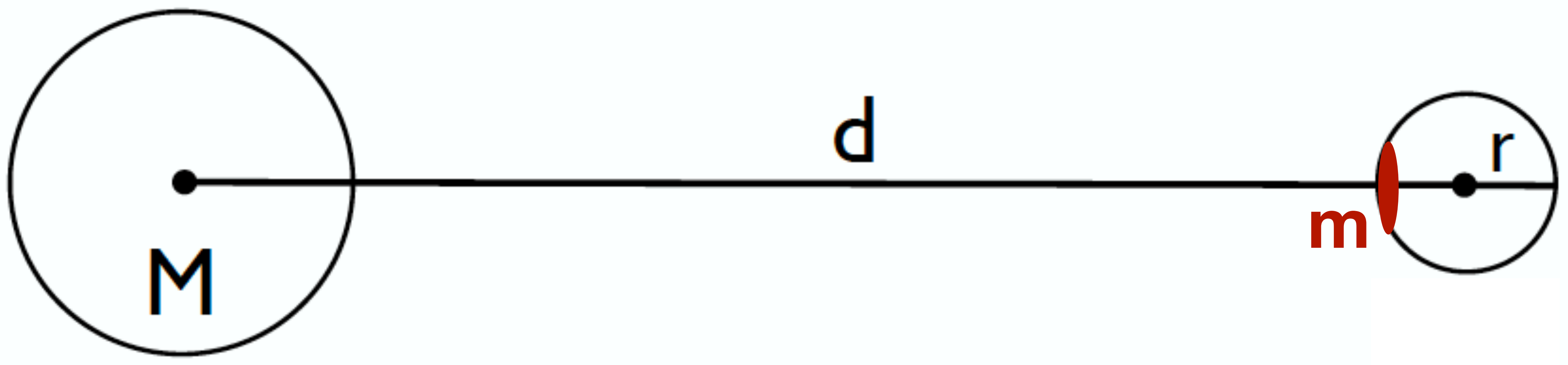
$$\Delta Dec = \frac{a(1-e^2)}{1+e\cos\nu}\,(-\cos\nu\cos i\sin\Omega\sin\omega + \cos\nu\cos\Omega\cos\omega - \sin\nu\cos i\sin\Omega\cos\omega - \sin\nu\cos\Omega\sin\omega)$$

Tidal Force

- The difference in gravitational force from one part of an object to another part is called the Tidal Force

$$F_{tidal} = \frac{GMm}{d^2} - \frac{GMm}{(d+r)^2}$$

- M: mass of the object causing the tide (say, the Moon)
- m: mass of the thing (say, a part of the ocean) feeling the tidal force
- (mass of the body feeling the tide does not appear)



Tidal Force

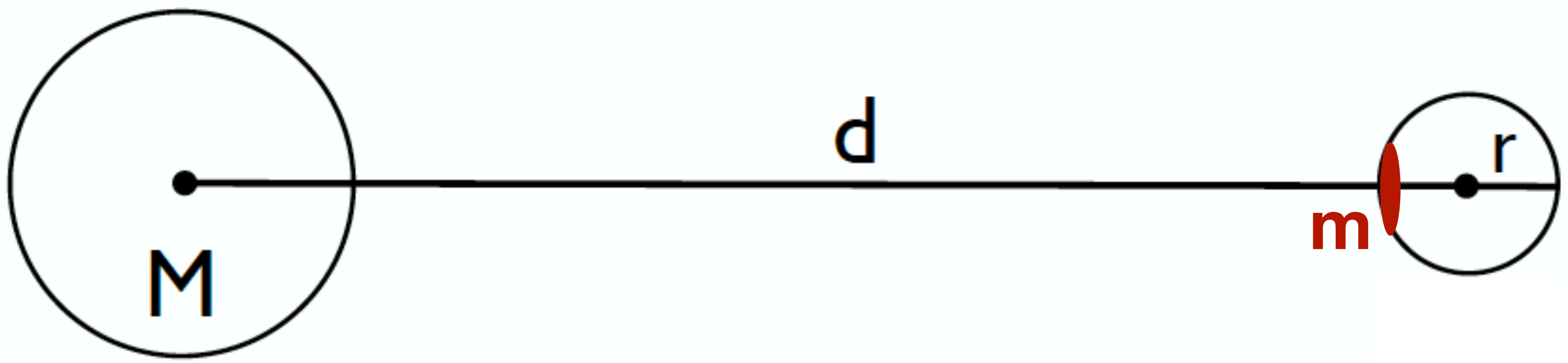
- The difference in gravitational force from one part of an object to another part is called the Tidal Force

$$F_{tidal} = \frac{GMm}{d^2} - \frac{GMm}{(d+r)^2}$$

- Get a common denominator:

