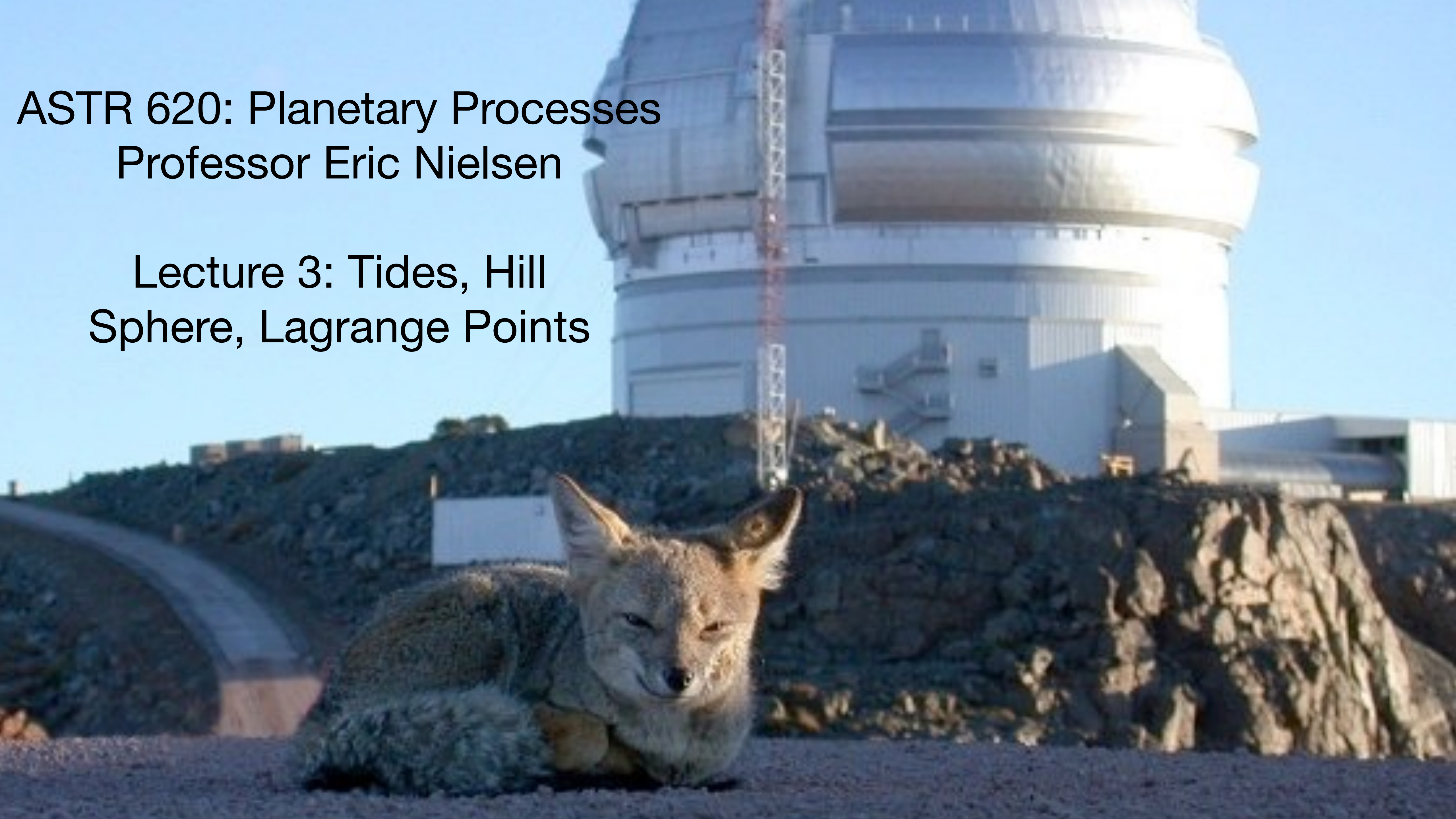


ASTR 620: Planetary Processes

Professor Eric Nielsen

Lecture 3: Tides, Hill Sphere, Lagrange Points



Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **You may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Remember to bring you response card to class
- Homework 1 will be due this Monday, August 29th, at 11:59:59pm on Canvas

Review of the last class

- Sedna has a semi-major axis of about 500 AU and eccentricity about 0.9: what is the perihelion and aphelion of Sedna?
 - (A) — 50 AU perihelion, 950 AU aphelion
 - (B) — 500 AU for both perihelion and aphelion
 - (C) — 450 AU perihelion, 550 AU aphelion
 - (D) — 5 AU perihelion, 995 AU aphelion
 - (E) — 1000 AU for both perihelion and aphelion

Review of the last class

- Saturn has a 10 AU semi-major axis. Therefore, it has an orbital period of:
 - (A) — 3 years
 - (B) — 10 years
 - (C) — 30 years
 - (D) — 100 years
 - (E) — 300 years

Review of the last class

- Escape velocity is given by:

- (A) — $v_{esc}(r) = \sqrt{\frac{2GM_{\odot}}{r}}$

- (B) — $v_{esc}(r) = \sqrt{\frac{GM_{\odot}}{r}}$

- (C) — $v_{esc}(r) = \frac{2GM_{\odot}}{r}$

- (D) — $v_{esc}(r) = \frac{GM_{\odot}}{r}$

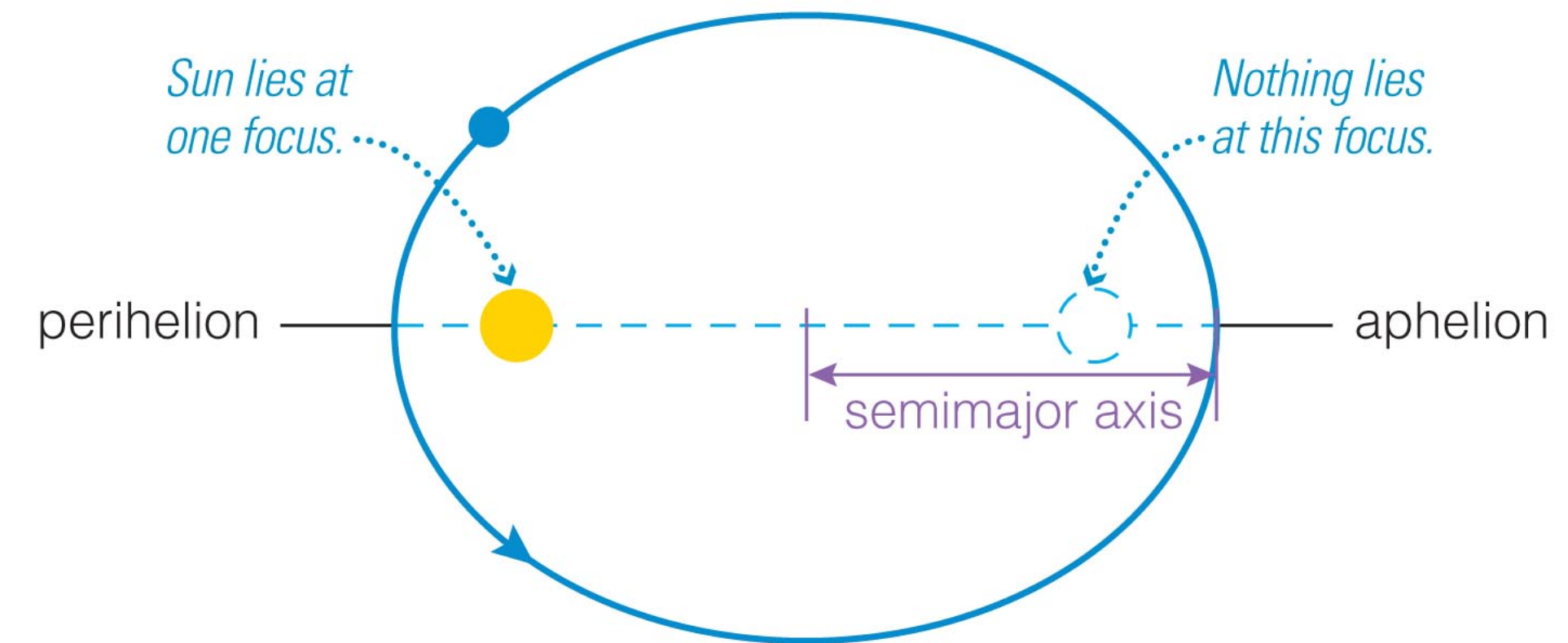
Review of the last class

- Which planet is traveling fastest?
 - (A) — A planet with a 1 AU semi-major axis, eccentricity of 0, when it's 1 AU from the star
 - (B) — A planet with a 10 AU semi-major axis, eccentricity of 0.9, when it's 1 AU from the star
 - (C) — A planet with a 0.53 AU semi-major axis, eccentricity of 0.9, when it's 1 AU from the star
 - (D) — They're all the same distance from the star, so they're all traveling at the same speed

Review of the last class

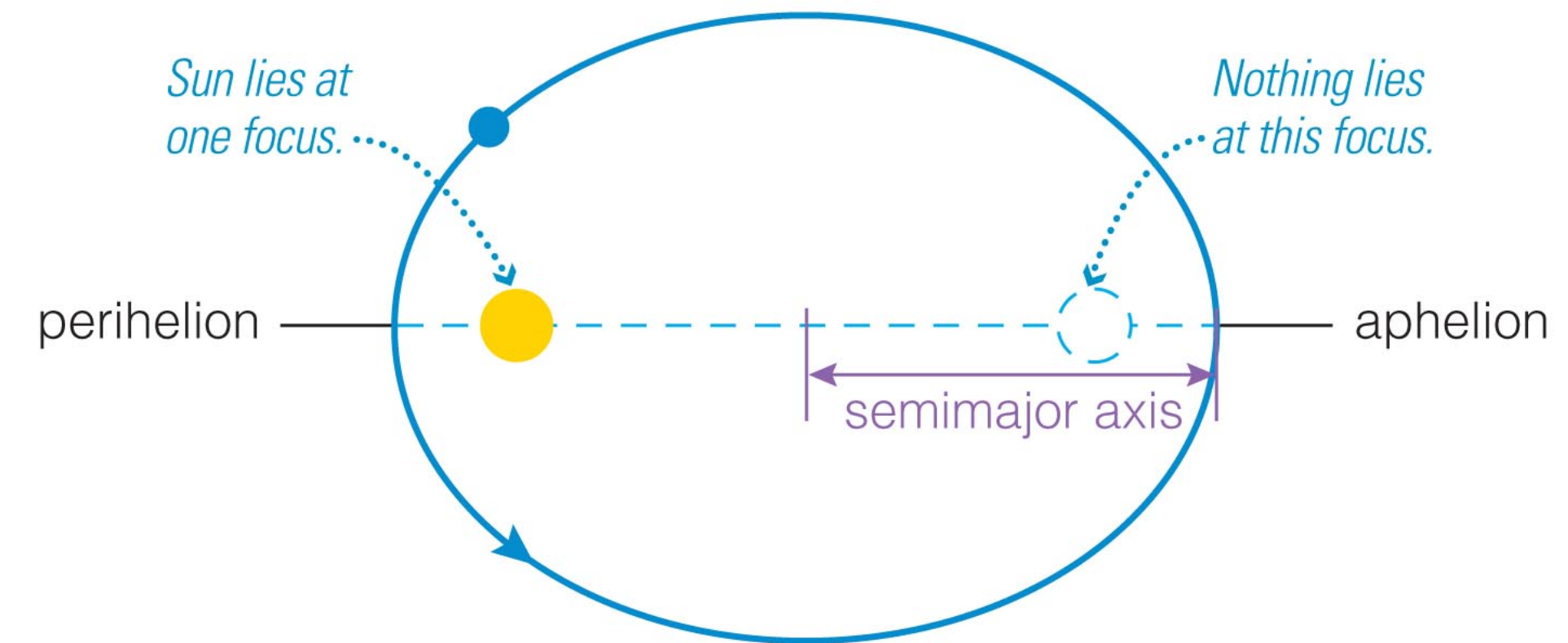
- How would a Hohmann Transfer orbit to Mercury from Earth differ from a Hohmann Transfer orbit to Jupiter from Earth?
 - (A) — It would be the same
 - (B) — At Earth, would have to slow down, then slow down again at Mercury
 - (C) — At Earth, would have to speed up, then slow down at Mercury
 - (D) — At Earth, would have to slow down, then speed up at Mercury
 - (E) — At earth, would have to speed up, then speed up again at Mercury

