ASTR 620: Planetary Processes
Professor Eric Nielsen

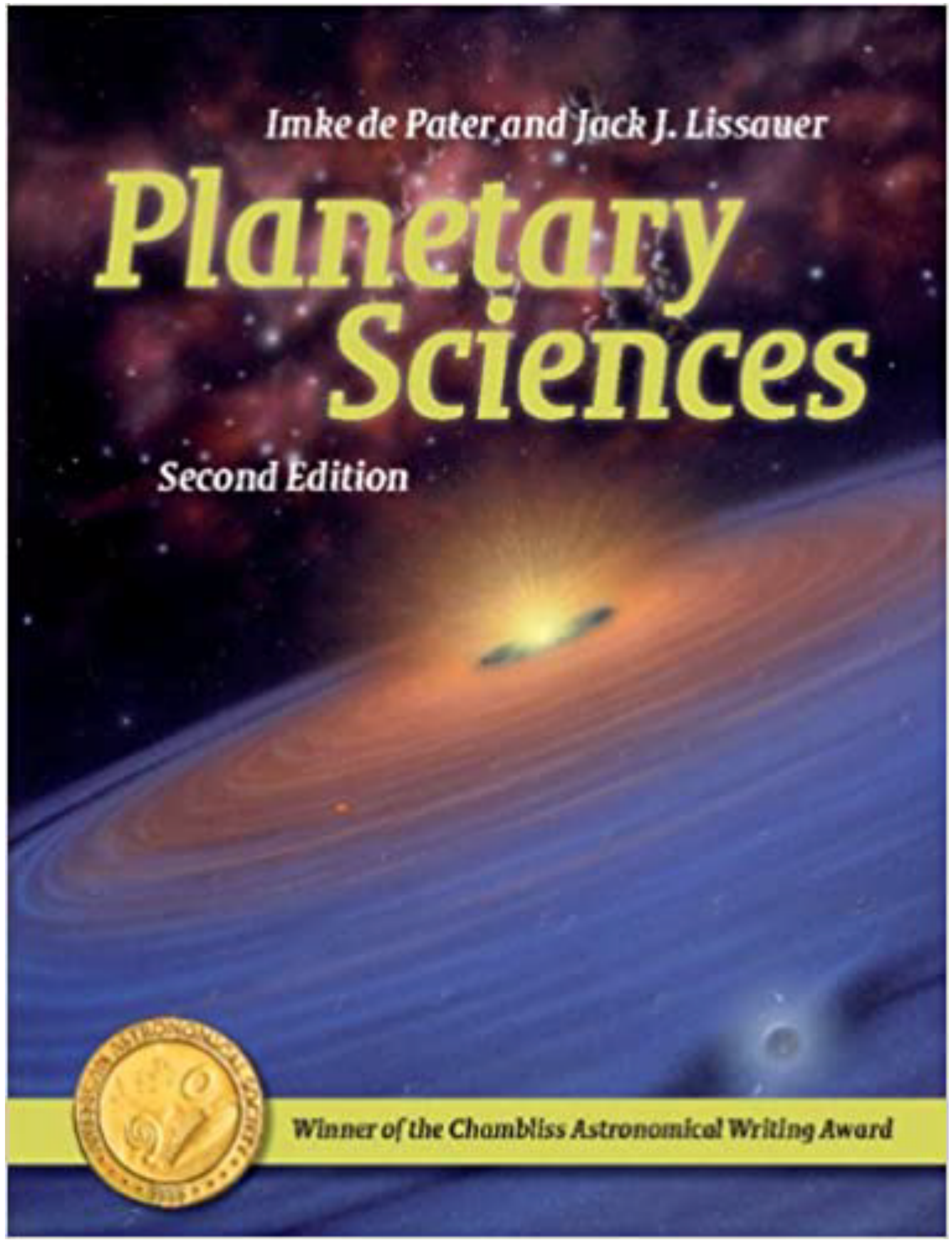
Lecture 2: Gravity and Orbits

Logistics

- Masks are encouraged
- No laptops, phones, or other electronic devices during class (I'll let you know in advance if we'll need laptops for an activity) **New: you may use a tablet to take notes if prefer, but please only use it for note-taking.**
- Things you should bring to lecture each day:
  - something to take notes on (including extra paper if there's a writing assignment),
  - something to write with
  - your response card (will be handed out later in class)
- Office hours: Monday 4-5pm, Astronomy 203

Textbook

- Readings and some homework will be assigned from the textbook
- Either “second edition” or “updated second edition” is fine
  - try to avoid the first edition, it’s a bit out of date



Review of the last class

- Which object contains most of the angular momentum (L = m*v*r) in our Solar System?
    - (A) — Earth
    - (B) — Jupiter
    - (C) — Saturn
    - (D) — Neptune
    - (E) — Sun

Review of the last class

- The region of space dominated by the Sun’s plasma and magnetic fields, separate from the ISM, is called the:
  - (A) — heliosphere, and contains all objects orbiting the Sun
  - (B) — heliosphere, and contains most objects orbiting the Sun
  - (C) — corona, and contains all objects orbiting the Sun
  - (D) — corona, and contains most objects orbiting the Sun
  - (E) — There is no Sun

Review of the last class

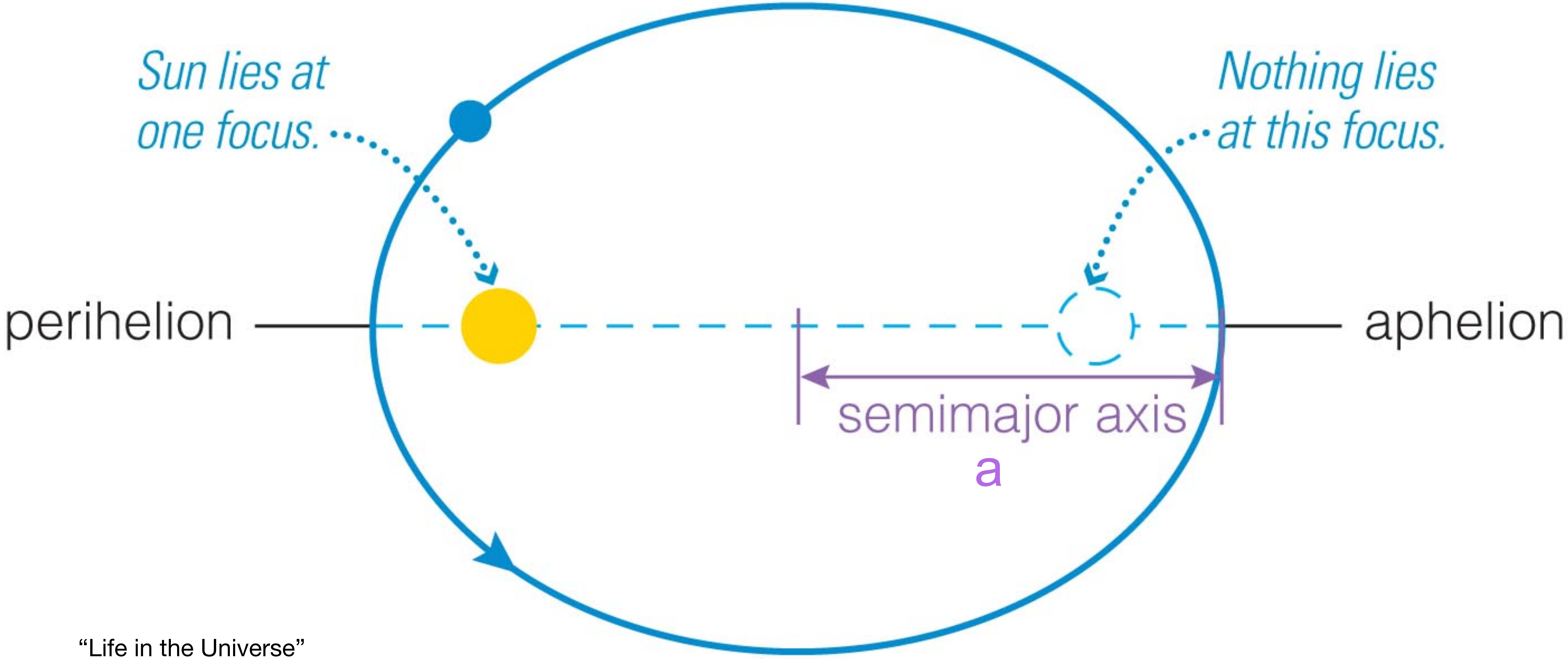
- How do we measure the masses of other objects in the solar system?
  - (A) — stellar occultations
  - (B) — radar mapping
  - (C) — light curves
  - (D) — orbits of moons and spacecrafts
  - (E) — Thermal IR spectra