$$F_{tidal} = \frac{GMm(d+r)^2}{d^2(d+r)^2} - \frac{GMmd^2}{d^2(d+r)^2}$$

$$F_{tidal} = \frac{GMm(d+r)^2 - GMmd^2}{d^2(d+r)^2}$$



Tidal Force

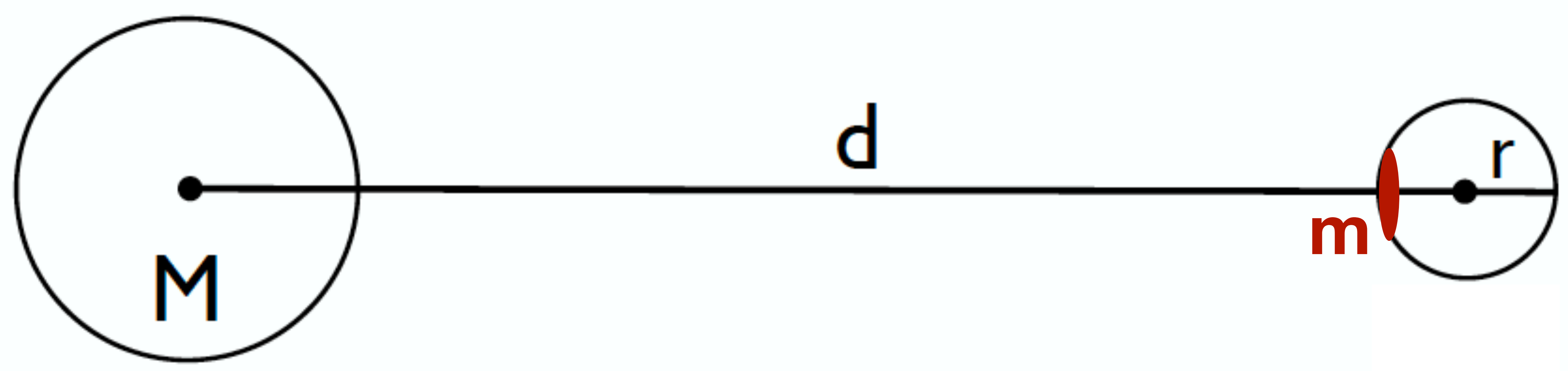
- $$F_{tidal} = \frac{GMm(d+r)^2 - GMmd^2}{d^2(d+r)^2}$$
- Simplify, then expand out terms:

$$F_{tidal} = GMm\frac{d^2 + 2rd + r^2 - d^2}{d^4 + 2rd^3 + d^2r^2}$$

$$F_{tidal} = GMm\frac{2rd + r^2}{d^4 + 2rd^3 + d^2r^2}$$

- If we assume $d \gg r$, then we can drop a few terms:

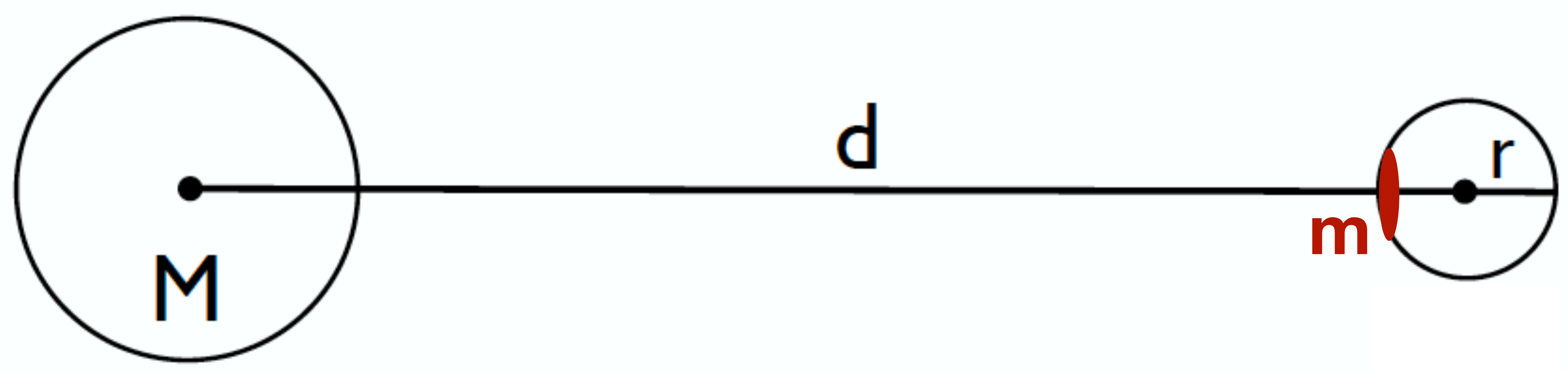
$$F_{tidal} = GMm\frac{2rd}{d^4} = \frac{2GMmr}{d^3}$$



Tidal Force

- $$F_{tidal} = \frac{2GMmr}{d^3}$$

- Magnitude falls off as $1/d^3$
- Magnitude decreases for smaller planets (smaller r)
- Magnitude is larger for the tides from a more massive body
- Magnitude does not depend on mass of the body feeling the tide