Orbital elements



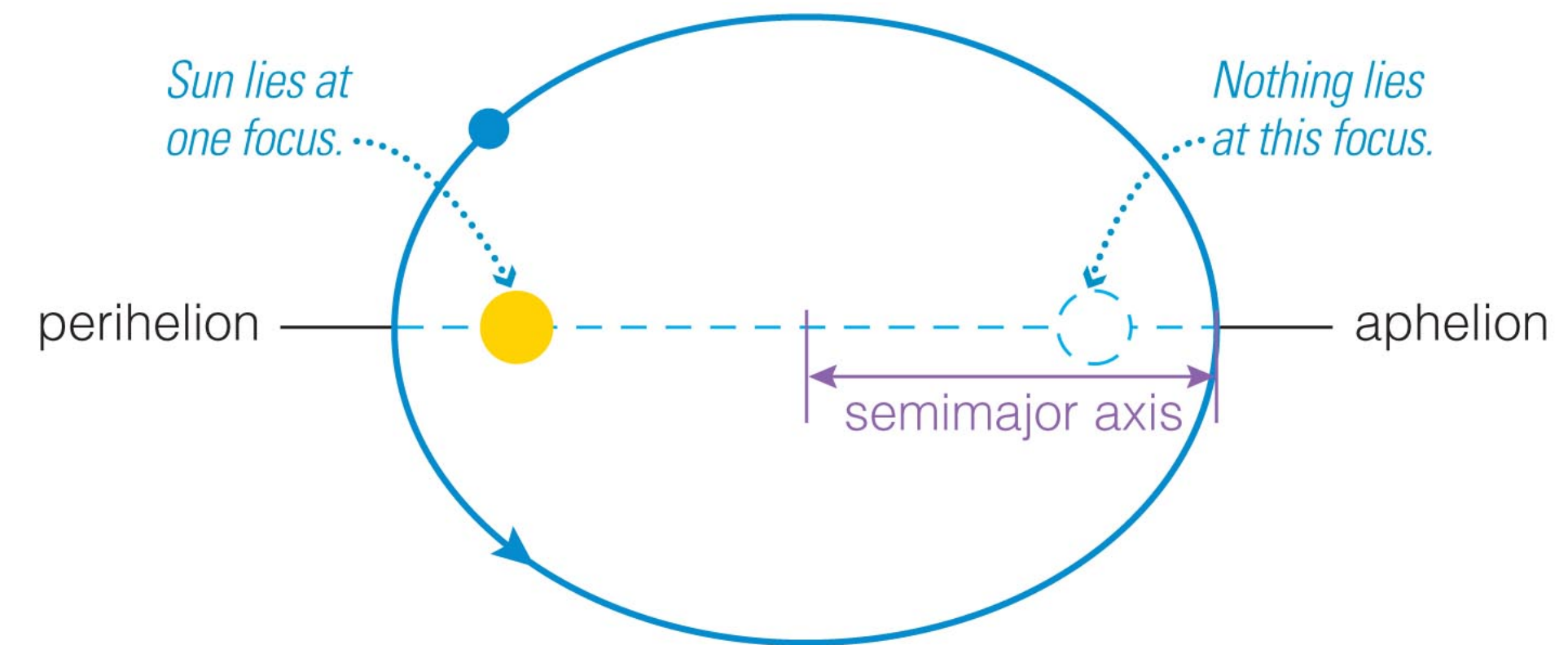
Orbital elements

semi-major axis (a)



Orbital elements

semi-major axis (a)
eccentricity (e)

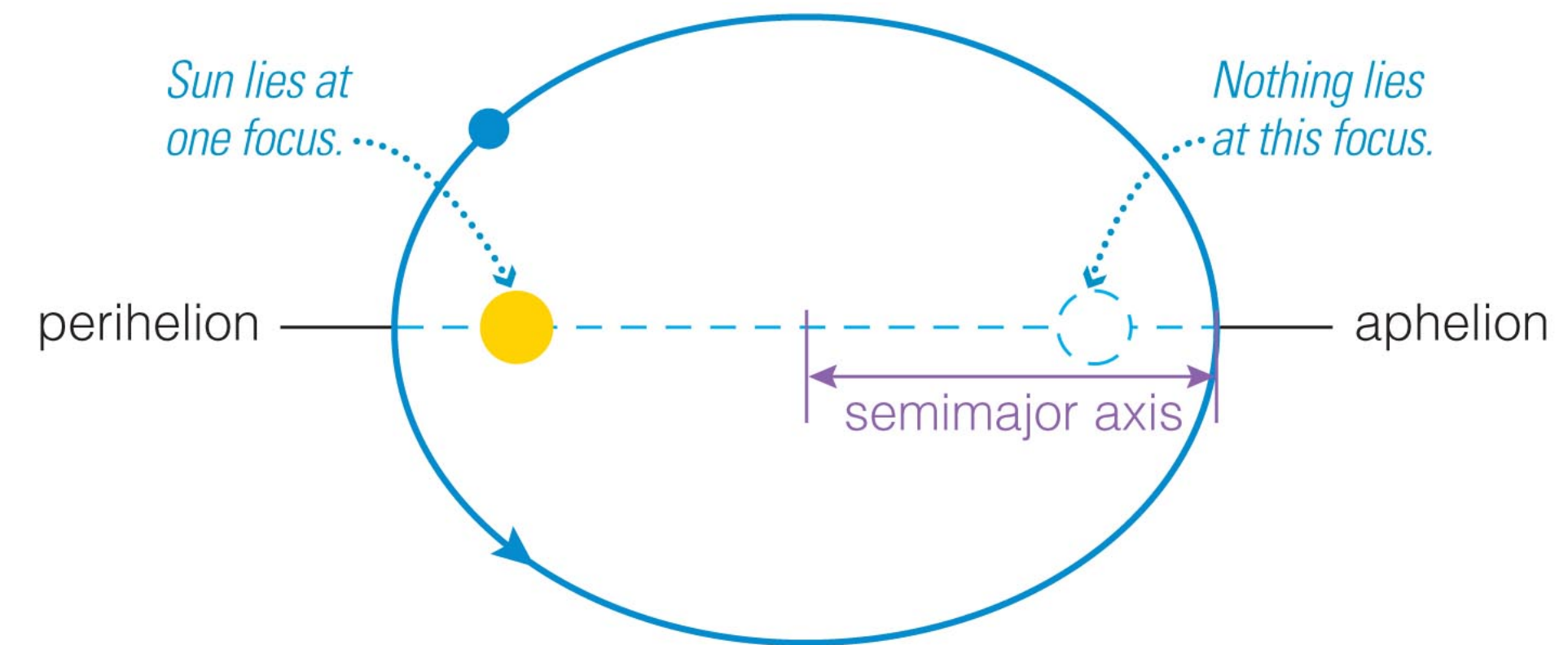


Orbital elements

semi-major axis (a)

eccentricity (e)

Period (P)



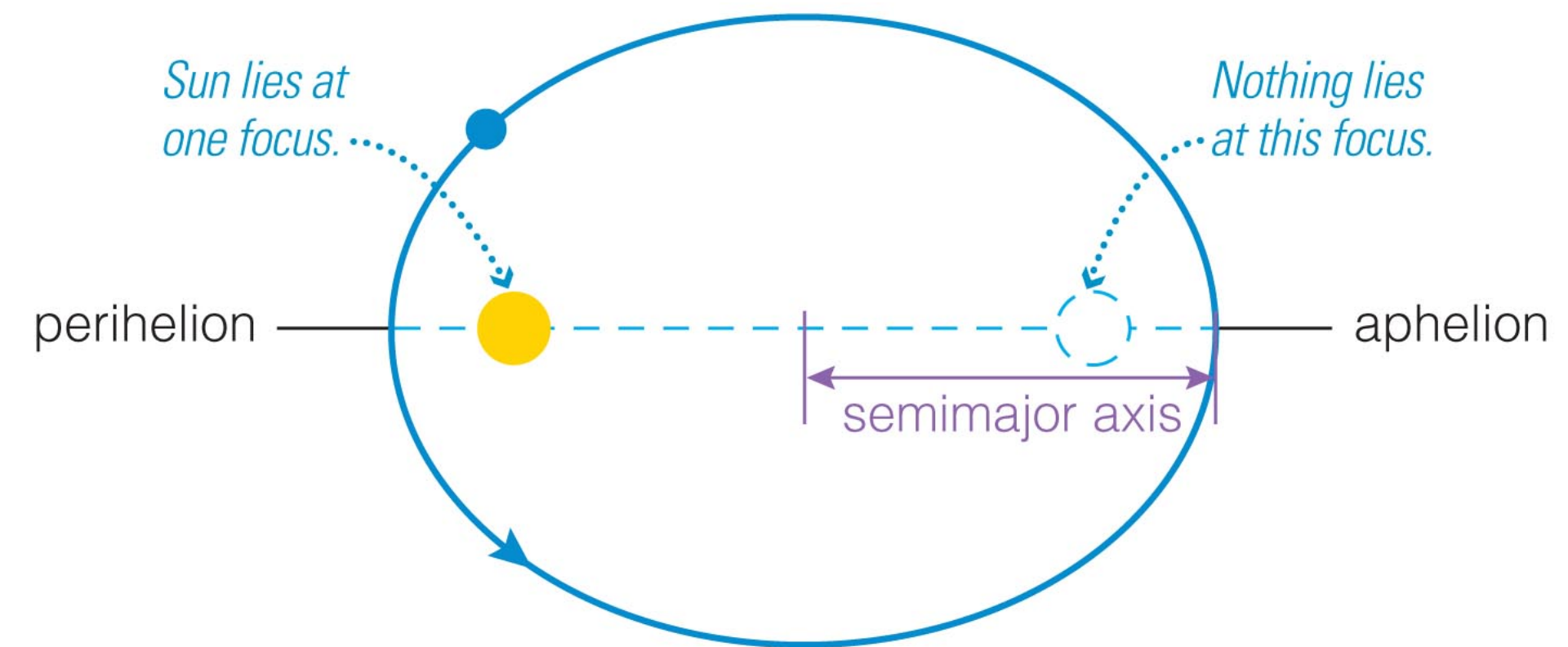
Orbital elements

semi-major axis (a)

eccentricity (e)

Period (P)

Epoch of Periastron Passage (T_0)



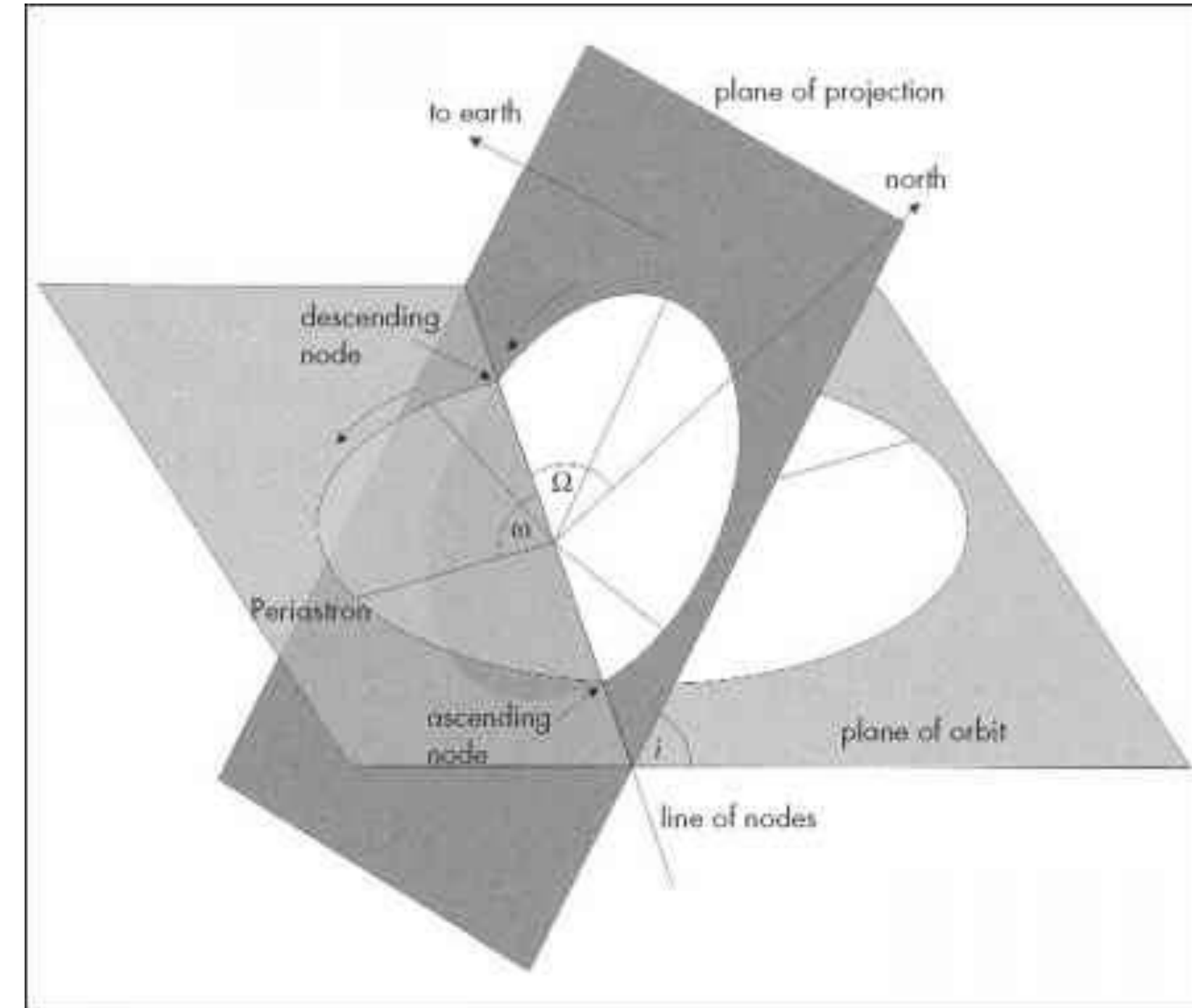
Orbital elements

semi-major axis (a)

eccentricity (e)

Period (P)

Epoch of Periastron Passage (T_0)