Review of the last class

- What is Jupiter's mass, in solar masses?
  - (A) — 10^{-5} solar masses
  - (B) — 10^{-4} solar masses
  - (C) — 10^{-3} solar masses
  - (D) — 10^{-2} solar masses
  - (E) — 10^{-1} solar masses

Kepler’s First Law

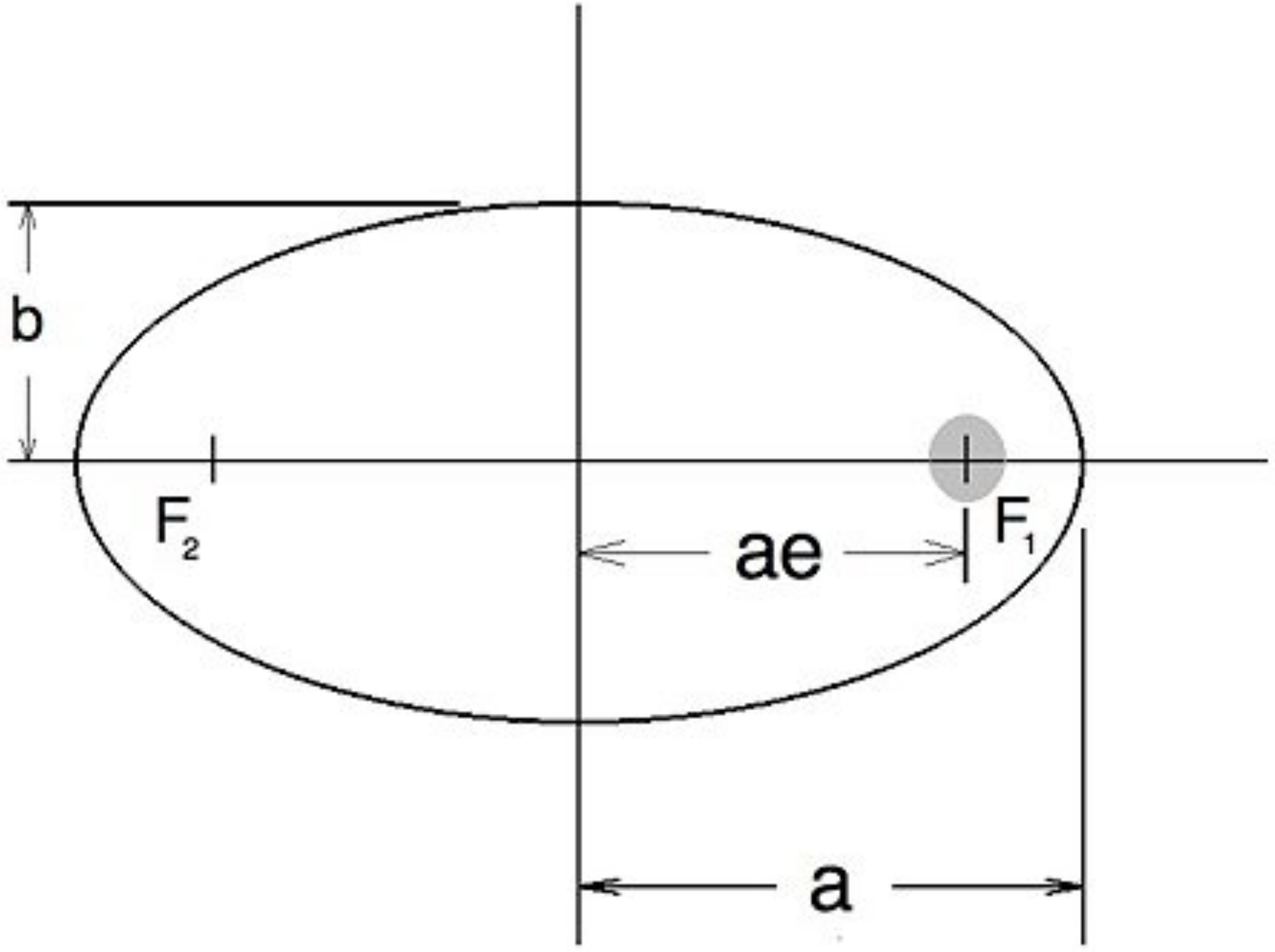
- The orbit of each planet around the Sun is an ellipse with the Sun at one focus.



“Life in the Universe”

Ellipses

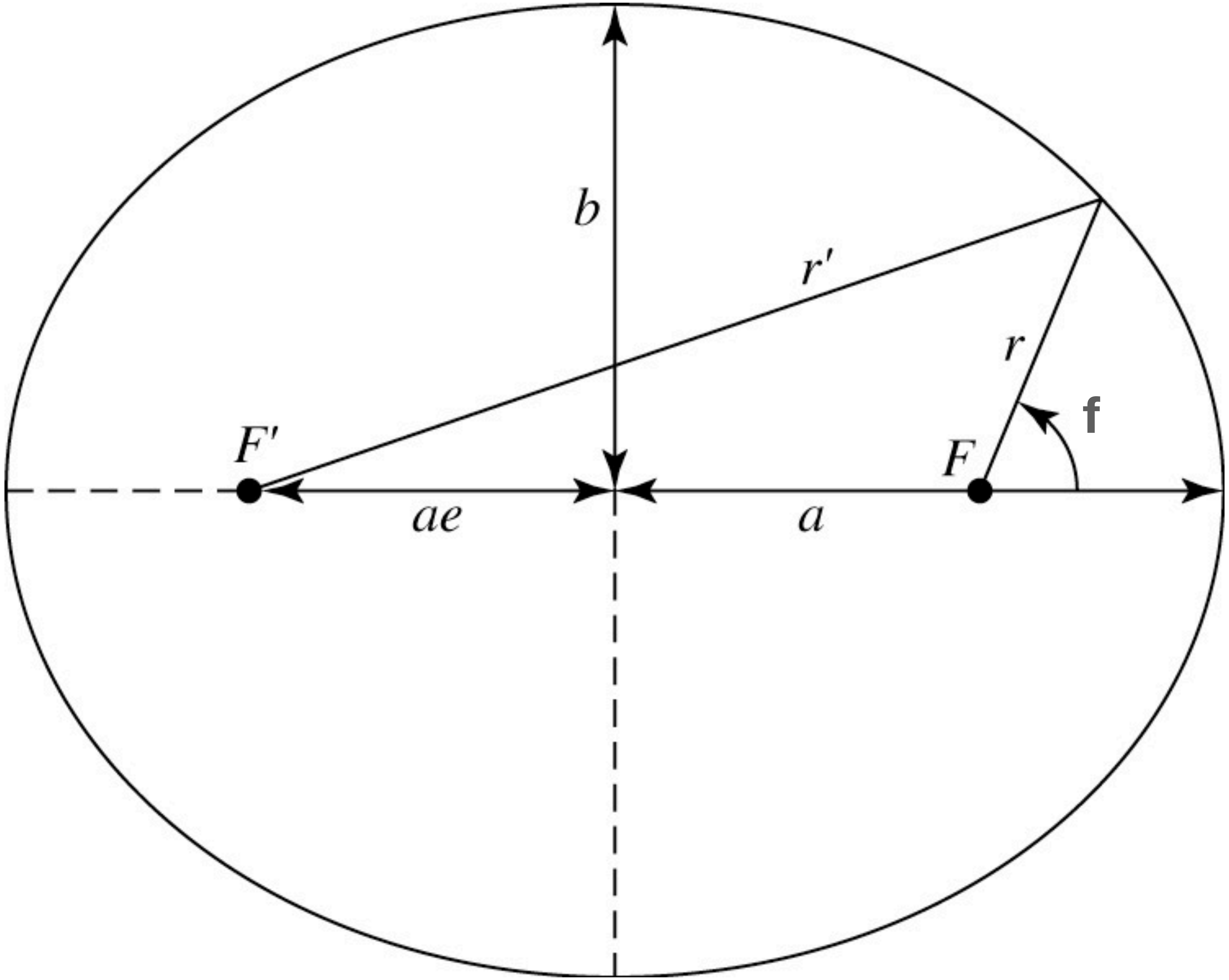
- Major axis: 2a (a = semi-major axis)
- Minor axis: 2b (b = semi-minor axis)
- Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$
- We describe elliptical orbits by their semi-major axis and eccentricity



wikipedia

Ellipses

- The set of points where the distance from one focus, to that point, to the other focus is equal defines the ellipse