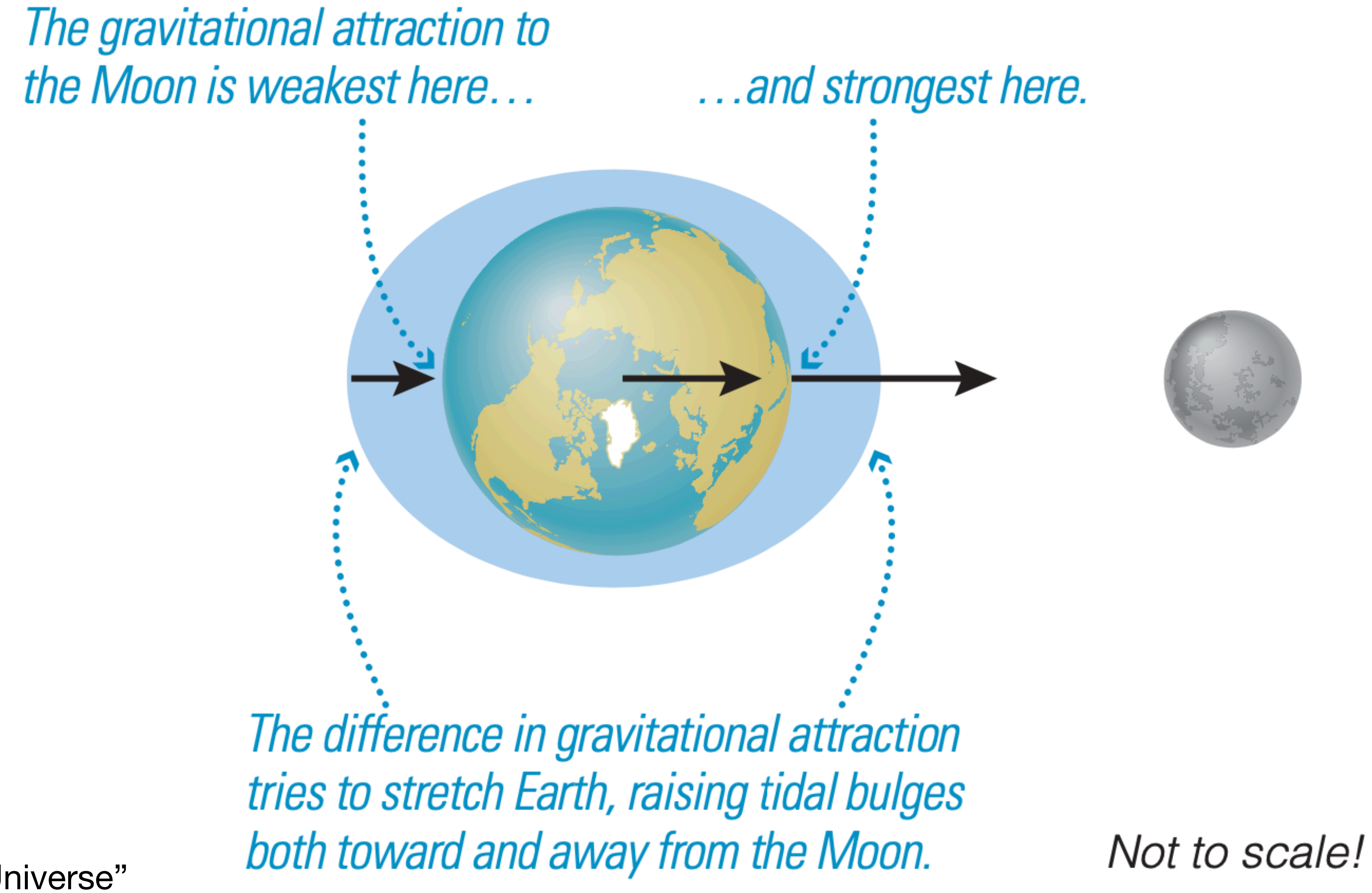


Response Card Question

- Consider two objects (say, the Earth and the Moon). What is the ratio between the tidal force ($F_{tidal} = \frac{2GMmr}{d^3}$) and the force of gravity felt by one piece of mass (m) on one body from the other body?
  - (A) — The force of gravity is $2\frac{r}{d}$ times the tidal force
  - (B) — The tidal force is $2\frac{r}{d}$ times the force of gravity
  - (C) — The force of gravity is $2\frac{r}{d^2}$ times the tidal force
  - (D) — The tidal force is $2\frac{r}{d^2}$ times the force of gravity
  - (E) — The two forces are the same

Gravity and Tides

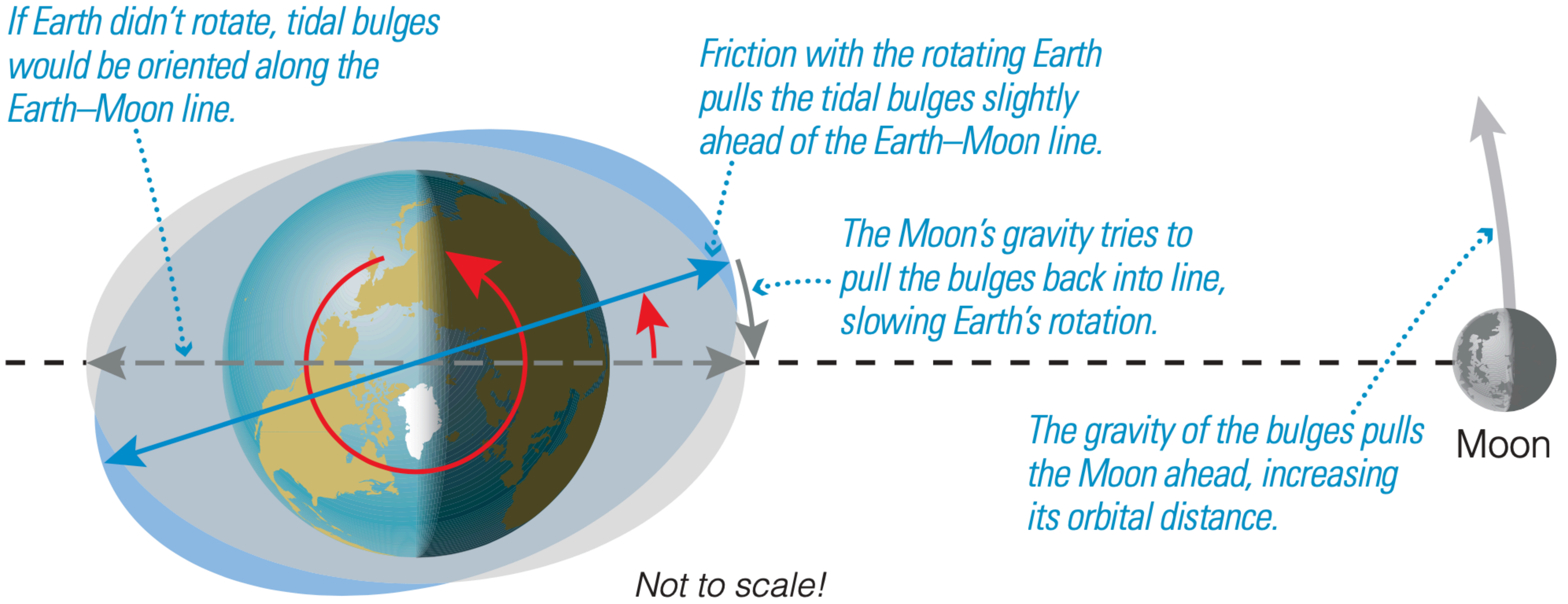
- Parts of the Earth closer to the Moon feel a stronger gravitational force than parts near the center
- Parts of the Earth furthest from the Moon feel a weaker gravitational force than parts near the center
- The oceans have a tidal bulge, with higher tides towards the Moon, and higher tides away from the Moon