Orbital elements

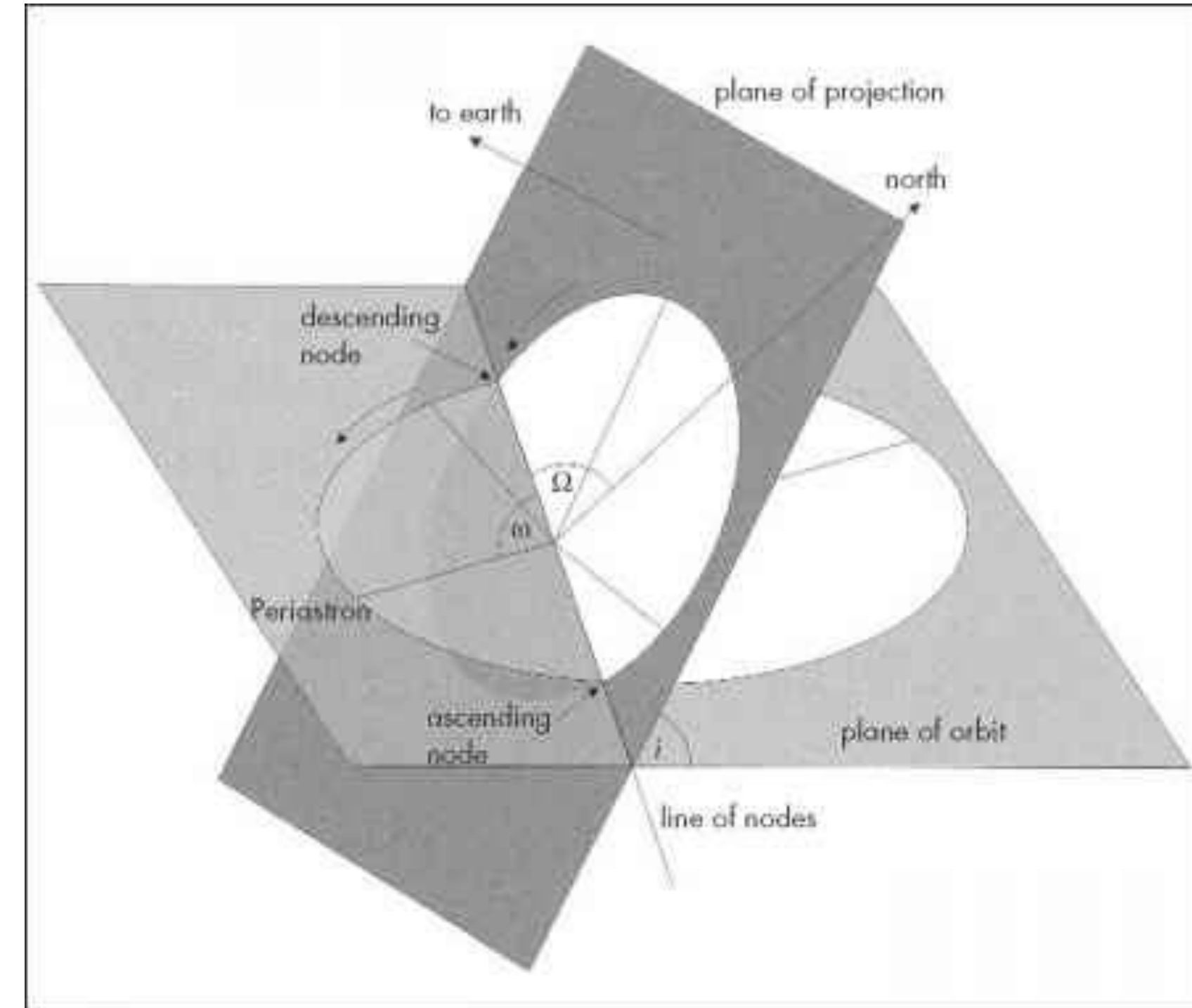
semi-major axis (a)

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Period (P)

Epoch of Periastron Passage (T_0)

Inclination Angle (i)



Orbital elements

semi-major axis (a)

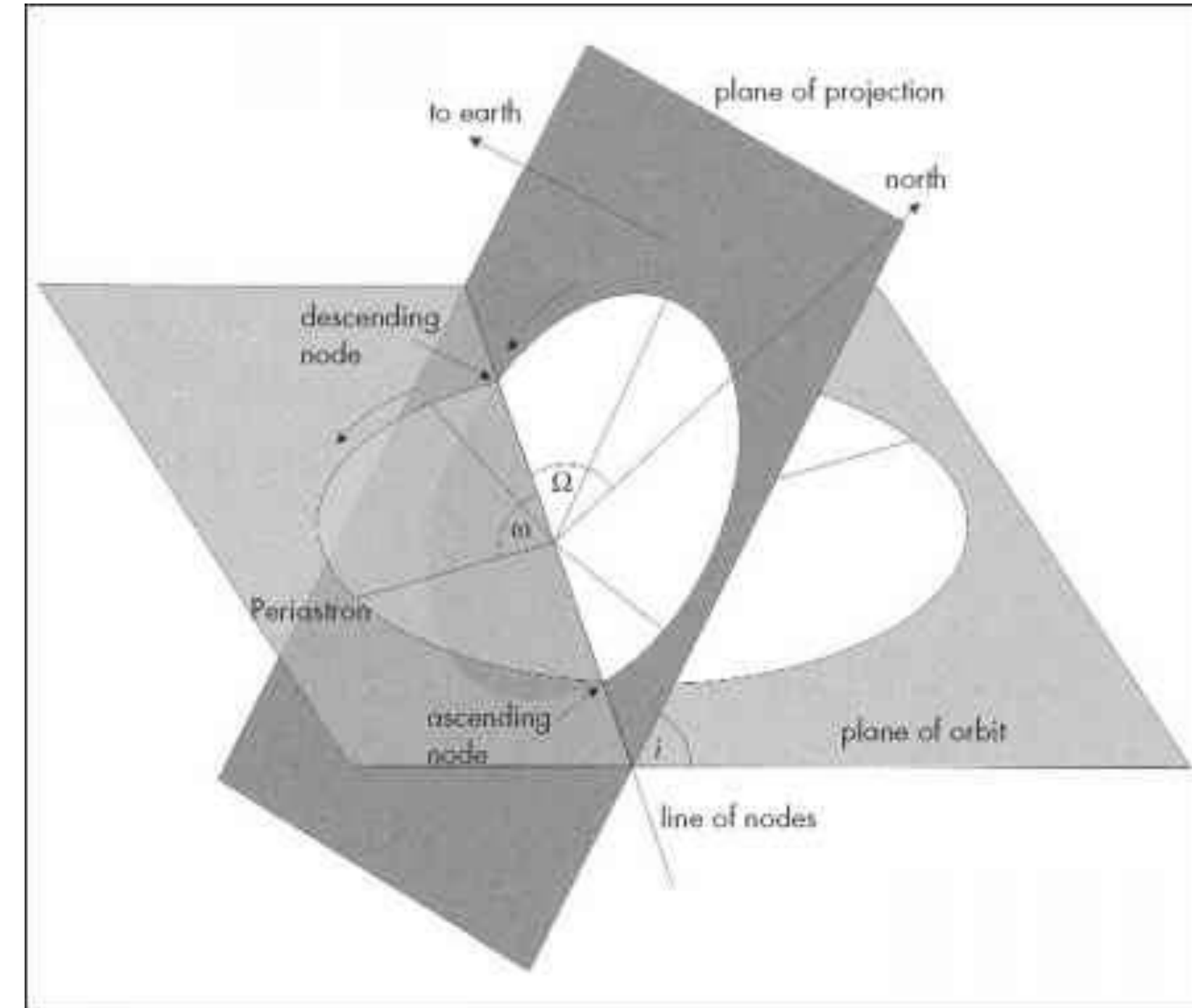
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Period (P)

Epoch of Periastron Passage (T_0)

Inclination Angle (i)

Position Angle of Nodes (Ω)



Orbital elements

semi-major axis (a)

eccentricity (e)

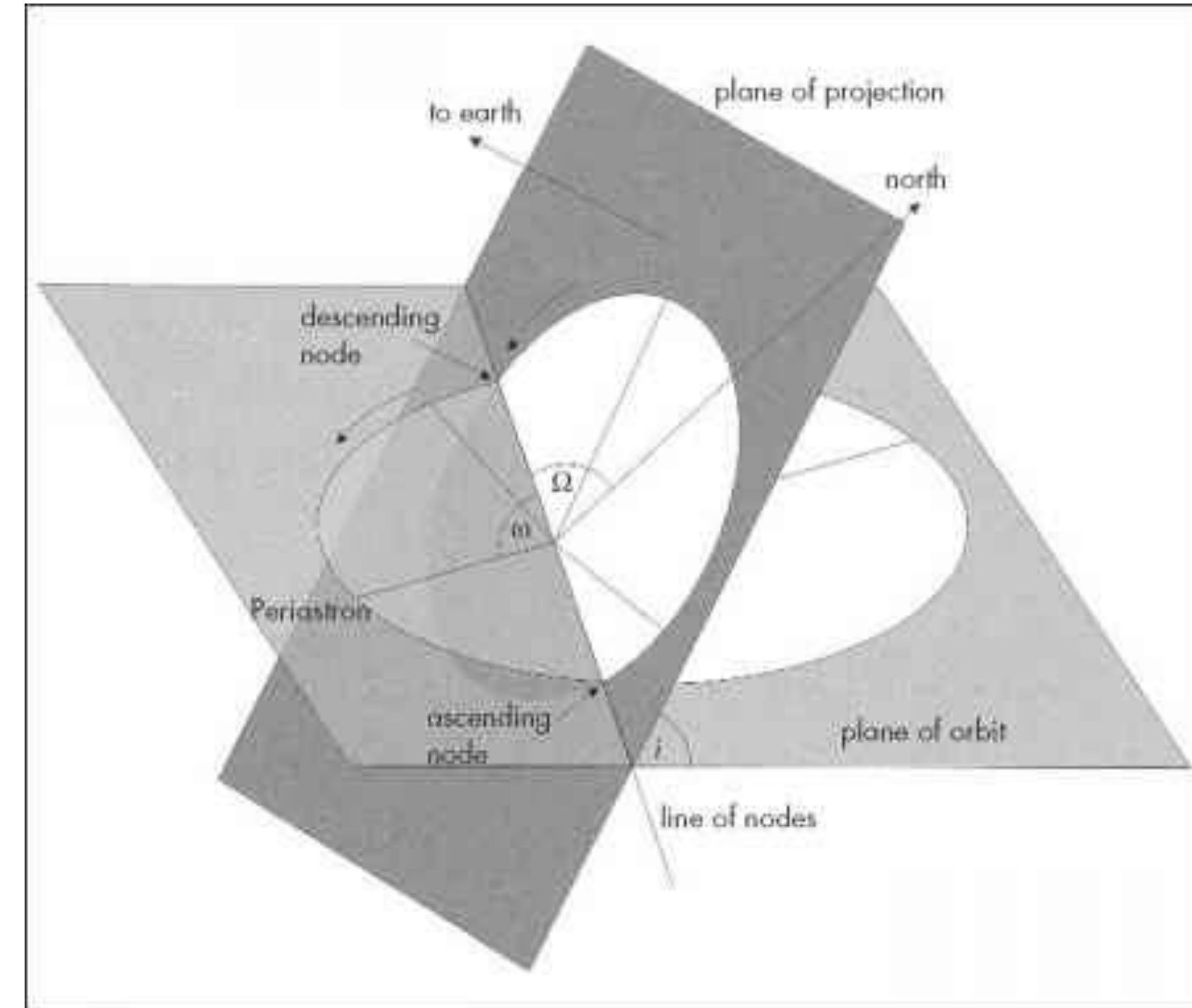
Period (P)

Epoch of Periastron Passage (T_0)

Inclination Angle (i)

Position Angle of Nodes (Ω)

Argument of Periastron (ω)