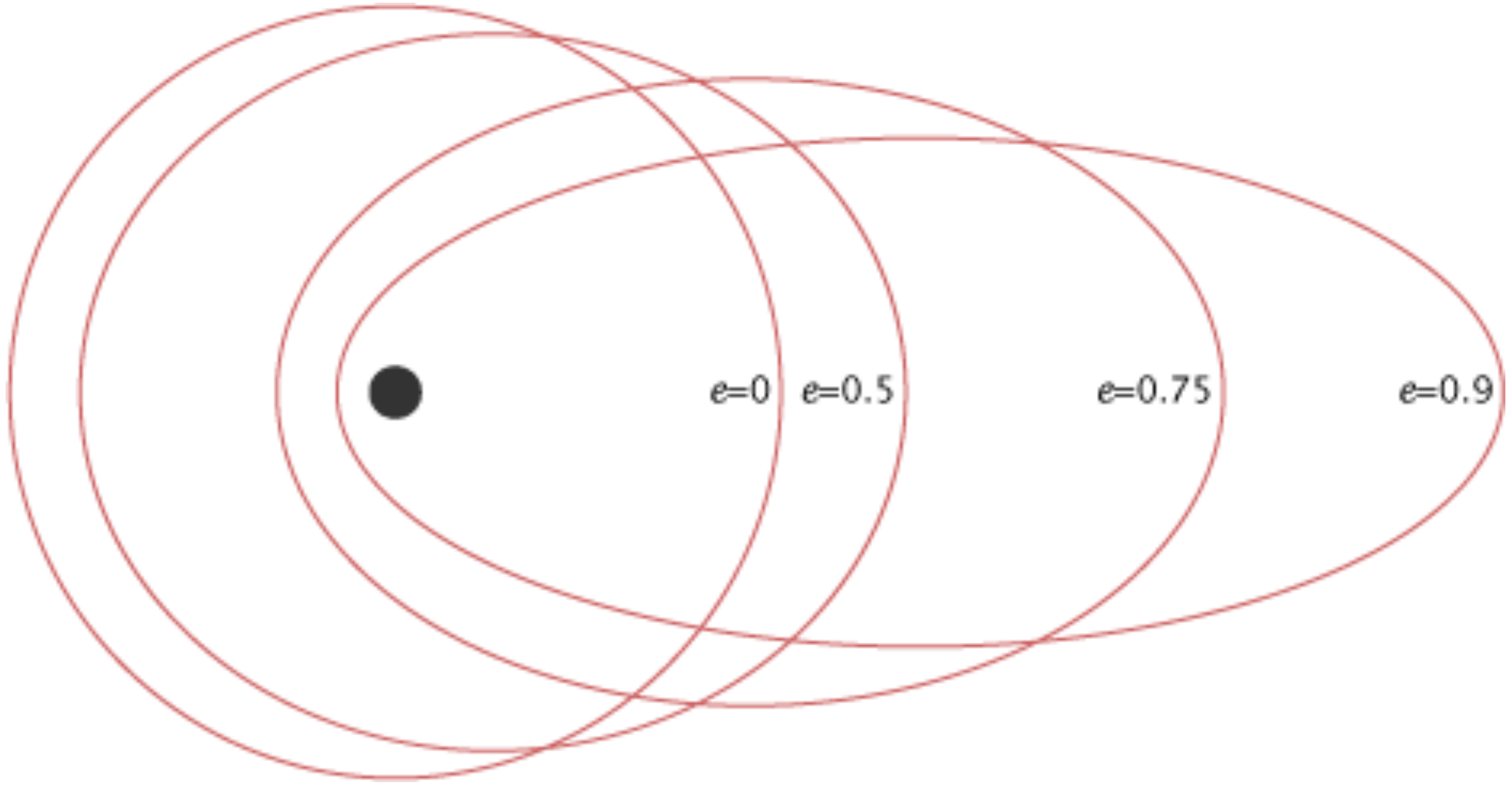


Response Card Question

- What shape do we call an ellipse when e=0, and what shape do we call an ellipse when e=1?
  - (A) — e=0: circle, e=1: oval
  - (B) — e=0: oval, e=1: circle
  - (C) — e=0: circle, e=1: parabola
  - (D) — e=0: parabola, e=1: circle
  - (E) — e=0: circle, e=1: line

Eccentricity

- Eccentricity (e) goes from 0 (circular orbits) to 1 (unbound orbit: a parabola)
- Closest approach, perihelion, is given by perihelion = (1-e) * a
- Furthest approach, aphelion = (1+e) * a
- -helion — Sun
- -gee — Earth
- -astron — Star