"Life in the Universe"

Gravity and Tides

- Tides also stretch the land on Earth, leading to “tidal friction” that heats up the planet
- Tidal friction results in the tidal bulge pointing about 20 degrees in front of the Moon’s path
- The Earth’s rotation rate is slowing down, and the Moon is moving further away (slowly!)

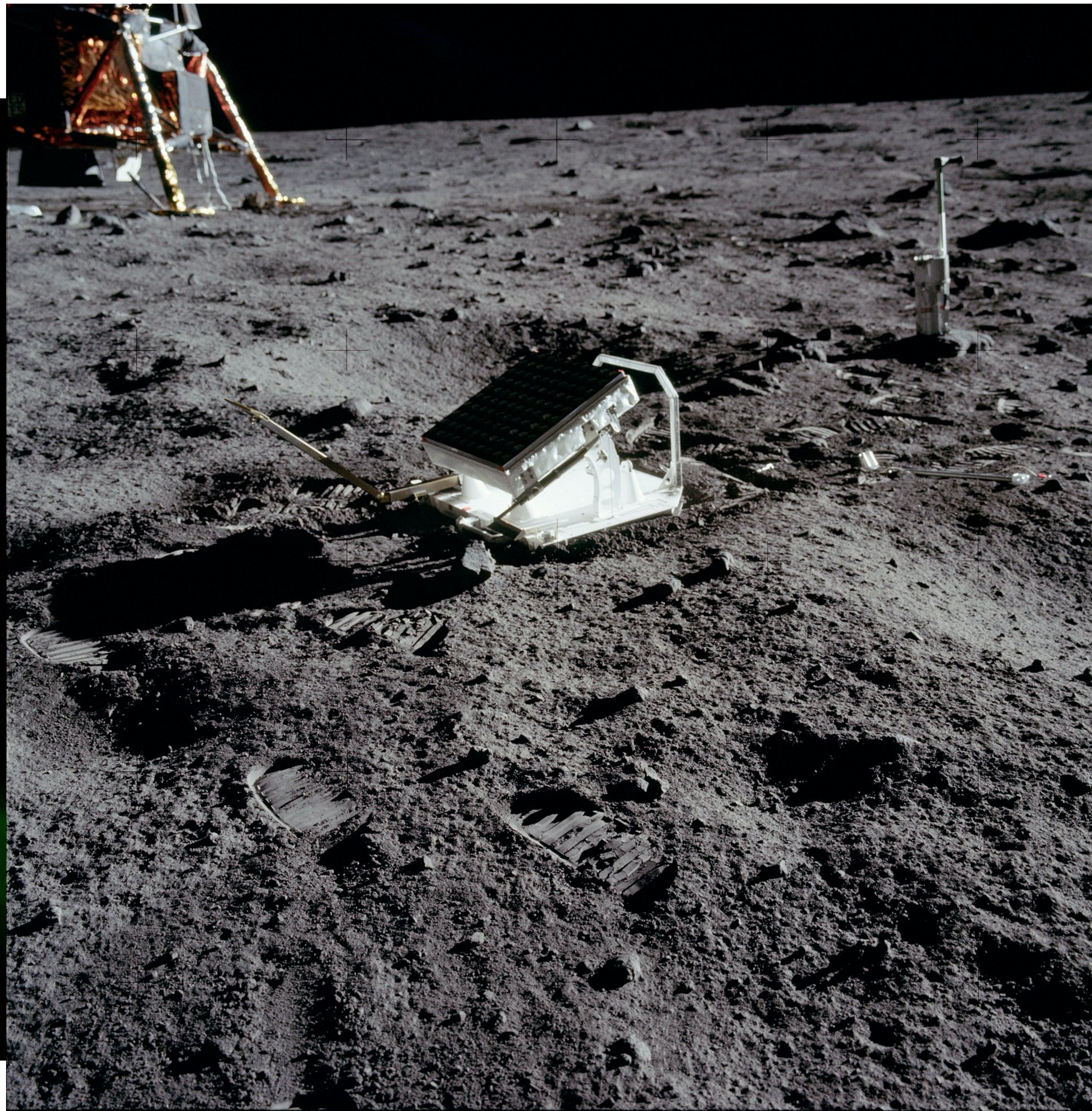


“Life in the Universe”

The distance to the Moon

- Apollo astronauts put "retroreflectors" on the Moon's surface
- Astronomers can bounce laser beams off those retroreflectors, and measure how much time it takes for light to return
- From Apache Point Observatory, we can measure the distance to the Moon to a precision of 1 mm
- The Moon is moving away from the Earth at 4 cm per year
- An Earth day gets longer by 1 second every 50,000 years

Dan Long (Apache Point Observatory)

NASA


Tides, the Moon, and synchronous rotation

- The early Moon probably rotated much faster than its orbital motion, and tidal forces from Earth were continually squeezing the Moon