Anomalies

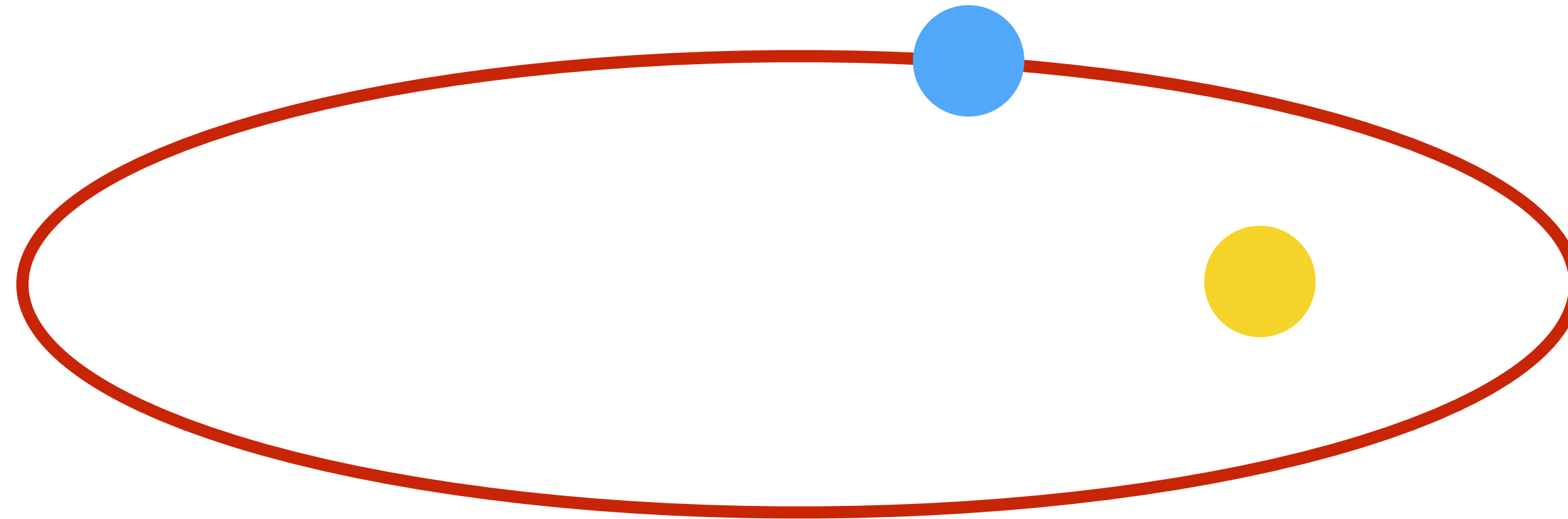
Anomalies

Mean Anomaly (M , $0-2\pi$) — Fraction of a period since last periastron

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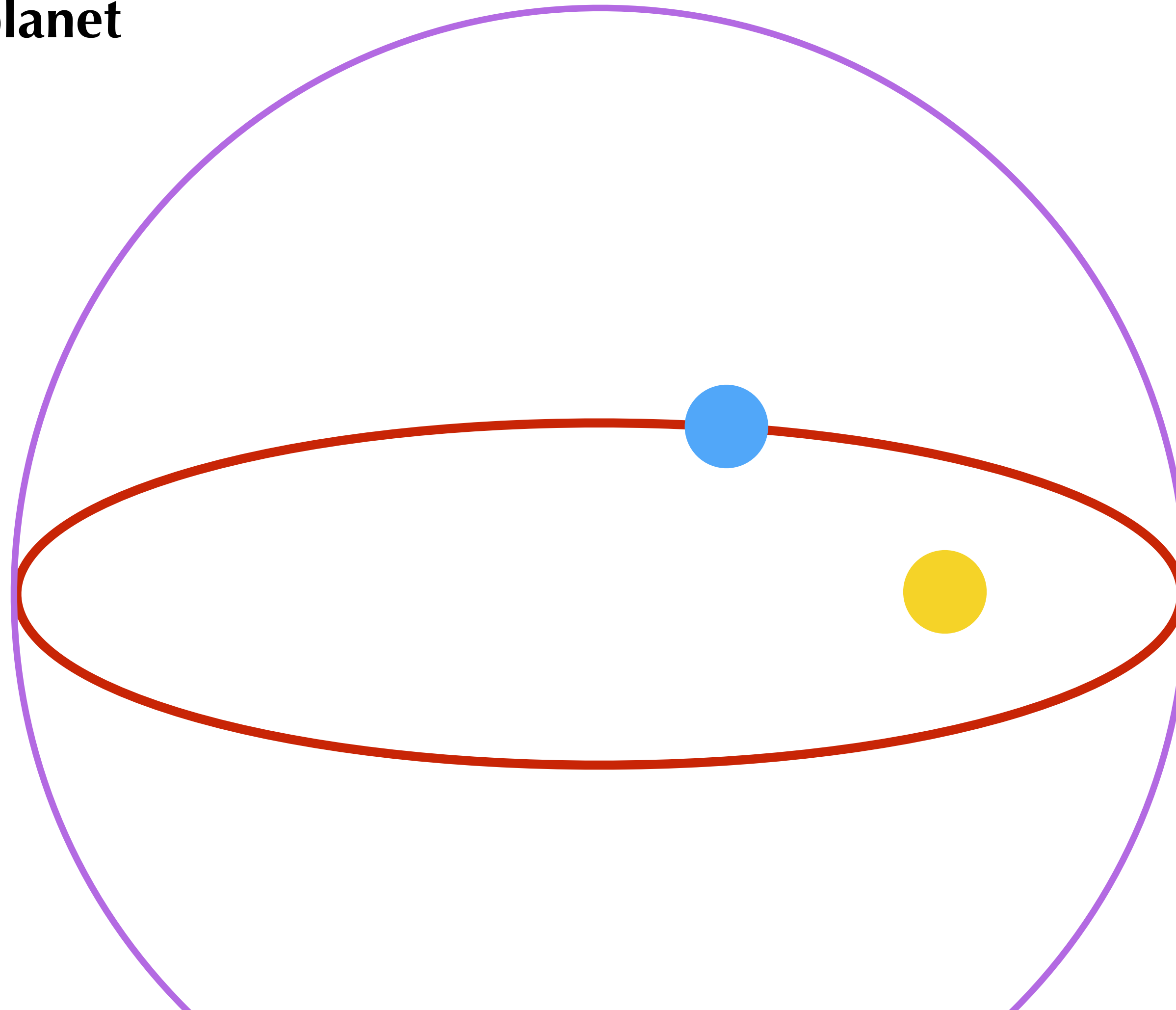
Eccentric Anomaly (E , $0-2\pi$) — Angle between periastron, center of ellipse, and (kind of) planet



Anomalies

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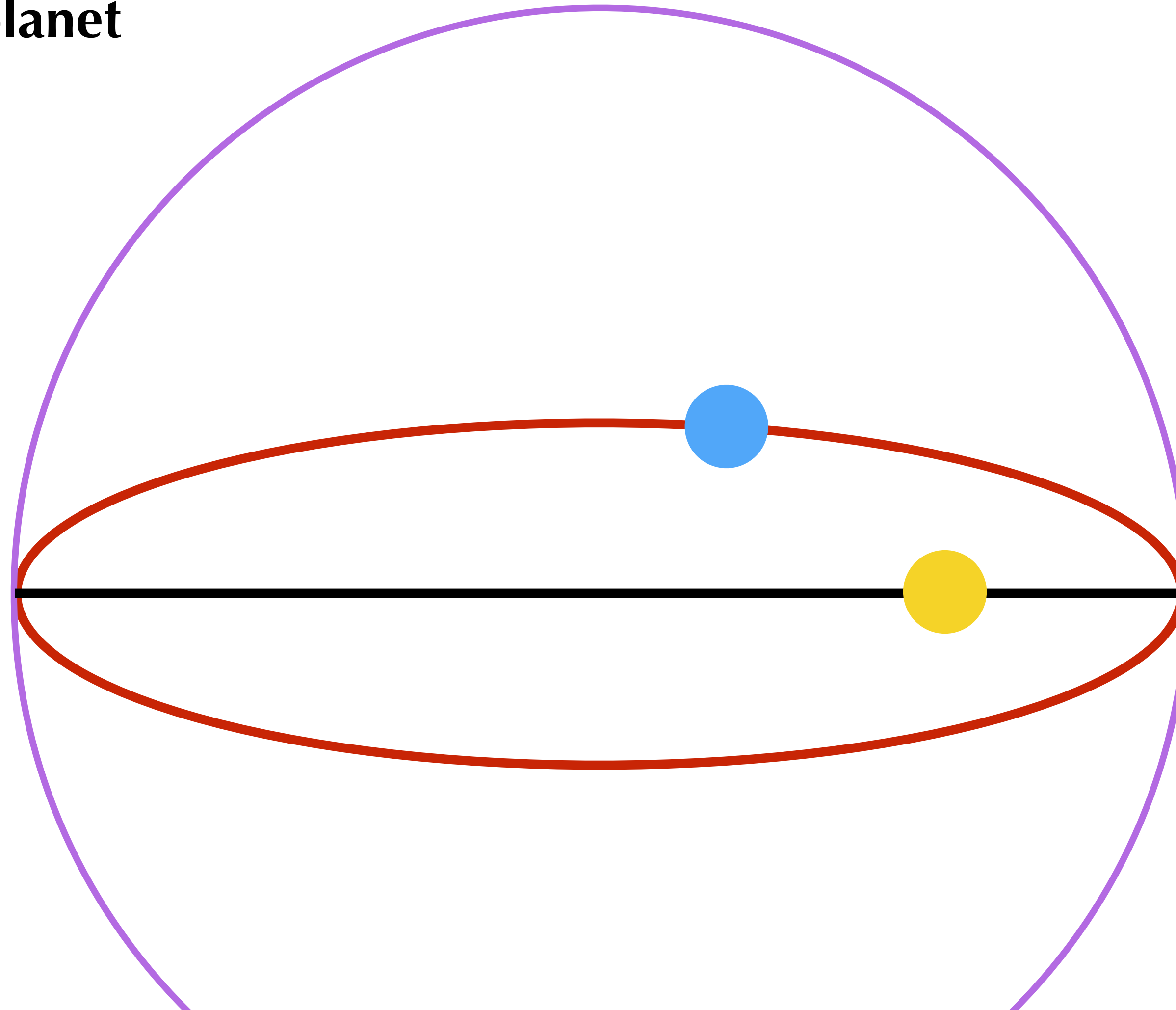
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Anomalies

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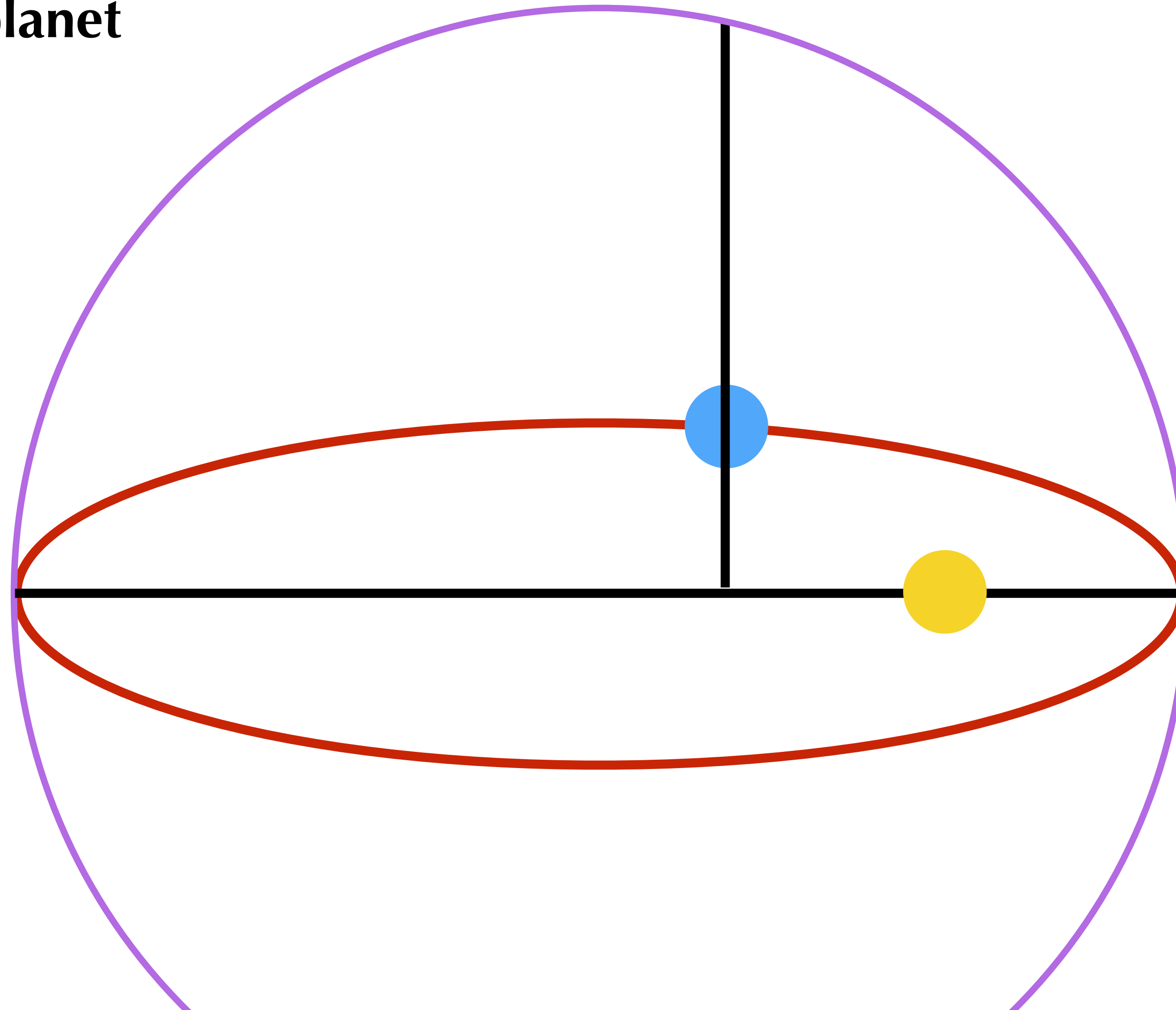
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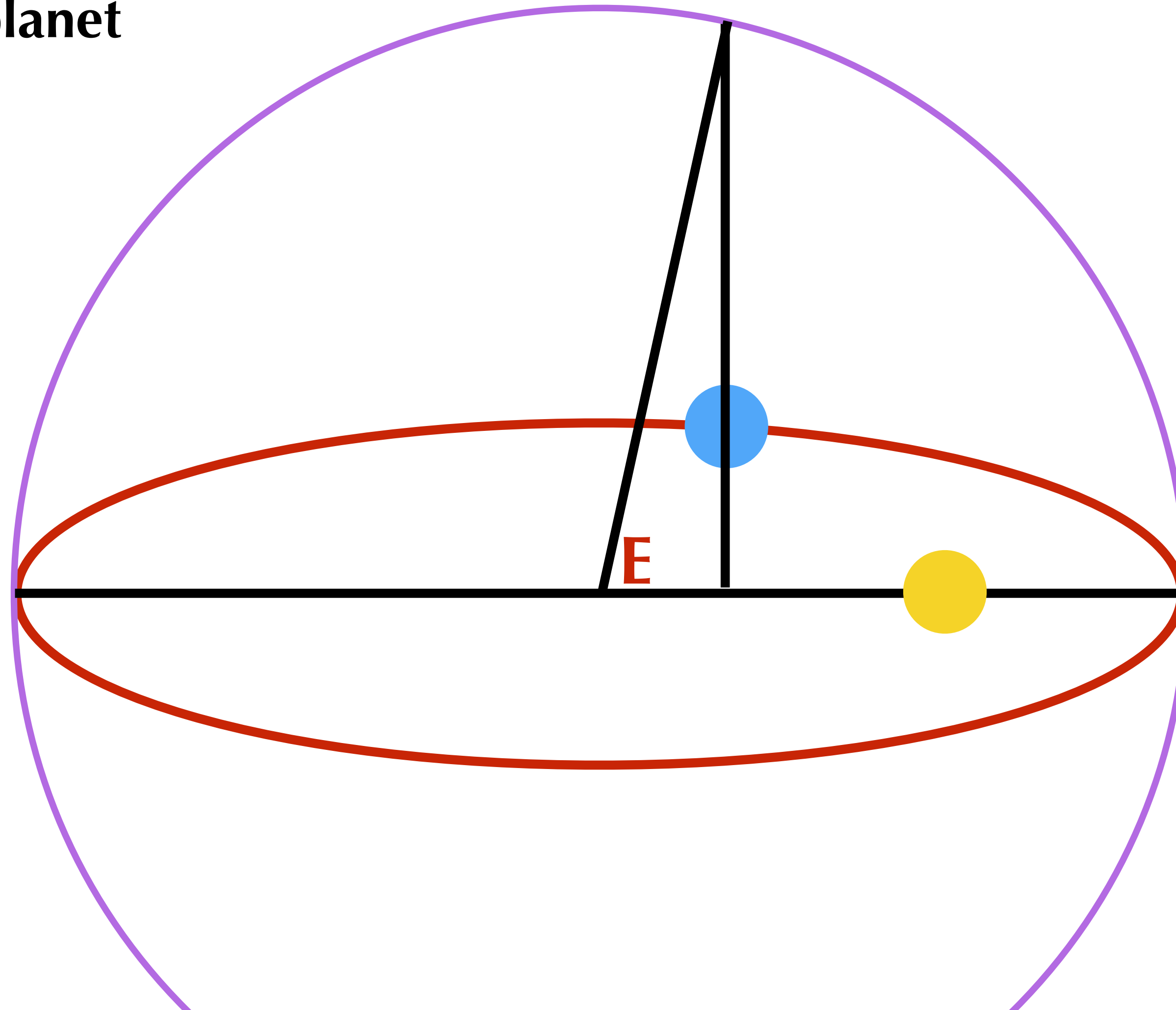
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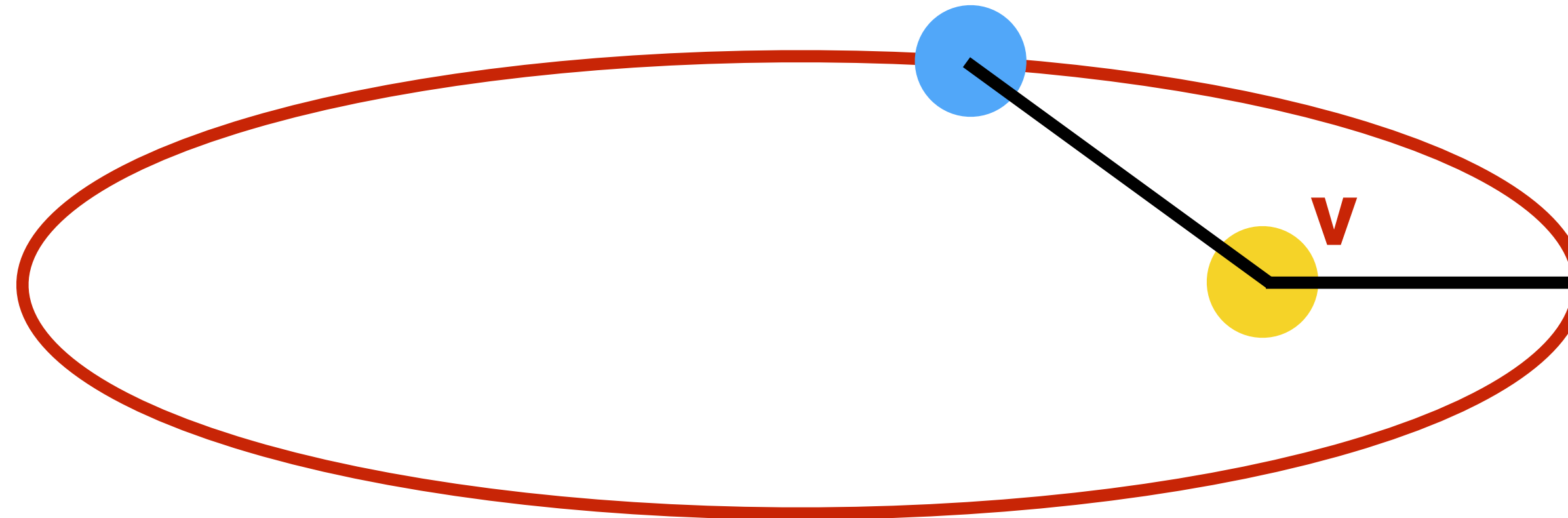