NASA

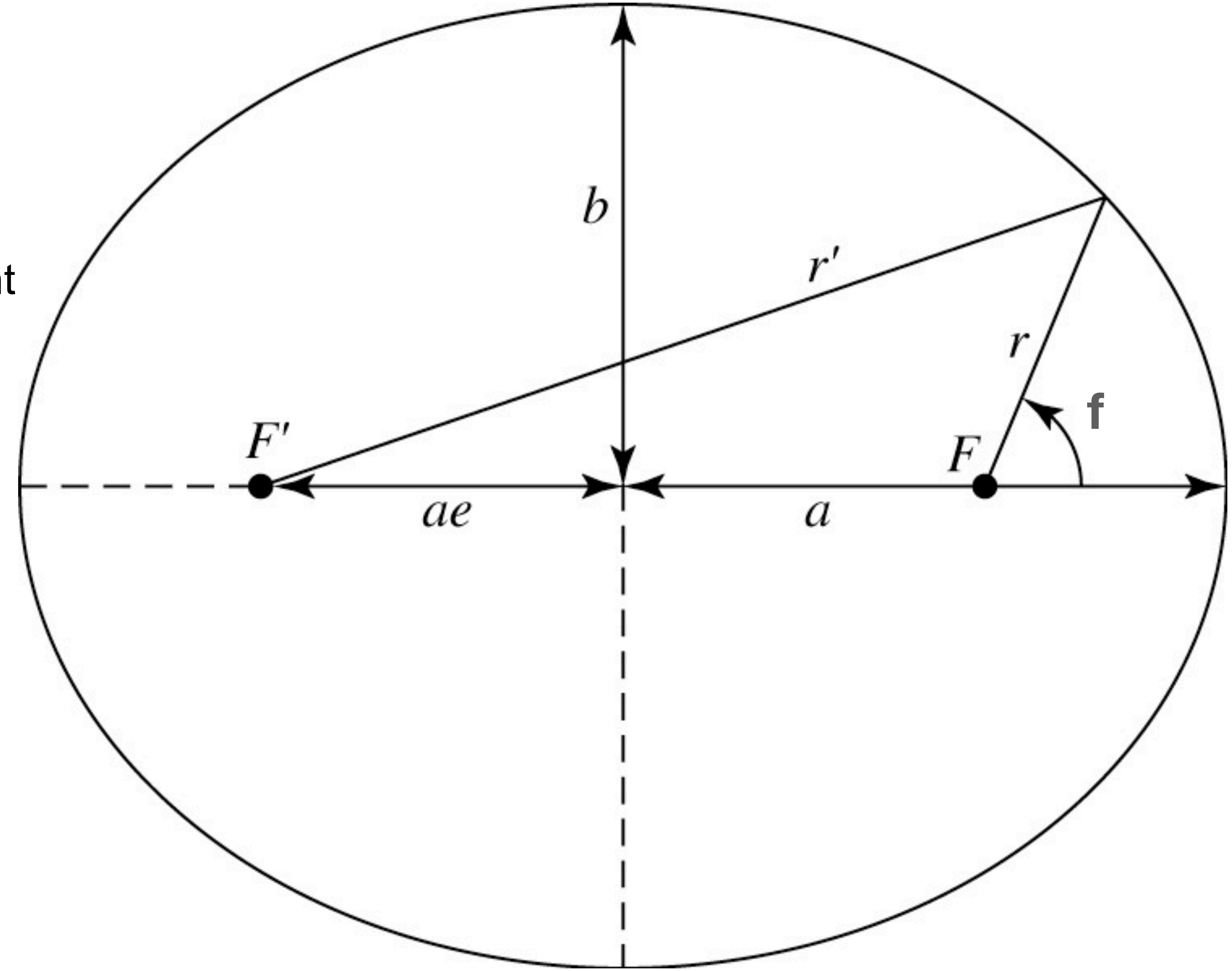
Eccentricity

Body	Closest approach	Farthest approach
General	Periapsis/Pericentre	Apoapsis
Galaxy	Perigalacticon	Apogalacticon
Star	Periastron	Apastron
Black hole	Perinigricon	Aponigricon
Sun	Perihelion	Aphelion
Mercury	Perihermion	Apohermion
Venus	Perikrition	Apokrition
Earth	Perigee	Apogee
Moon	Perilune	Apolune
Mars	Periareion	Apoareion
Jupiter	Perizene/Perijove	Apozene/Apojove
Saturn	Perikrone/Perisaturnium	Apokrone/Aposaturnium
Uranus	Periuranion	Apouranion
Neptune	Periposeidon	Apoposeidon
Pluto	Perihadion	Apohadion

Ellipses

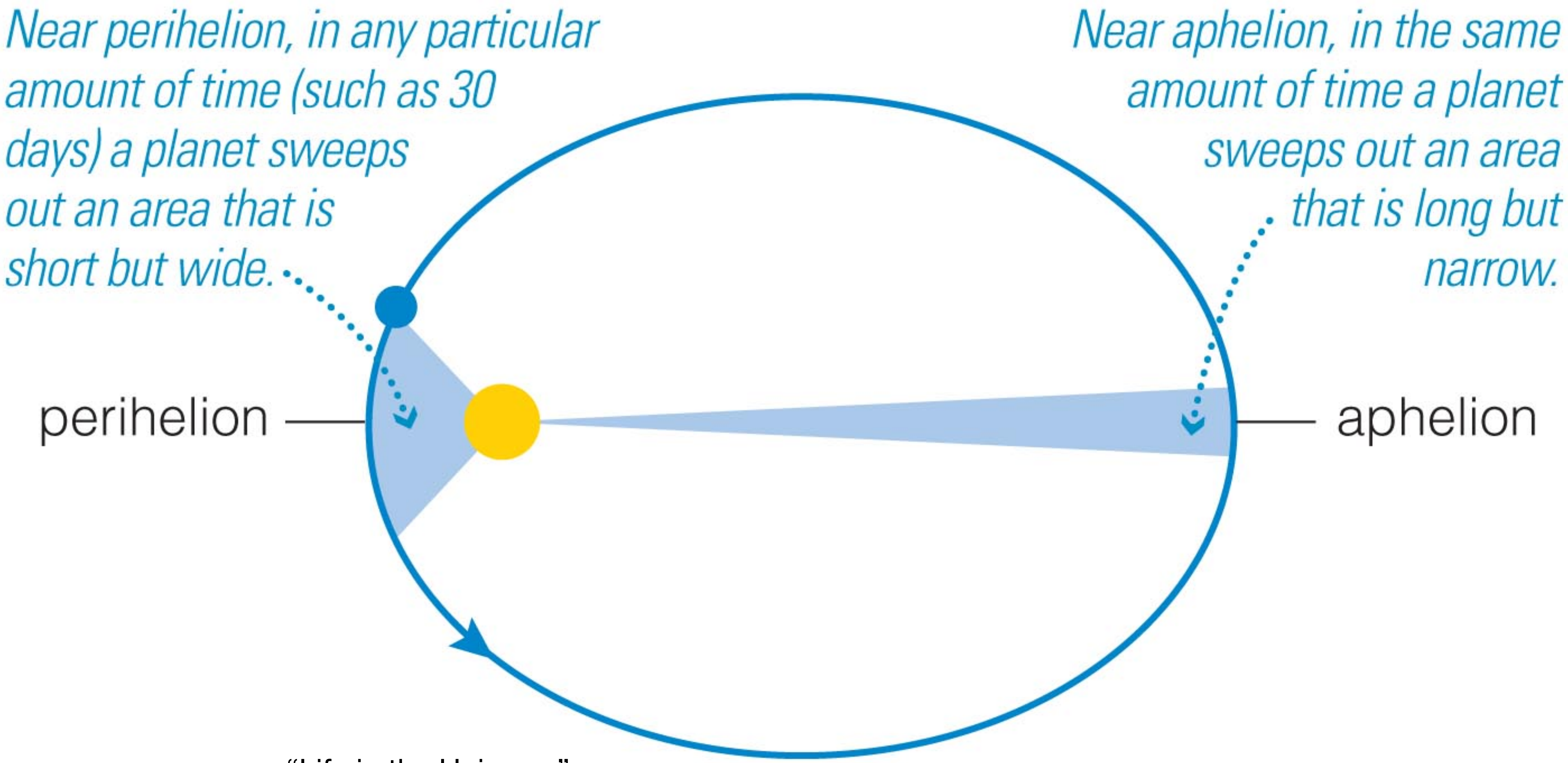
- The set of points where the distance from one focus, to that point, to the other focus is equal defines the ellipse
- True anomaly (f): angle in the plane of the orbit, between periastron, the central star, and the current position of the planet
- Current planet/star separation:

$$r = \frac{a(1-e^2)}{1+e\cos f}$$



Kepler's Second Law

- As a planet moves around its orbit, it sweeps out equal areas in equal times.
- This means that a planet travels faster when it is nearer to the Sun and slower when it is farther from the Sun.
- Angular momentum (m * v * r) is conserved.

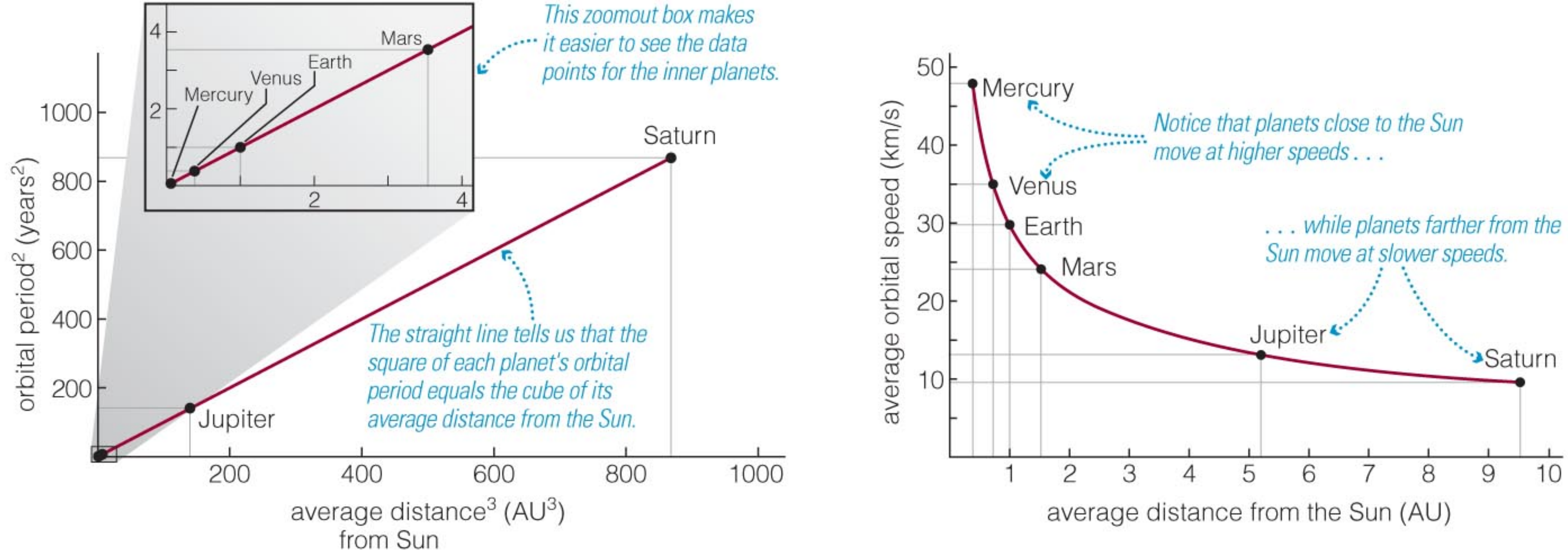


"Life in the Universe"