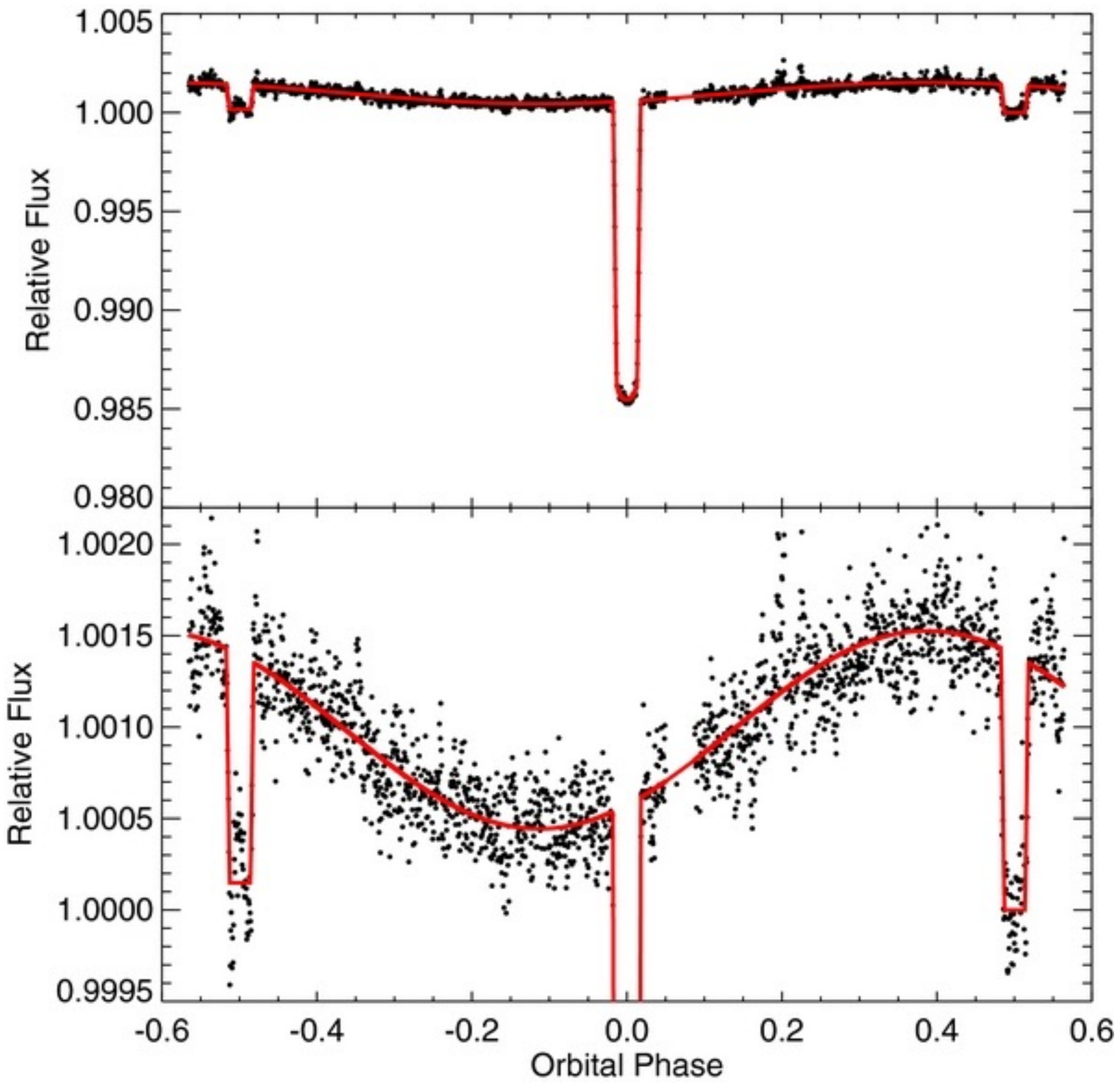
- This tidal friction slowed down the Moon’s rotation rate until it matched the orbital rate

- From that point on, the Moon was in synchronous rotation (also called being “tidally locked”)

Gregory Revera

Tidal locking

- Small objects orbiting close to a larger object tend to either be in synchronous rotation, or are moving toward synchronous rotation
- Moons of giant planets tend to be tidally locked
- Exoplanets close to their star tend to be tidally locked