Anomalies

Mean Anomaly (M , $0-2\pi$) — Fraction of a period since last periastron

Eccentric Anomaly (E , $0-2\pi$) — Angle between periastron, center of ellipse, and (kind of) planet

True Anomaly (v , $0-2\pi$) — Angle between periastron, star, and planet



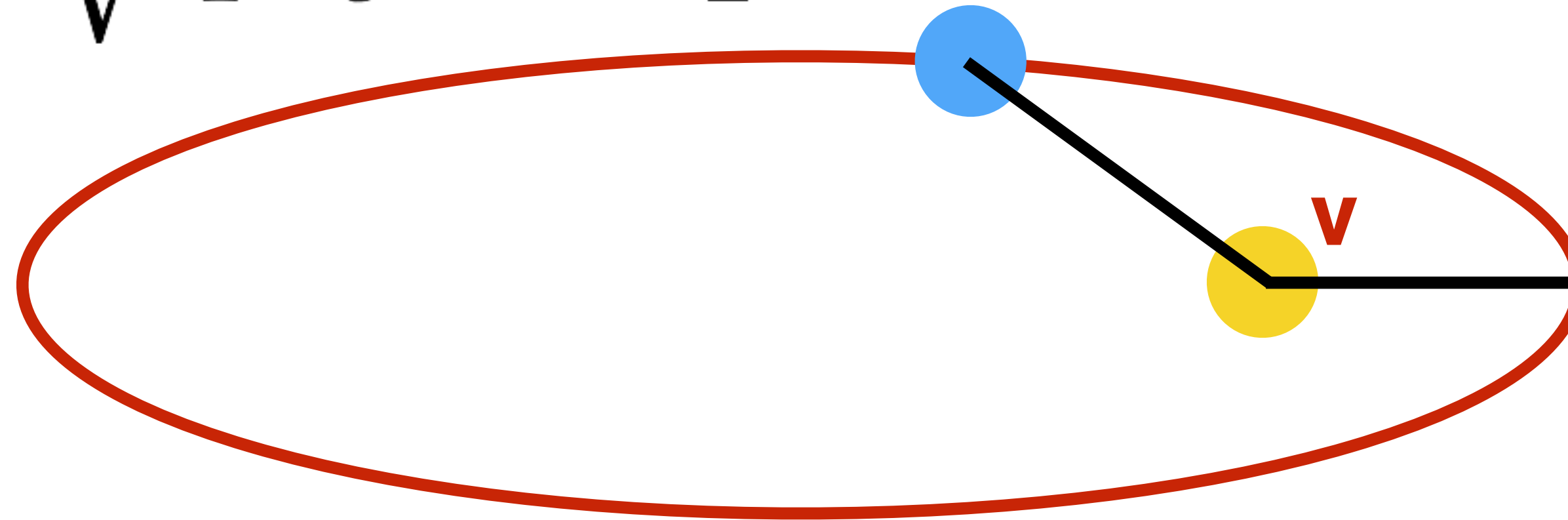
Anomalies

Mean Anomaly (M , $0-2\pi$) — Fraction of a period since last periastron

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True Anomaly (ν , $0-2\pi$) — Angle between periastron, star, and planet

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$



Anomalies

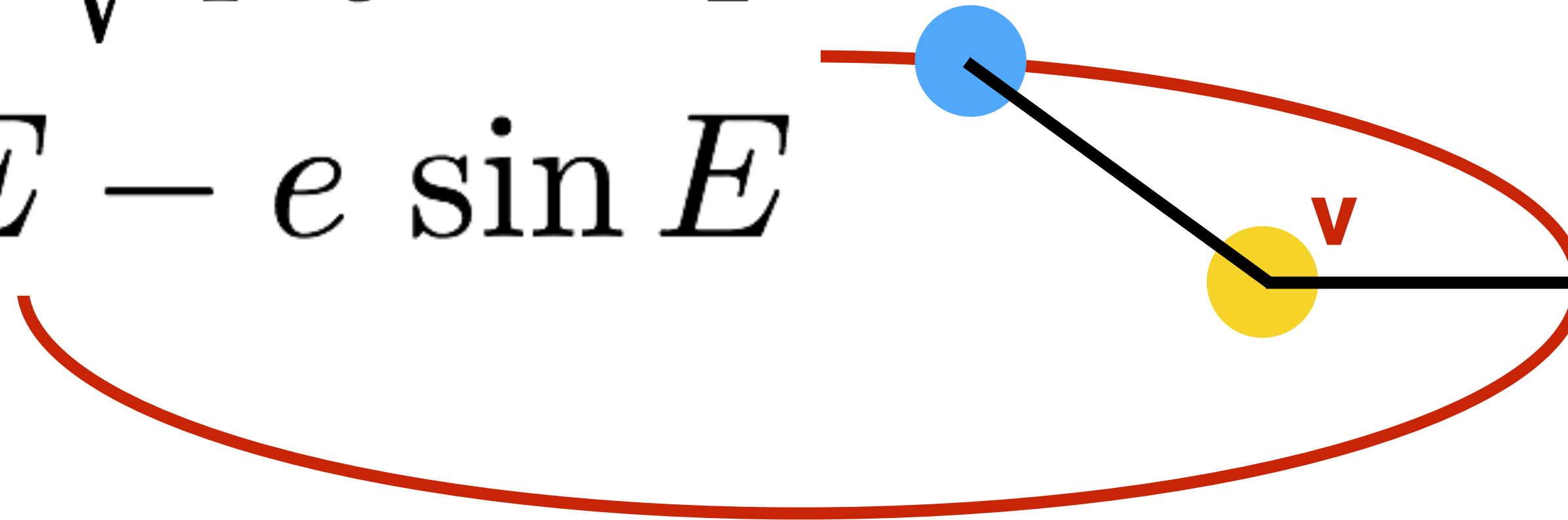
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$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

$$M = E - e \sin E$$



Anomalies

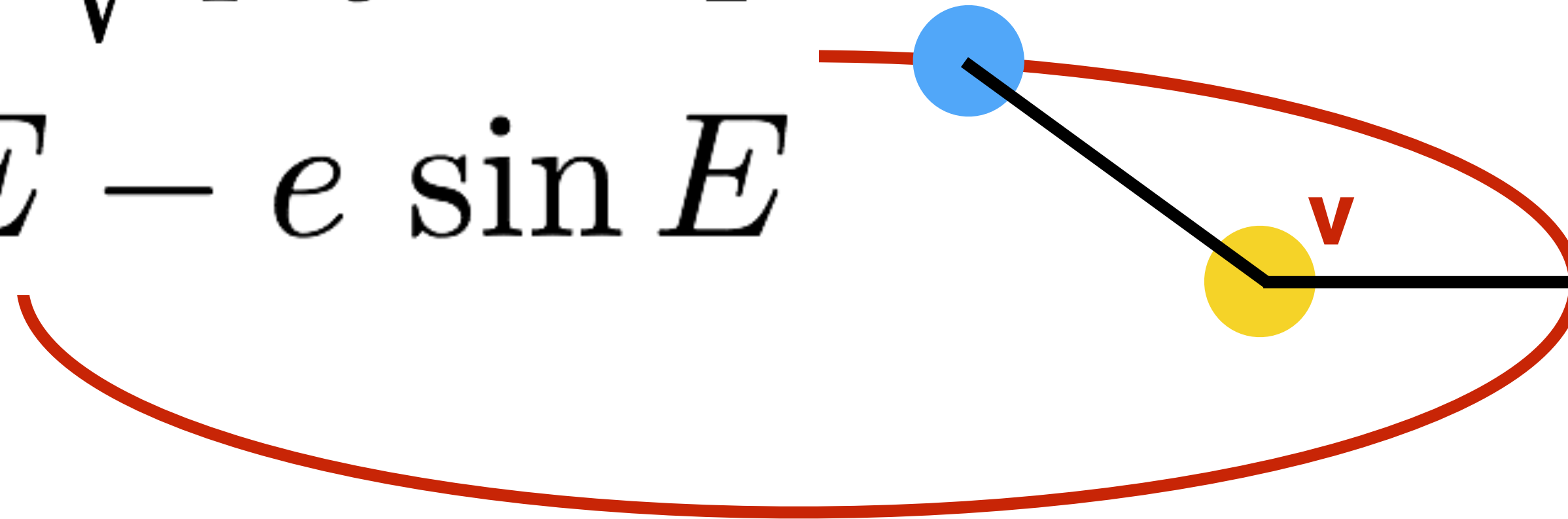
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$$M = E - e \sin E$$



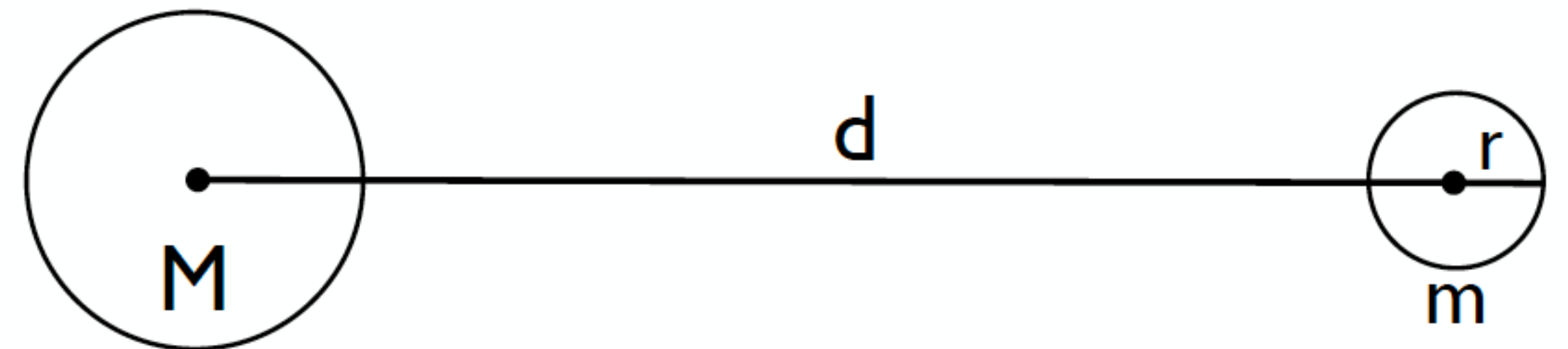
$$\Delta RA = \frac{a(1-e^2)}{1+e \cos(\nu)} (\cos \nu \cos i \cos \Omega \sin \omega + \cos \nu \sin \Omega \cos \omega + \sin \nu \cos i \cos \Omega \cos \omega - \sin \nu \sin \Omega \sin \omega)$$

$$\Delta Dec = \frac{a(1-e^2)}{1+e \cos \nu} (-\cos \nu \cos i \sin \Omega \sin \omega + \cos \nu \cos \Omega \cos \omega - \sin \nu \cos i \sin \Omega \cos \omega - \sin \nu \cos \Omega \sin \omega)$$