Kepler's Third Law

- More distant planets orbit the Sun at slower average speeds, obeying the relationship
- $p^2 = a^3$
- p = orbital period in years
- a = average distance from Sun in AU
- Astronomical Unit (AU): semi-major axis of the Earth's orbit around the Sun

Kepler's Third Law

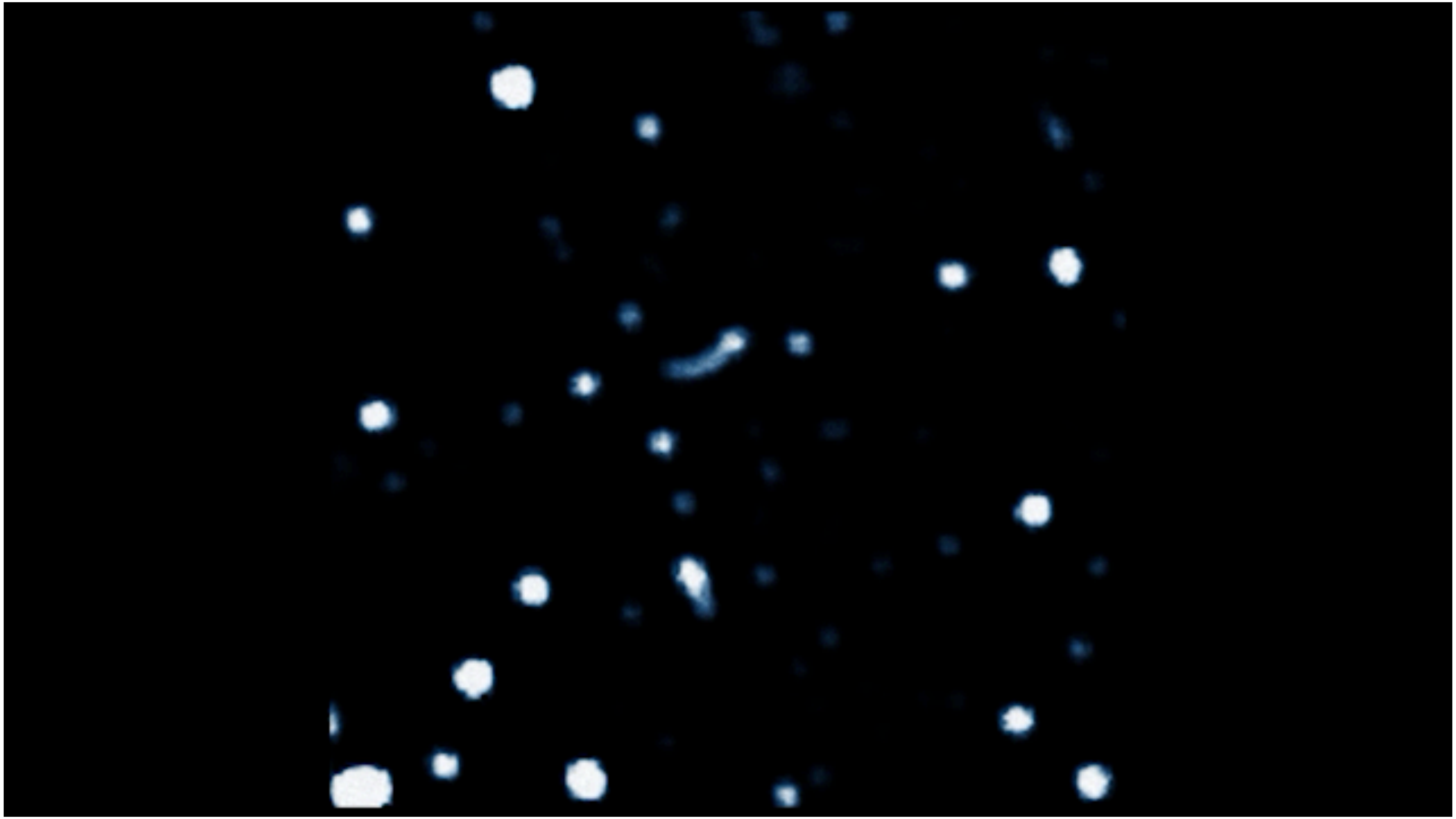


a This graph shows that Kepler's third law ($p^2 = a^3$) holds true; the graph shows only the planets known in Kepler's time.

b This graph, based on Kepler's third law and modern values of planetary distances, shows that more distant planets orbit the Sun more slowly.

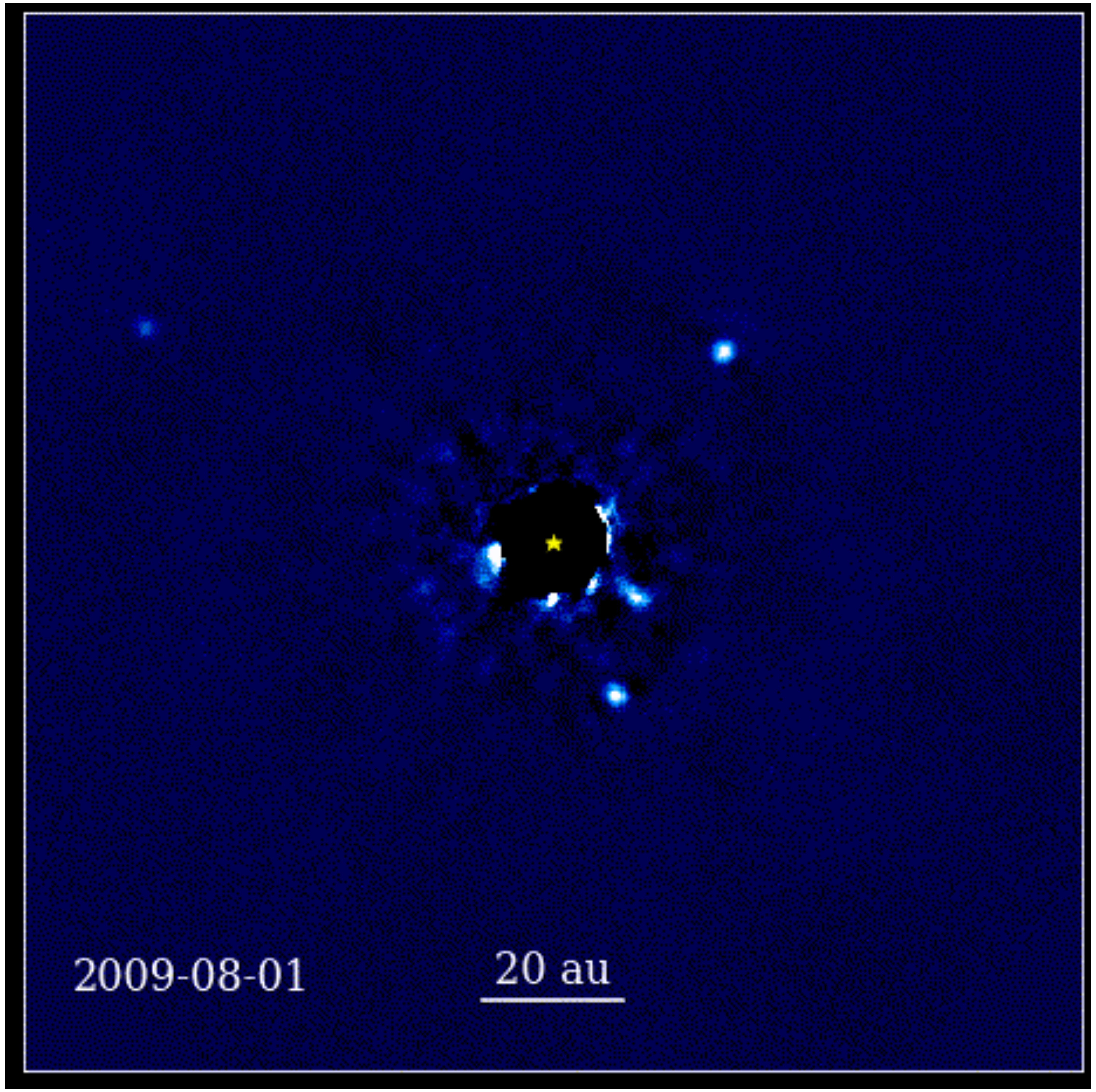
"Life in the Universe"