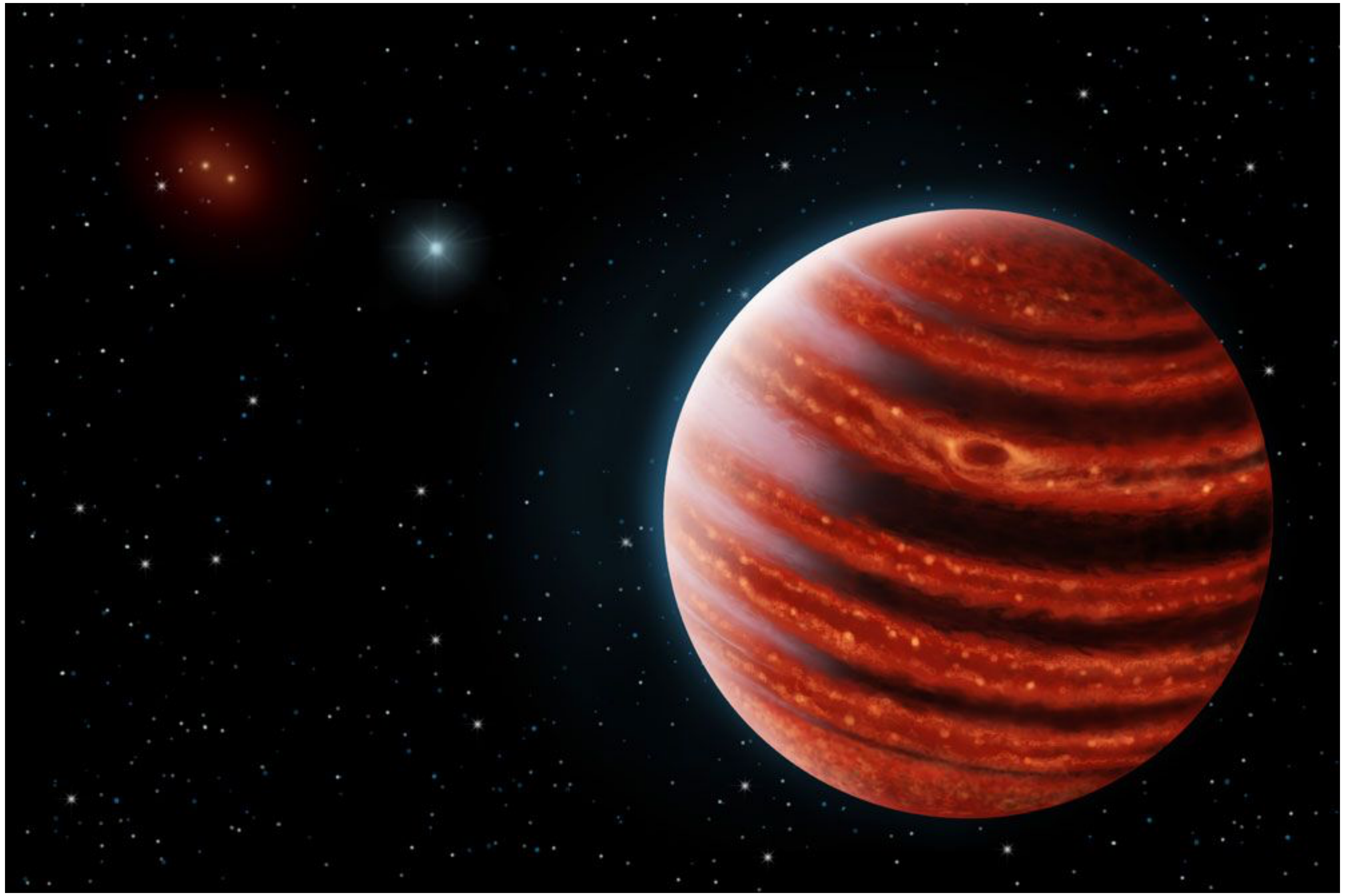


HD 209458 b, Zellem et al. 2014

Break

The 3-body problem

- There is an exact solution to the 2-body problem (how two objects move as a function of initial conditions and their own gravity)
- There is not an analytic solution to the 3-body problem
- There are approximations in special cases:
    - The restricted 3-body problem (When 1 mass is much smaller than the other 2 masses)
    - Hill's problem (When 1 mass is much larger than the other 2 masses)