Tidal Force

- The difference in gravitational force from one part of an object to another part is called the Tidal Force

$$F_{tidal} = \frac{GMm}{d^2} - \frac{GMm}{(d+r)^2}$$



Tidal Force

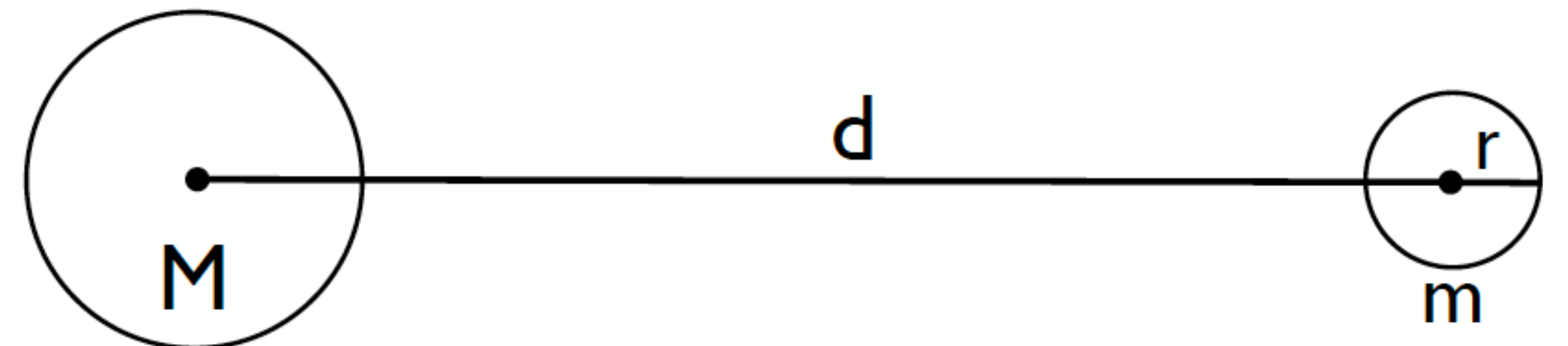
- The difference in gravitational force from one part of an object to another part is called the Tidal Force

$$F_{tidal} = \frac{GMm}{d^2} - \frac{GMm}{(d+r)^2}$$

- Get a common denominator:

$$F_{tidal} = \frac{GMm(d+r)^2}{d^2(d+r)^2} - \frac{GMmd^2}{d^2(d+r)^2}$$

$$F_{tidal} = \frac{GMm(d+r)^2 - GMmd^2}{d^2(d+r)^2}$$



Tidal Force

-

$$F_{tidal} = \frac{GMm(d+r)^2 - GMmd^2}{d^2(d+r)^2}$$

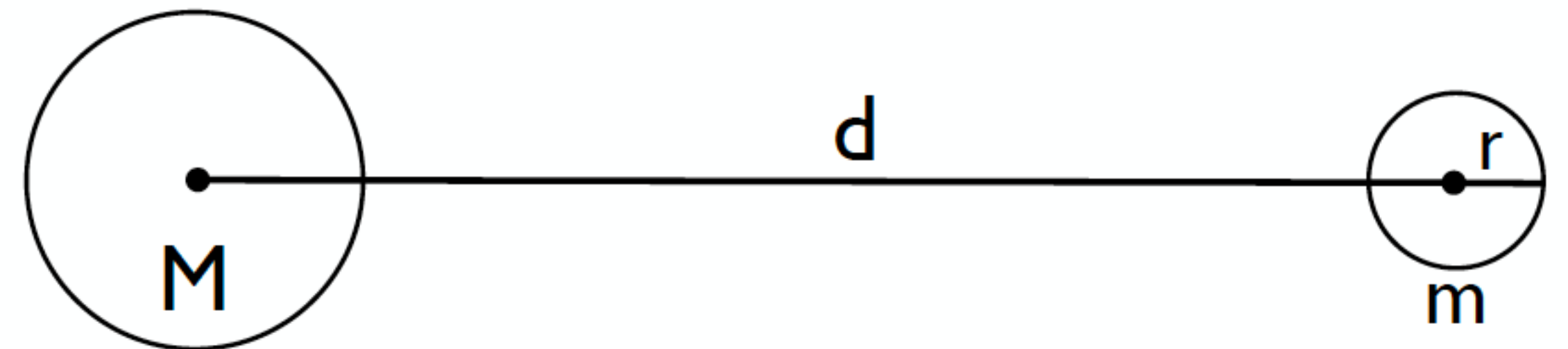
- Simplify, then expand out terms:

$$F_{tidal} = GMm \frac{d^2 + 2rd + r^2 - d^2}{d^4 + 2rd^3 + d^2r^2}$$

$$F_{tidal} = GMm \frac{2rd + r^2}{d^4 + 2rd^3 + d^2r^2}$$

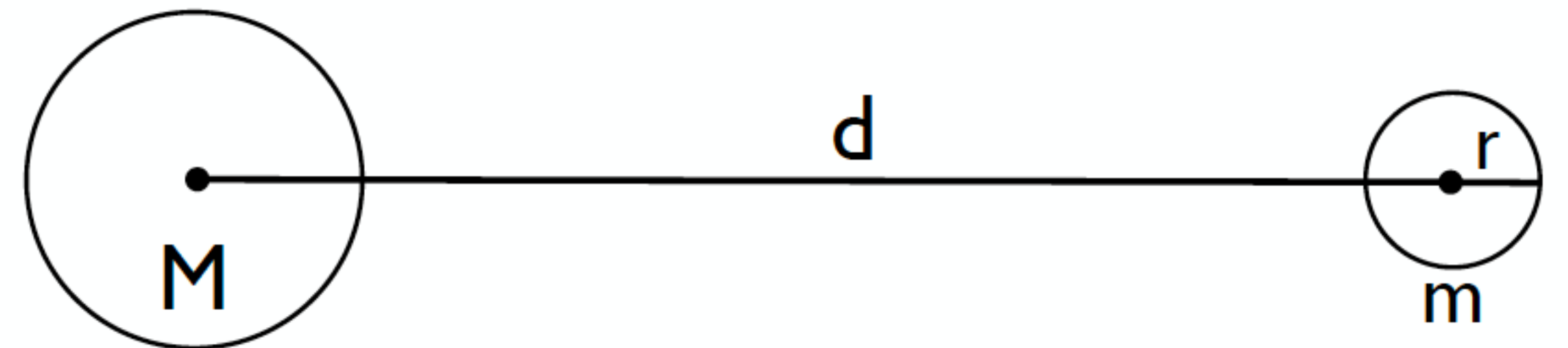
- If we assume $d \gg r$, then we can drop a few terms:

$$F_{tidal} = GMm \frac{2rd}{d^4} = \frac{2GMmr}{d^3}$$



Tidal Force

- $$F_{tidal} = \frac{2GMmr}{d^3}$$
- Magnitude falls off as $1/d^3$
- Magnitude decreases for smaller planets (smaller r)



Response Card Question

- Consider two objects (say, the Earth and the Moon). What is the ratio between the tidal force ($F_{tidal} = \frac{2GMmr}{d^3}$) and the force of gravity felt by the two objects?
 - (A) — The force of gravity is $2\frac{r}{d}$ times the tidal force
 - (B) — The tidal force is $2\frac{r}{d}$ times the force of gravity
 - (C) — The force of gravity is $2\frac{r}{d^2}$ times the tidal force
 - (D) — The tidal force is $2\frac{r}{d^2}$ times the force of gravity
- (E) — The two forces are the same

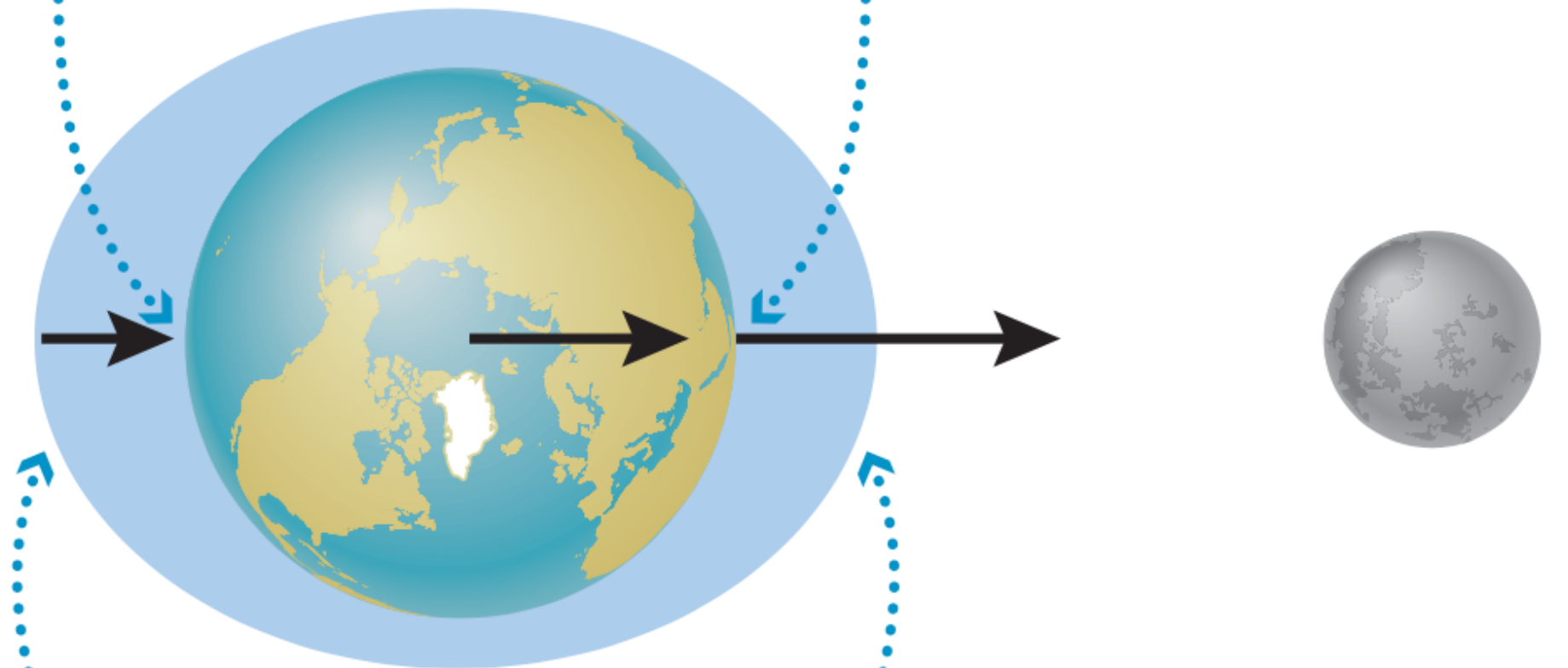
Gravity and Tides

- Parts of the Earth closer to the Moon feel a stronger gravitational force than parts near the center
- Parts of the Earth furthest from the Moon feel a weaker gravitational force than parts near the center
- The oceans have a tidal bulge, with higher tides towards the Moon, and higher tides away from the Moon

The gravitational attraction to the Moon is weakest here...

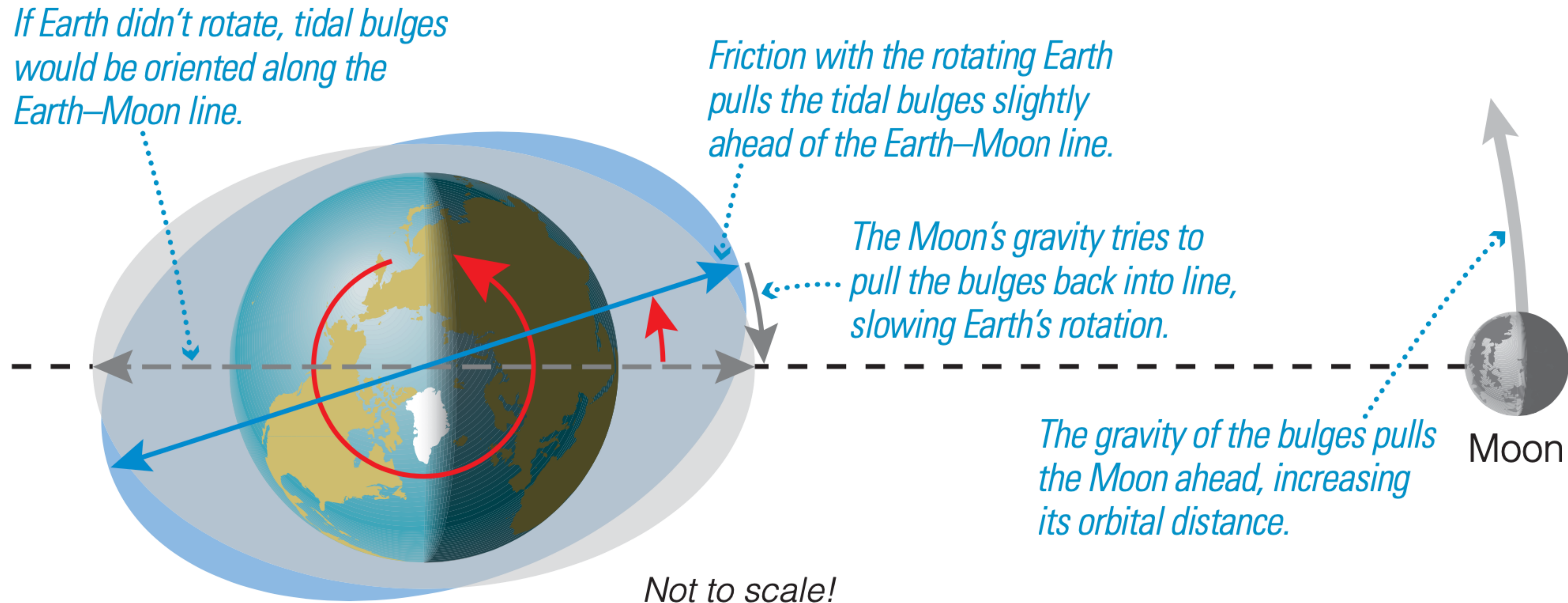
...and strongest here.