Kepler’s Laws and Black Holes



ESO/MPE

Kepler's Laws and Exoplanets



Movie from Jason Wang and Christian Marois

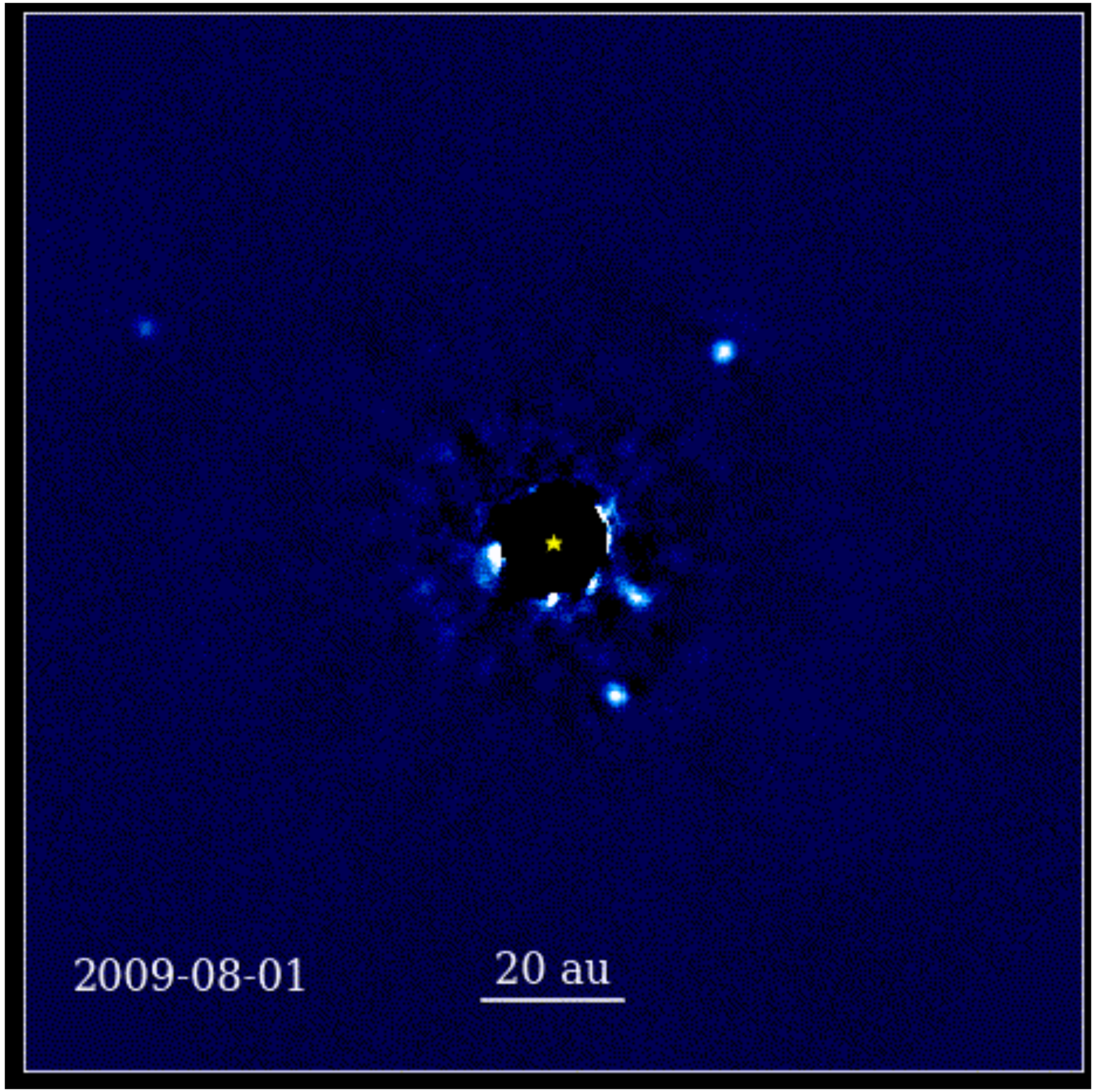
Newton Derives Kepler's Laws

- Needed to invent calculus to solve the "two body problem"
  - Combining Newton's 3 laws with the law of universal gravitation resulted in Kepler's 3 laws
- Kepler's third law, specific to objects orbiting the Sun:

$P^2 = a^3$ (P in years, a in AU)

- Newton's version of Kepler's third law, can be applied to any orbit:

$$P^2 = \frac{4\pi^2 a^3}{GM_{total}}$$



Movie from Jason Wang and Christian Marois

Newton's Version of Kepler's Third Law

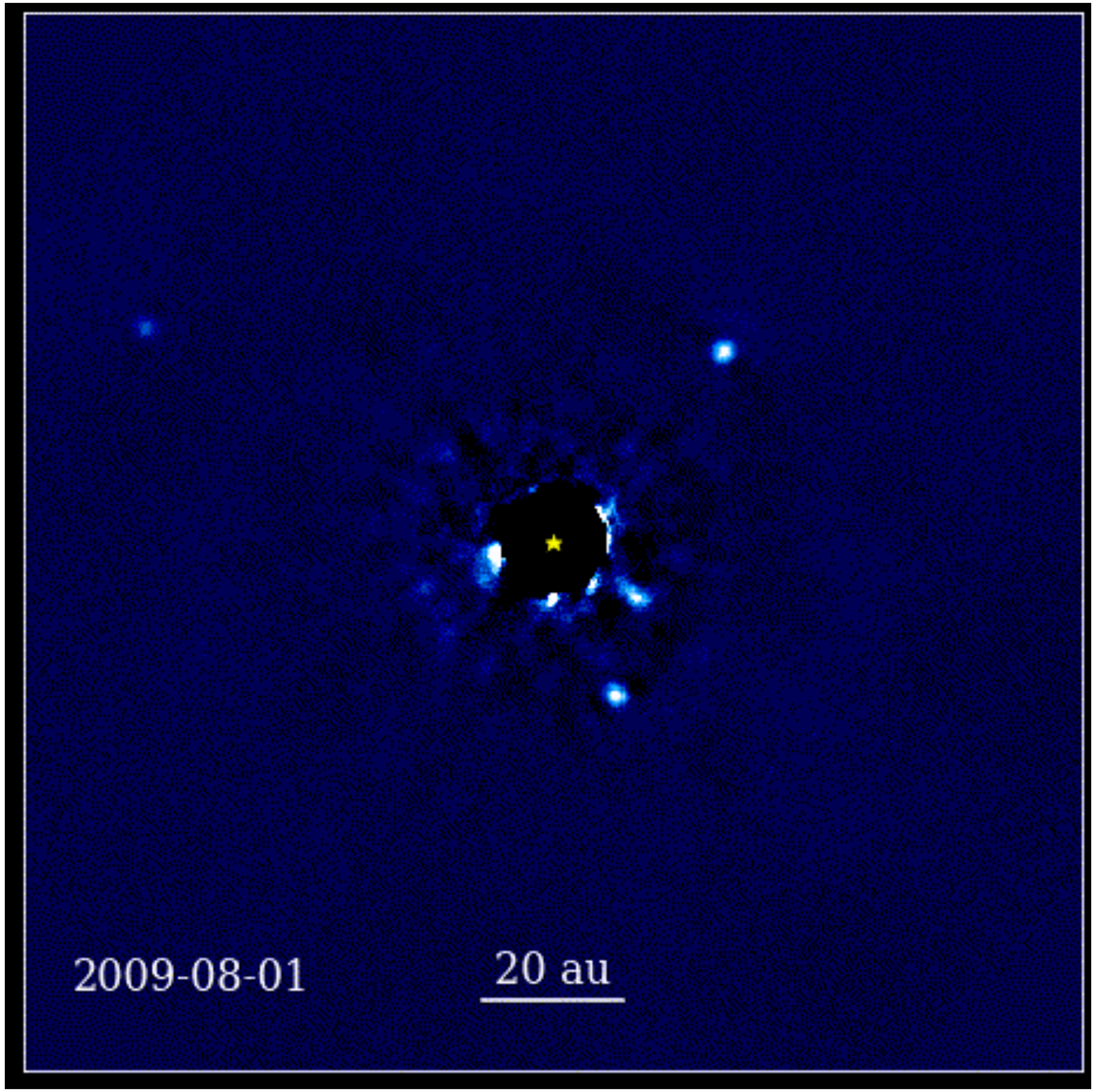
- Newtons' version of Kepler's third law, dumb units:

$$P^2 = \frac{4\pi^2 a^3}{GM_{total}}$$

- Newton's version of Kepler's third law, smarter units:

$$P^2 = \frac{a^3}{M_{total}}$$

(P in years, a in AU, M_{total} in solar masses)



Movie from Jason Wang and Christian Marois

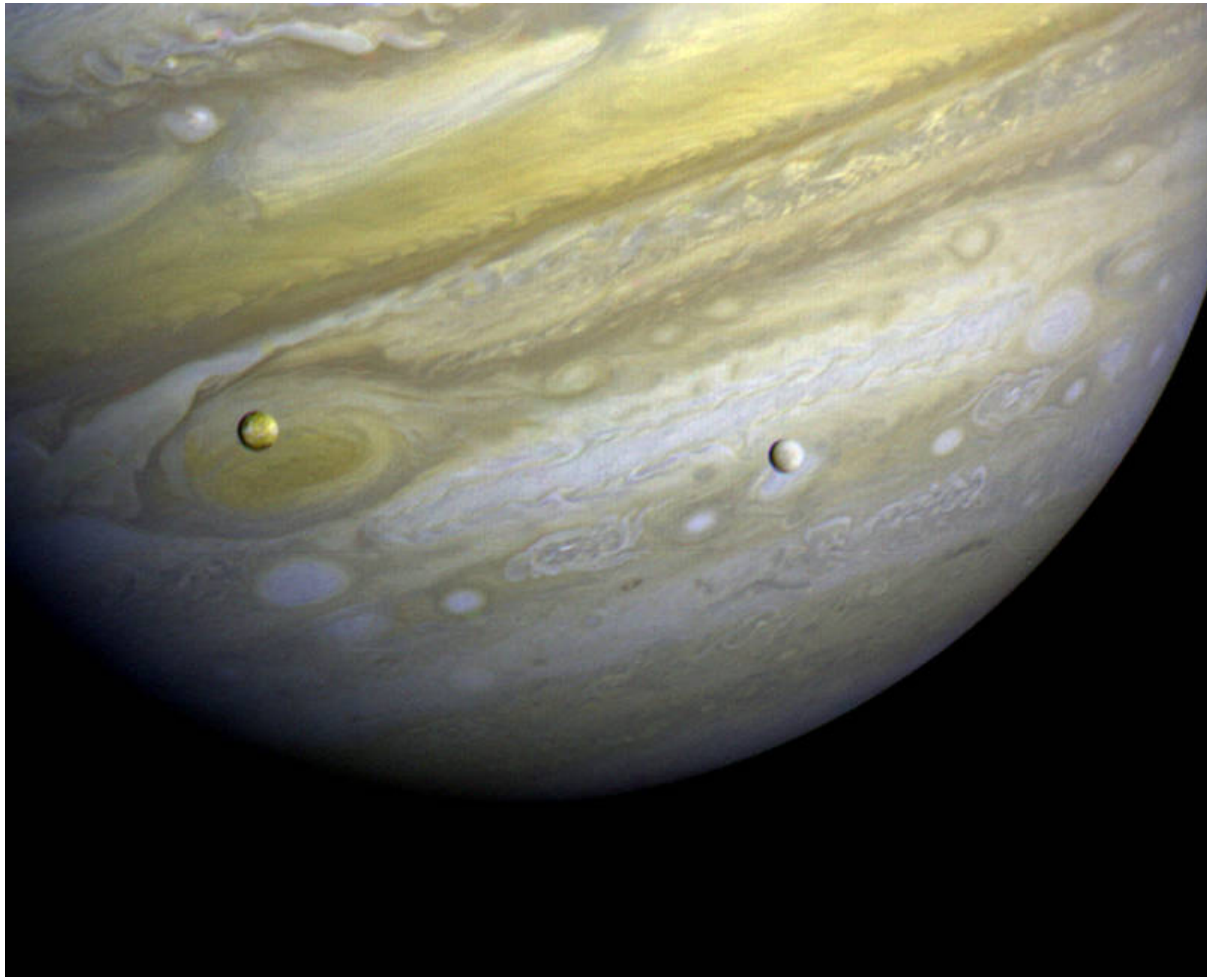
In-Class Activity
Order of Magnitude

- $$P^2 = \frac{a^3}{M_{total}}$$ (P in years, a in AU, M_{total} in solar masses)

- (1) If Earth orbited Jupiter at 1 AU, instead of orbiting the Sun, what would Earth's orbital period be?

- (2) Io orbits Jupiter with a period of 42 hours. What is the semi-major axis of Io, in AU?

- (3) What is the speed of Io in its orbit around Jupiter, in km/s? (hint: Io's orbital eccentricity is very, very small)



NASA/Voyager

In-Class Activity
Order of Magnitude

- (1) Jupiter has about 1/1000 (10^{-3}) the mass of the Sun, so in solar masses: 10^{-3}
- $P^2 = \frac{a^3}{M}$ So plugging in M and a=1 AU: $P = \sqrt{\frac{a^3}{M}} = \sqrt{\frac{1^3}{10^{-3}}} = \sqrt{10^3} = 30$ years
- (2) 42 hours = 48 hours = 2 days = 2/365 years = 1/180 = $5x10^{-3}$ years

$$a = (P^2M)^{\frac{1}{3}} = ((5x10^{-3})^2 * 10^{-3})^{\frac{1}{3}} = (25x10^{-9})^{\frac{1}{3}} = 3x10^{-3} \text{ AU}$$

- (3) 1 AU = $1.5x10^{13}$ cm = $1.5x10^{8}$ km 1 year = $3x10^{7}$ seconds Circumference of orbit: $2\pi a$

$$v = \frac{2\pi a}{P} = \frac{2\pi * 3x10^{-3}AU * \frac{1.5x10^8 km}{1AU}}{5x10^{-3}year * \frac{3x10^7 s}{1year}} = \frac{10 * 10^{-3}10^8 km}{10x10^{-3}x10^7 s} = \frac{10^6 km}{10^5 s} = 10\, km/s$$