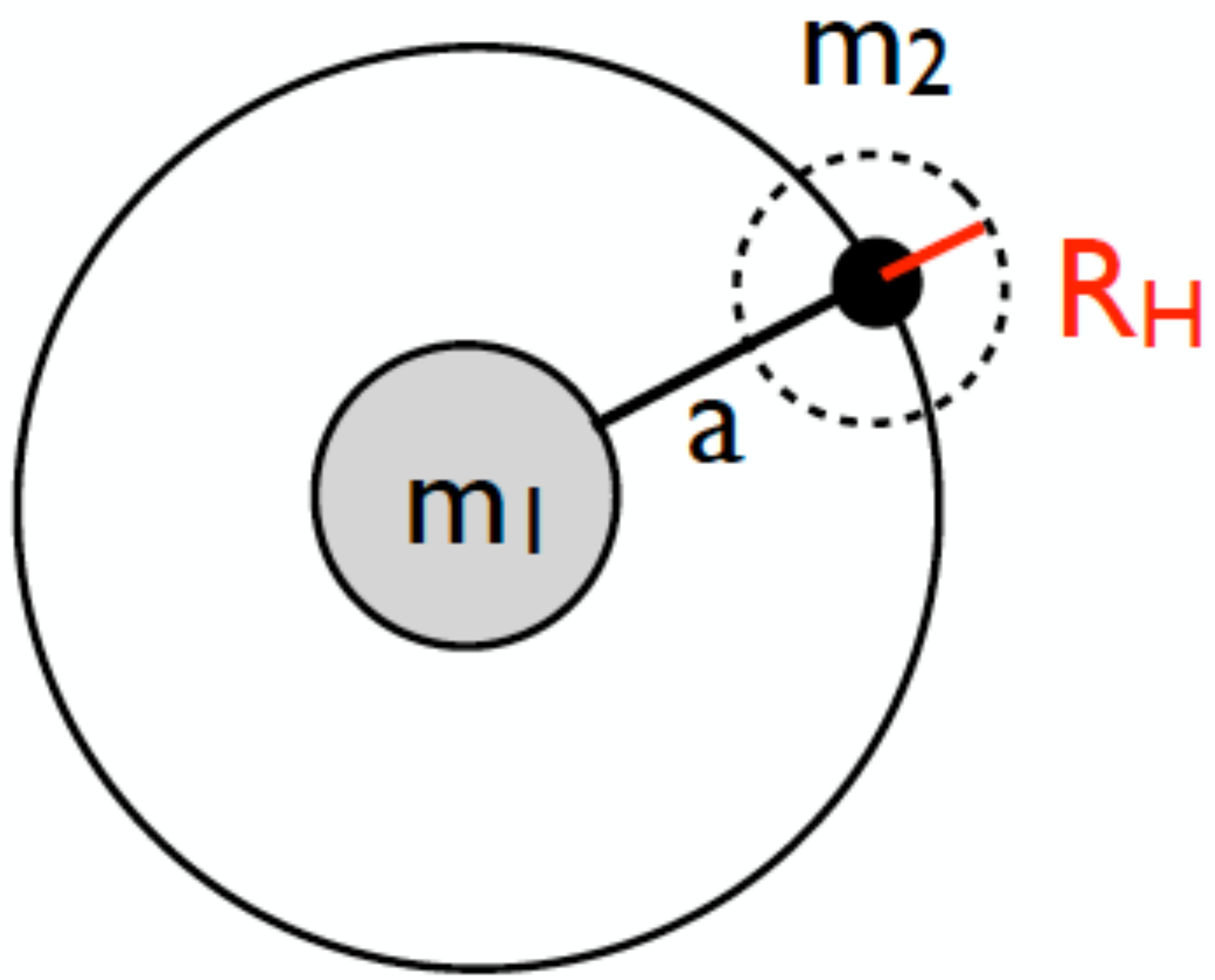


Danielle Futselaar & Franck Marchis

Hill Sphere

- Gravitational “sphere of influence” around a secondary
- Inside a planet’s Hill sphere, the planet’s gravity is more important to a test particle
- Outside the Hill sphere, the Sun’s gravity is more important

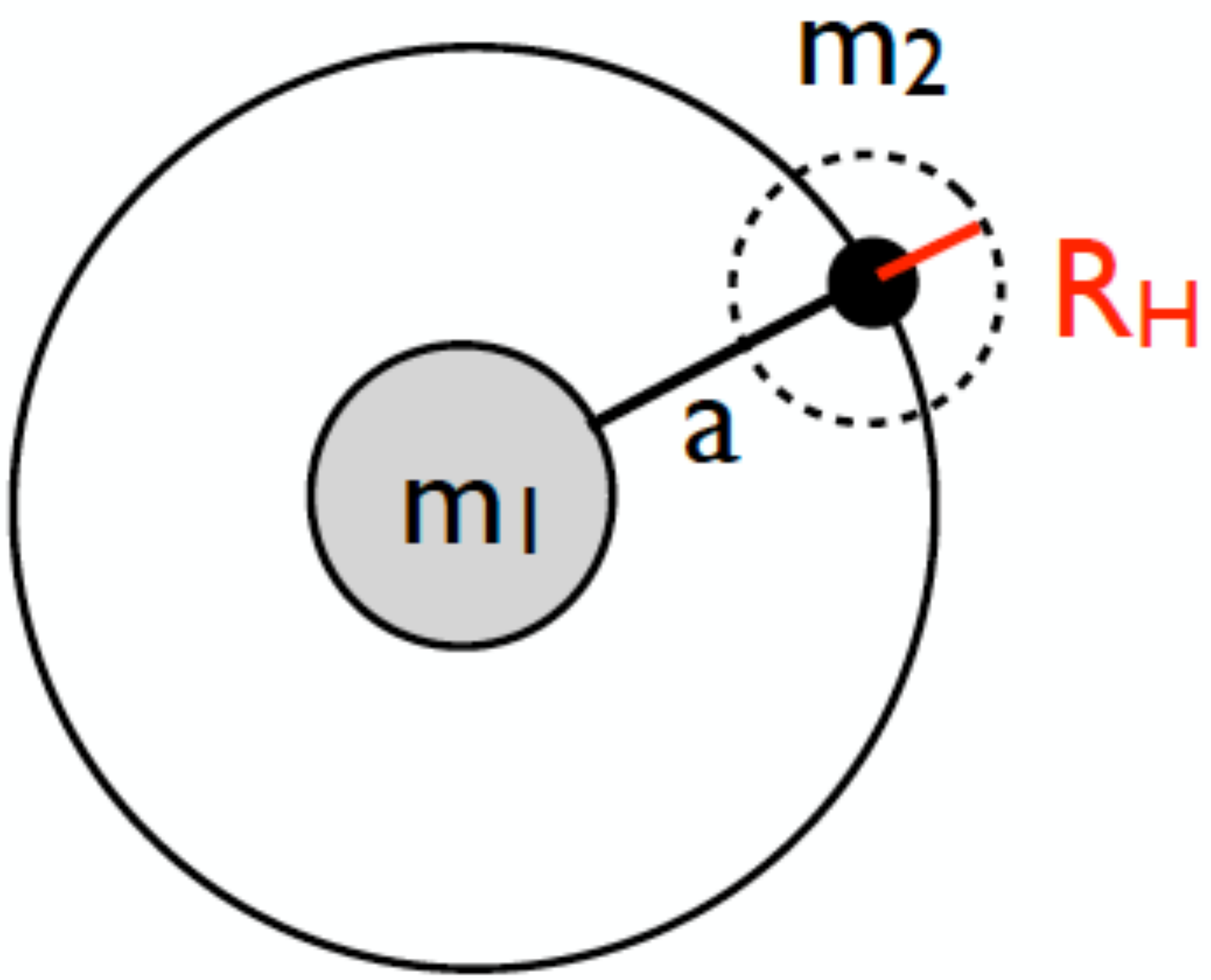
$$R_H = \left(\frac{M_2}{3(M_1 + M_2)}\right)^{\frac{1}{3}} a * (1 - e)$$



Hill Sphere

-

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)}\right)^{\frac{1}{3}} a * (1 - e)$$