The difference in gravitational attraction tries to stretch Earth, raising tidal bulges both toward and away from the Moon.



Gravity and Tides

- Tides also stretch the land on Earth, leading to “tidal friction” that heats up the planet
- Tidal friction results in the tidal bulge pointing about 20 degrees in front of the Moon’s path
- The Earth’s rotation rate is slowing down, and the Moon is moving further away (slowly!)



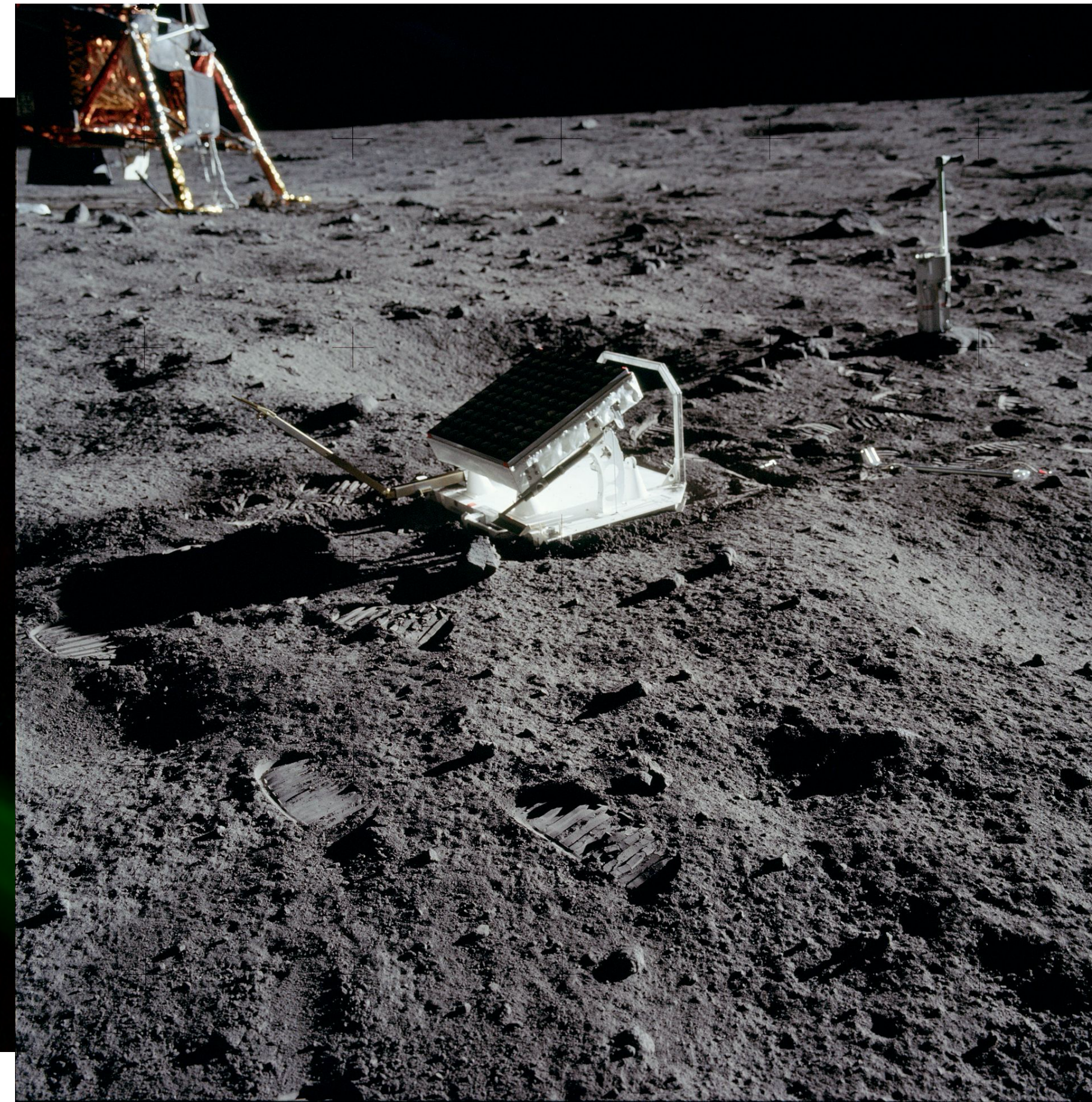
The distance to the Moon

- Apollo astronauts put “retroreflectors” on the Moon’s surface
- Astronomers can bounce laser beams off those retroreflectors, and measure how much time it takes for light to return
- From Apache Point Observatory, we can measure the distance to the Moon to a precision of 1 mm
- The Moon is moving away from the Earth at 4 cm per year
- An Earth day gets longer by 1 second every 50,000 years



Dan Long (Apache Point Observatory)

NASA



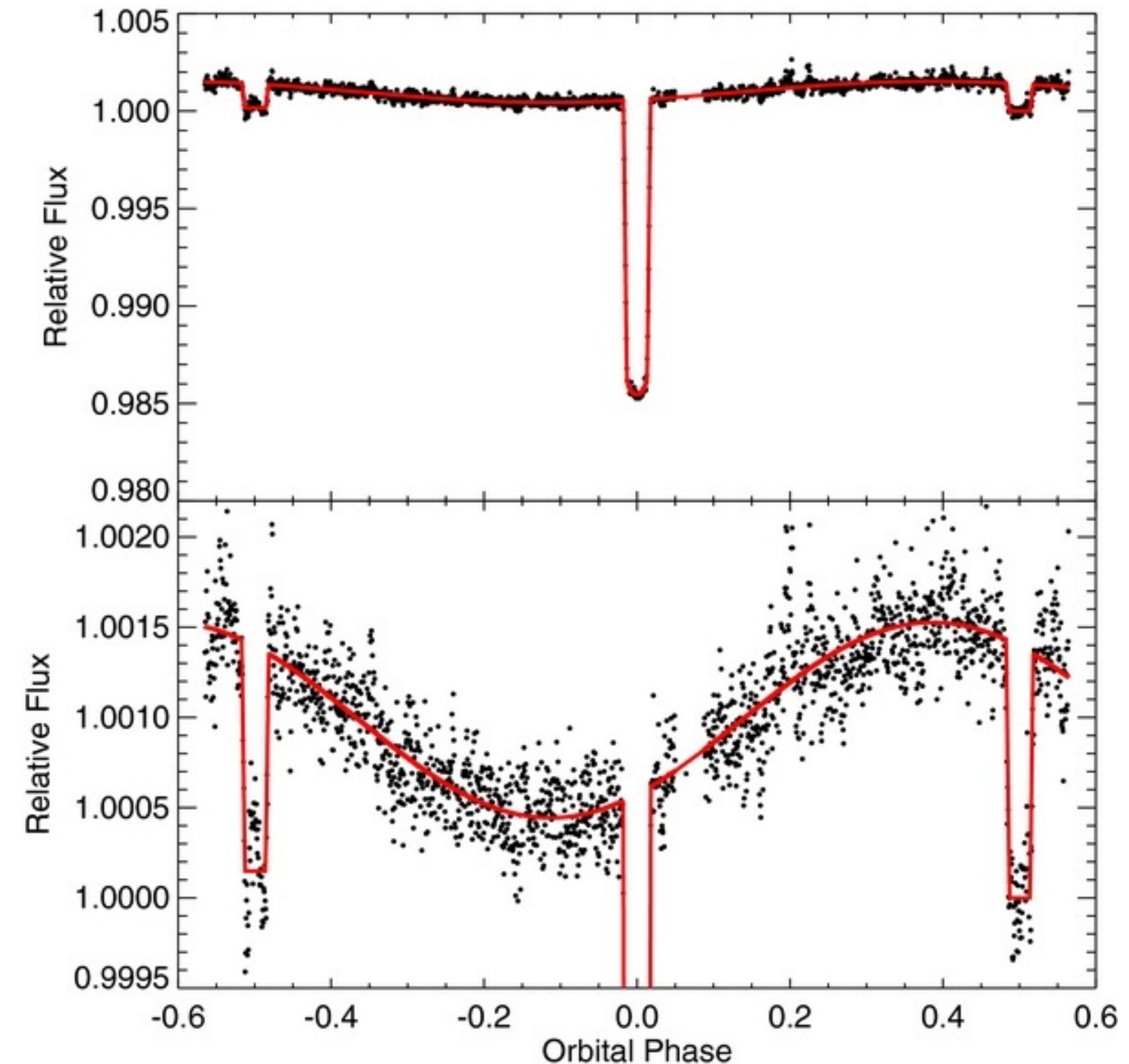
Tides, the Moon, and synchronous rotation

- The early Moon probably rotated much faster than its orbital motion, and tidal forces from Earth were continually squeezing the Moon
- This tidal friction slowed down the Moon's rotation rate until it matched the orbital rate
- From that point on, the Moon was in synchronous rotation (also called being "tidally locked")



Tidal locking

- Small objects orbiting close to a larger object tend to either be in synchronous rotation, or are moving toward synchronous rotation
- Moons of giant planets tend to be tidally locked
- Exoplanets close to their star tend to be tidally locked



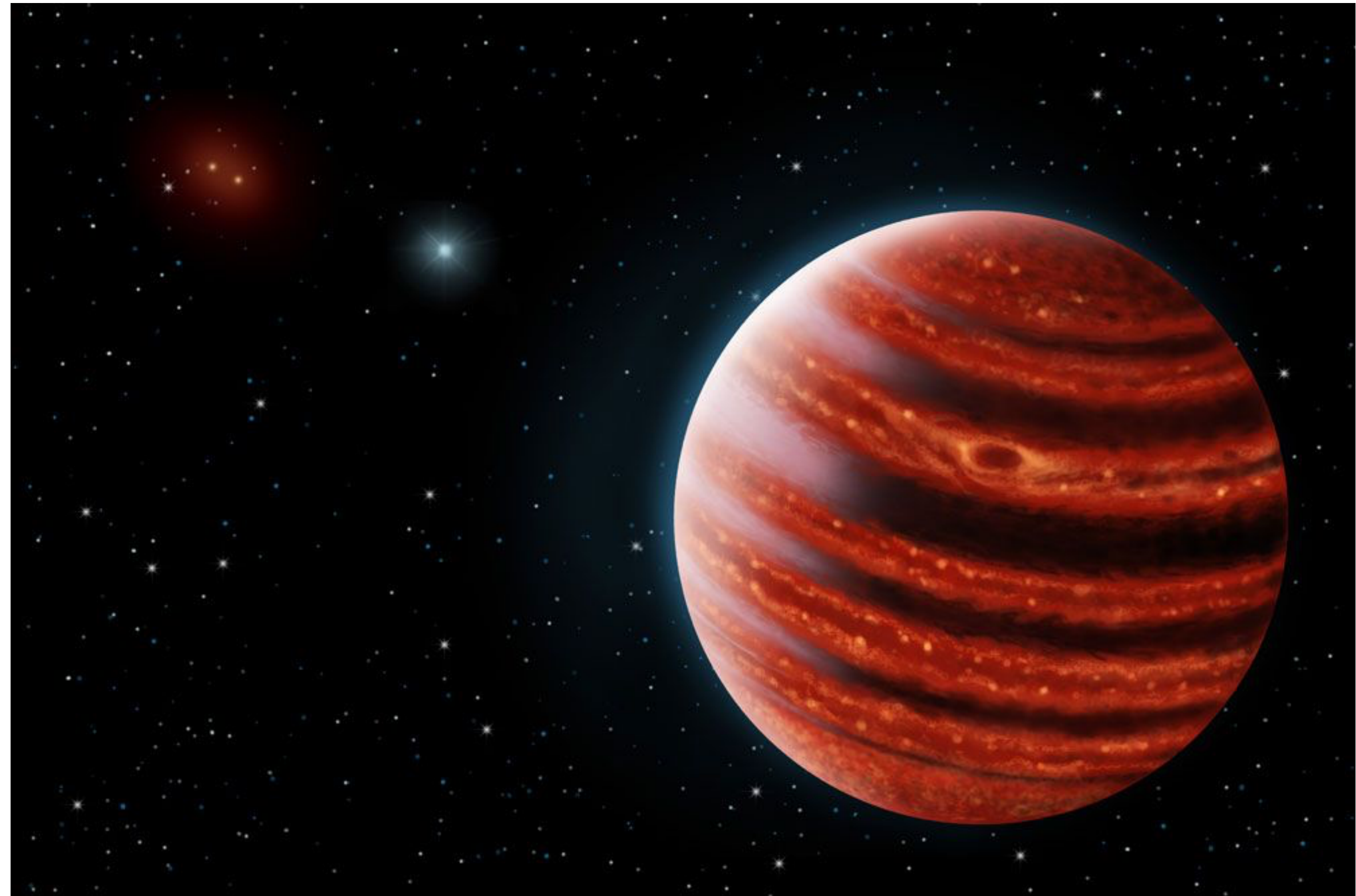
HD 209458 b, Zellem et al. 2014

Break

05:00

The 3-body problem

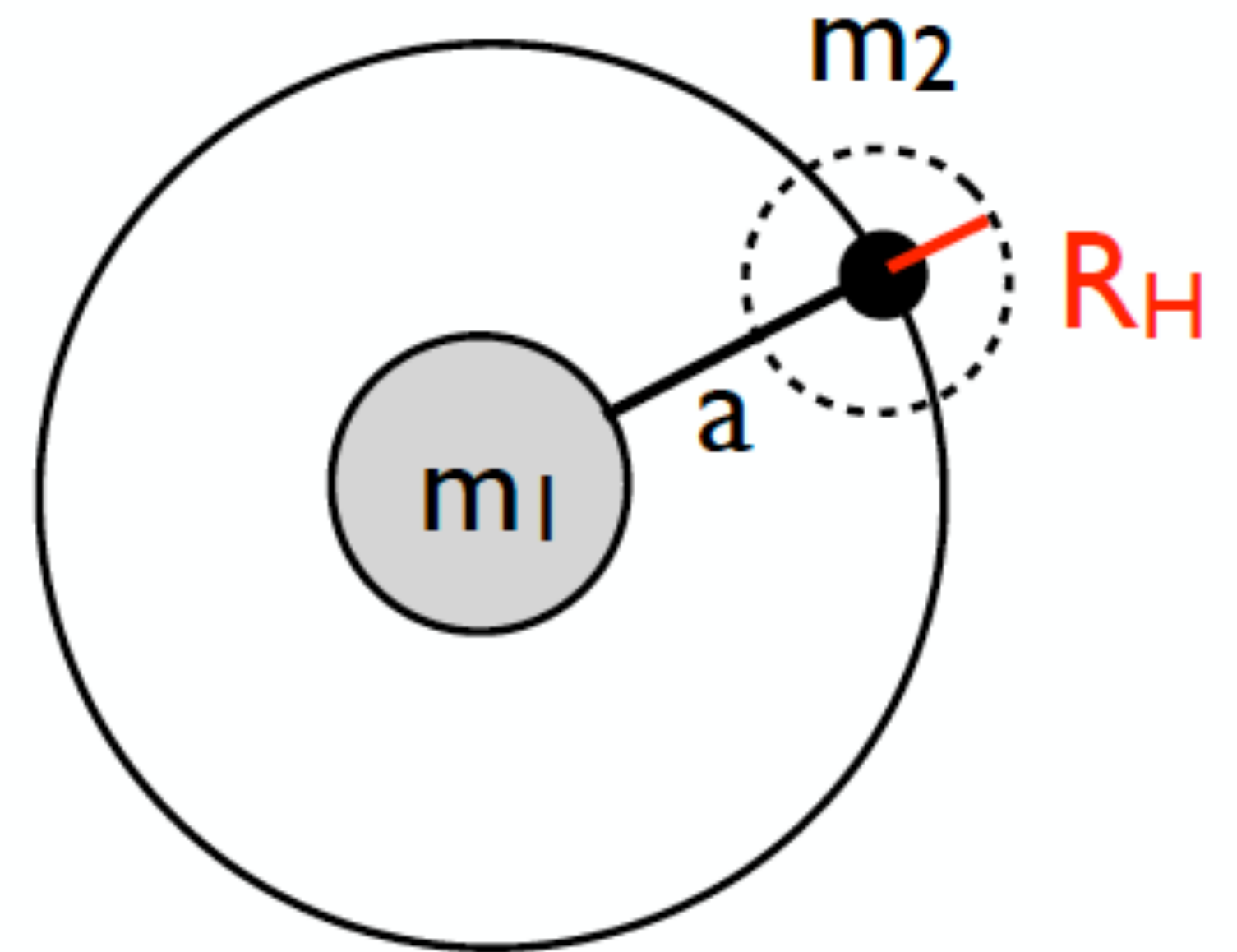
- There is an exact solution to the 2-body problem (how two objects move as a function of initial conditions and their own gravity)
- There is not an analytic solution to the 3-body problem
- There are approximations in special cases:
 - The restricted 3-body problem (When 1 mass is much smaller than the other 2 masses)
 - Hill's problem (When 1 mass is much larger than the other 2 masses)



Hill Sphere

- Gravitational “sphere of influence” around a secondary
- Inside a planet’s Hill sphere, the planet’s gravity is more important to a test particle
- Outside the Hill sphere, the Sun’s gravity is more important

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)} \right)^{\frac{1}{3}} a * (1 - e)$$

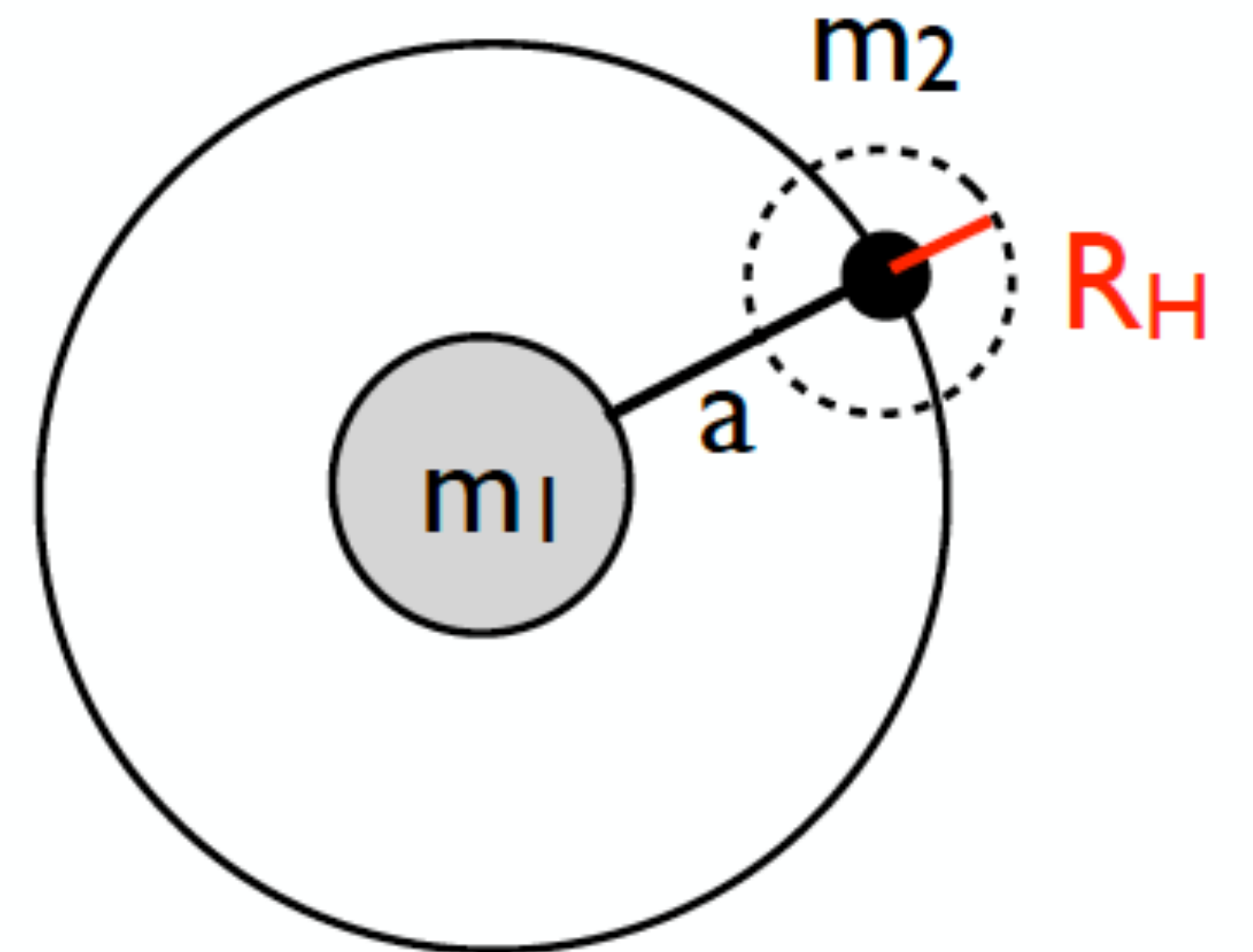


Hill Sphere

-

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)} \right)^{\frac{1}{3}} a * (1 - e)$$

- A planet's moons and rings will be within that planet's Hill Sphere

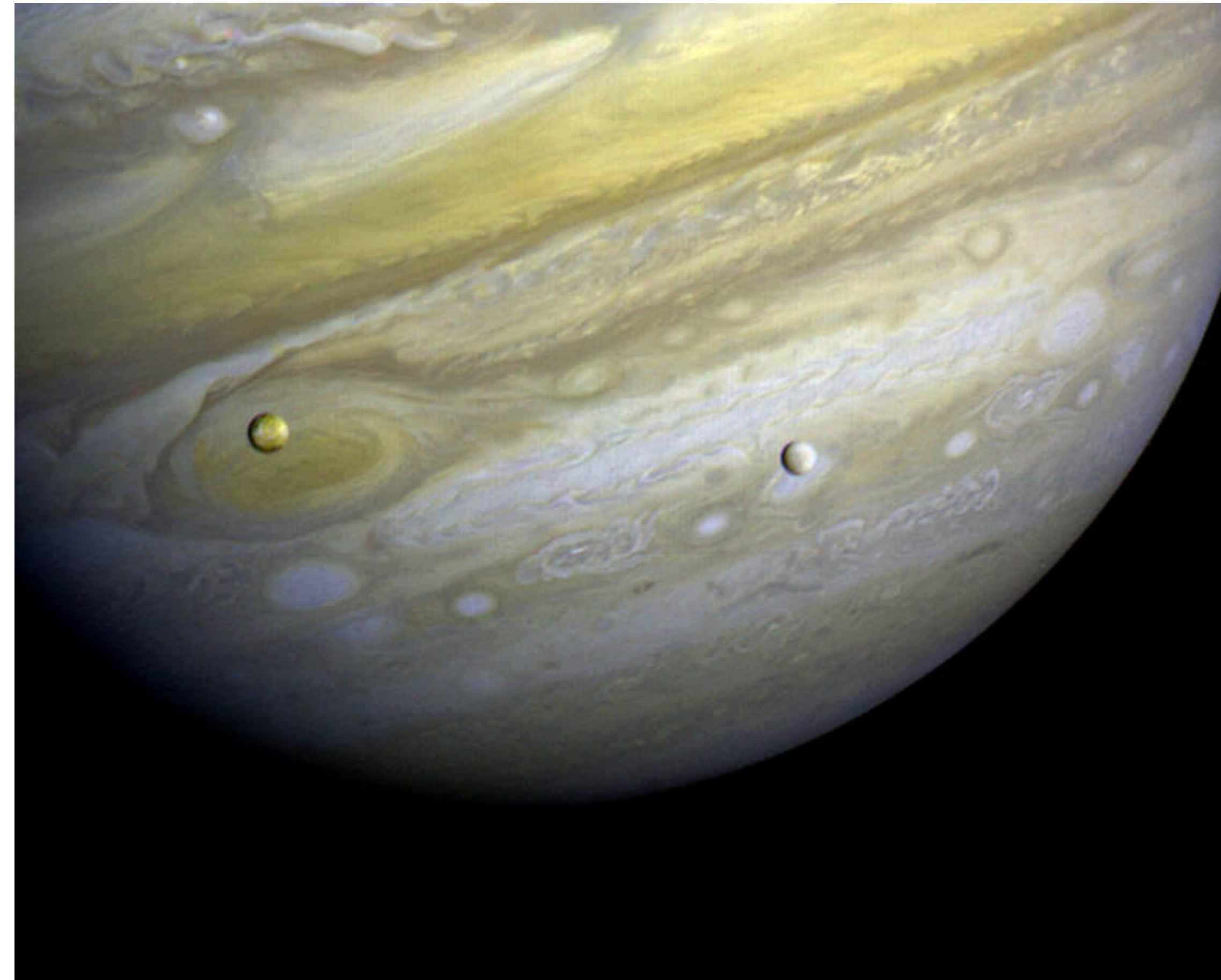


In-Class Activity

Order of Magnitude

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)} \right)^{\frac{1}{3}} a * (1 - e)$$

- (1) What is the radius of Jupiter's Hill Sphere, in AU?
- (2) If Jupiter had a moon at the edge of (but just inside) its Hill Sphere, what would its orbital period be?
- (3) Think of the solar system as a 2-dimensional disk. What fraction of that disk within Jupiter's orbit around the Sun is contained within Jupiter's Hill Sphere?



In-Class Activity

Order of Magnitude

- (1) Jupiter has about 1/1000 (10^{-3}) the mass of the Sun, so in solar masses: 10^{-3} The semi-major axis of Jupiter is 5.2 AU, and has a roughly circular ($e=0$) orbit. Plugging in numbers:

$$R_H = \left(\frac{M_2}{3(M_1 + M_2)} \right)^{\frac{1}{3}} a * (1 - e) = \left(\frac{10^{-3}}{3(1 + 10^{-3})} \right)^{\frac{1}{3}} 5.2 * (1 - 0) = \left(\frac{1}{3} \right)^{\frac{1}{3}} * (10^{-3})^{\frac{1}{3}} * 5.2 = 1 * 10^{-1} * 5.2 = 0.5 AU$$

- (2) Using Kepler's third law:

$$P^2 = \frac{a^3}{M} \quad \text{So plugging in M and } a=0.5 \text{ AU:} \quad P = \sqrt{\frac{a^3}{M}} = \sqrt{\frac{0.5^3}{10^{-3}}} = \sqrt{0.125 * 10^3} = \sqrt{10^2} = 10 \text{ years}$$

- (3) Area of a 2D disk is just area of a circle: $A = \pi R^2$, so the ratio of areas of Jupiter's Hill Sphere and the Solar System out to Jupiter is:

$$\frac{A_H}{A_{SS}} = \frac{\pi R_H^2}{\pi d_J^2} = \left(\frac{0.5 AU}{5.2 AU} \right)^2 = (0.1)^2 = 0.01 = 1 \%$$

For next time

- Find a copy of the textbook (remember: either version of the 2nd edition works — NOT the first edition)
- Reading: de Pater & Lissaeuer Chaper 2, section 2.6.1, 2.7, 2.7.1
- Homework 1 due in 5 days, August 29 at 11:59:59 on Canvas (reminder, late homework loses 10% of possible points each day)