- A planet's moons and rings will be within that planet's Hill Sphere

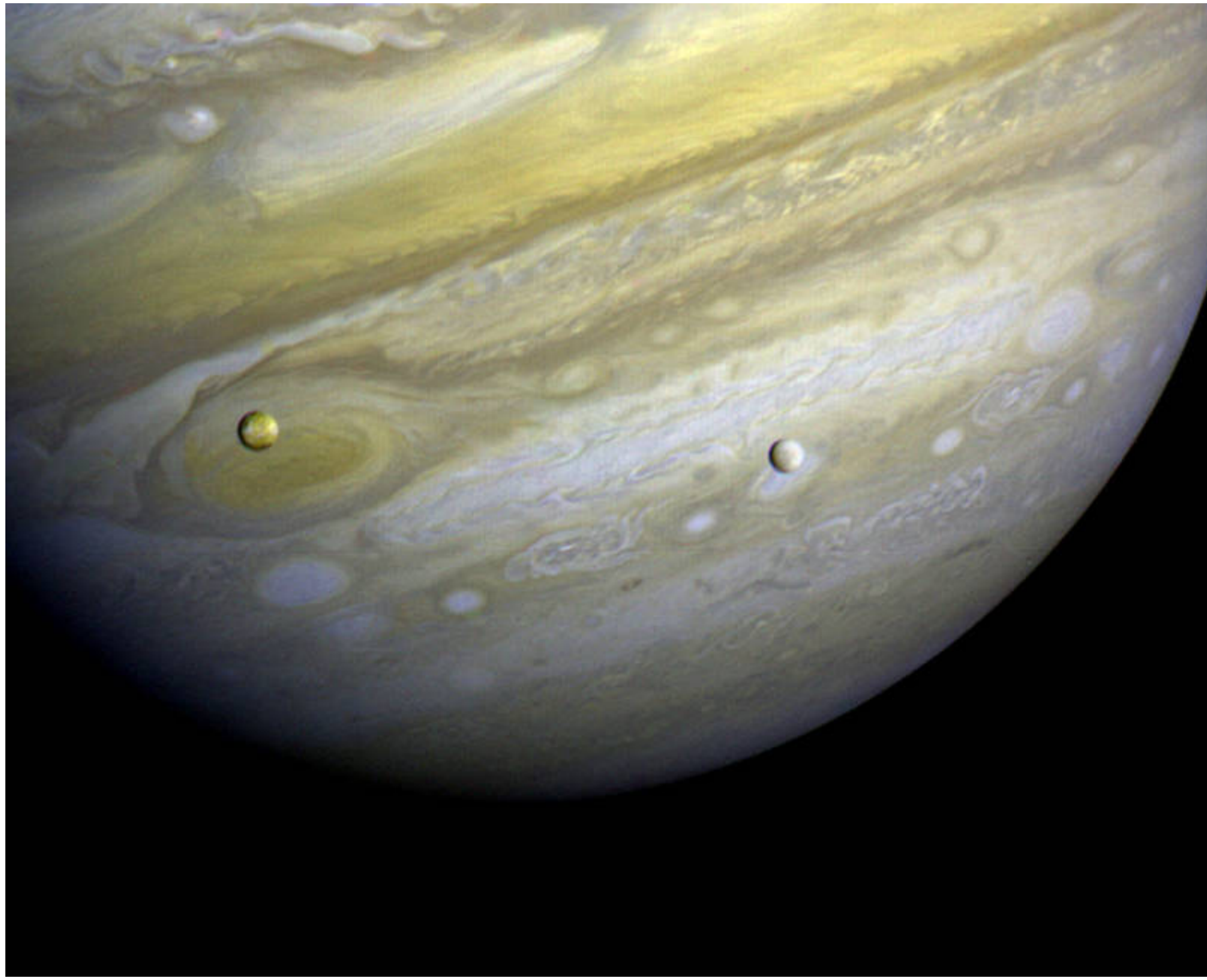
In-Class Activity
Order of Magnitude

- $$R_H = \left(\frac{M_2}{3(M_1 + M_2)}\right)^{\frac{1}{3}} a * (1 - e)$$

- (1) What is the radius of Jupiter's Hill Sphere, in AU?

- (2) If Jupiter had a moon at the edge of (but just inside) its Hill Sphere, what would its orbital period be?

- (3) Think of the solar system as a 2-dimensional disk. What fraction of that disk within Jupiter's orbit around the Sun is contained within Jupiter's Hill Sphere?



NASA/Voyager

In-Class Activity
Order of Magnitude

- (1) Jupiter has about 1/1000 (10^{-3}) the mass of the Sun, so in solar masses: 10^{-3} The semi-major axis of Jupiter is 5.2 AU, and has a roughly circular (e=0) orbit. Plugging in numbers:

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)}\right)^{\frac{1}{3}} a * (1 - e) = \left(\frac{10^{-3}}{3(1 + 10^{-3})}\right)^{\frac{1}{3}} 5.2 * (1 - 0) = (\frac{1}{3})^{\frac{1}{3}} * (10^{-3})^{\frac{1}{3}} * 5.2 = 1 * 10^{-1} * 5.2 = 0.5AU$$

- (2) Using Kepler's third law:

- $$P^2 = \frac{a^3}{M}$$ So plugging in M and a=0.5 AU: $$P = \sqrt{\frac{a^3}{M}} = \sqrt{\frac{0.5^3}{10^{-3}}} = \sqrt{0.125 * 10^3} = \sqrt{10^2} = 10 \text{ years}$$

- (3) Area of a 2D disk is just area of a circle: $A = \pi R^2$, so the ratio of areas of Jupiter's Hill Sphere and the Solar System out to Jupiter is:

$$\frac{A_H}{A_{SS}} = \frac{\pi R_H^2}{\pi d_J^2} = \left(\frac{0.5AU}{5.2AU}\right)^2 = (0.1)^2 = 0.01 = 1\,\%$$

For next time

- Find a copy of the textbook (remember: either version of the 2nd edition works — NOT the first edition)
- Reading: de Pater & Lissaeuer Chaper 2, section 2.6.1, 2.7, 2.7.1
- Homework 1 due in 5 days, August 29 at 11:59:59 on Canvas (reminder, late homework loses 10% of possible points each day)