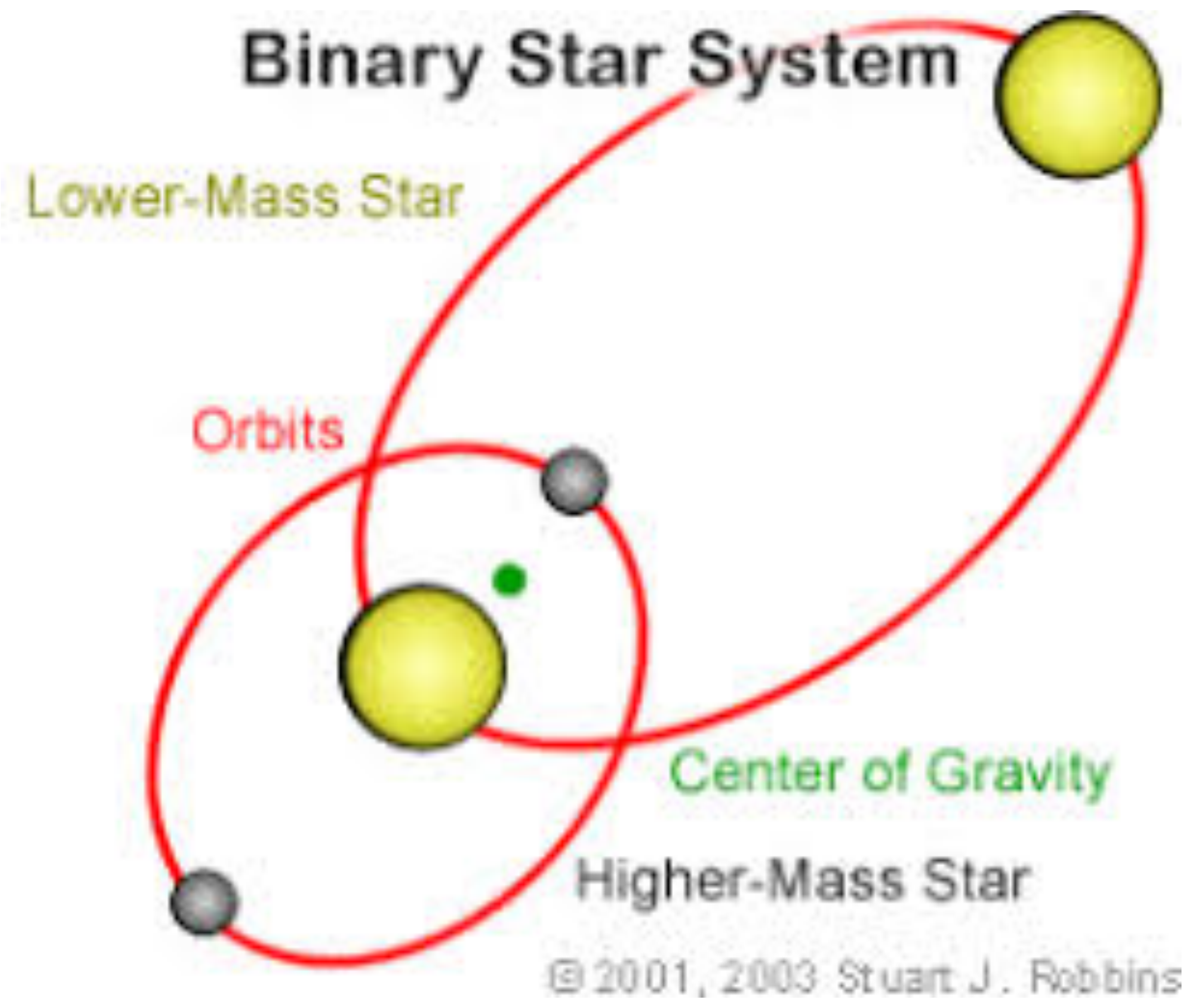
Center of Mass

- Really, both objects in an orbit move around their common center of mass

- The two ellipses have different semi-major axes, but the center of mass is at a focus of each

$$a_1 M_1 = a_2 M_2$$

- The two orbits have the same period

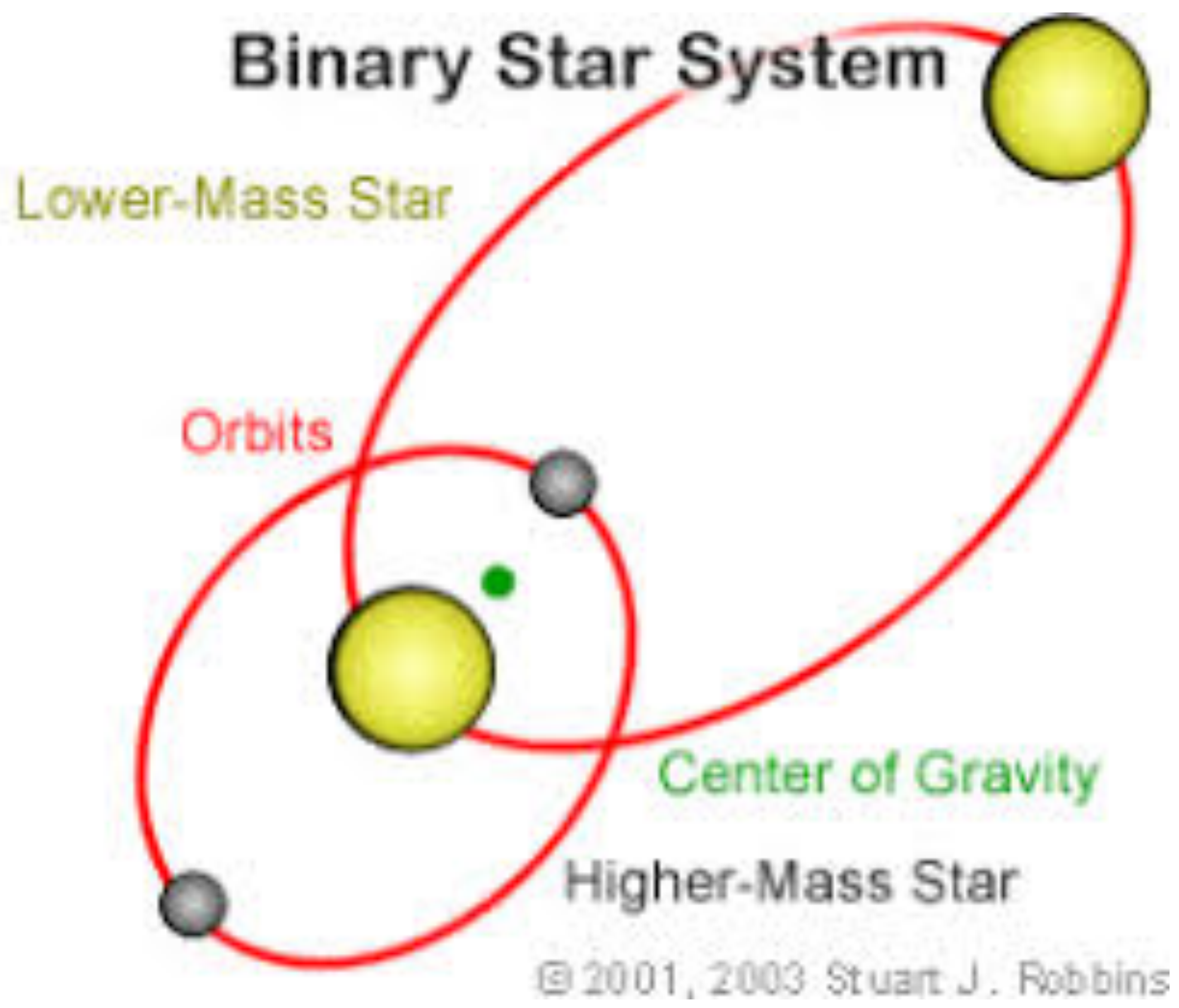


Response Card Question

- Two objects in orbit move in ellipses around their common center of mass, with semi-major axis a_1 and a_2. Which semi-major axis do we plug into Kepler's third law, $P^2 = \frac{a^3}{M_{total}}$?
  - (A) — The semi-major axis of the more massive object: a_1
  - (B) — The semi-major axis of the less massive object: a_2
  - (C) — The sum of the two semi-major axes: $a_1 + a_2$
  - (D) — The average of the two semi-major axes: $\frac{a_1 + a_2}{2}$

Center of Mass

- For solar system objects and exoplanets, we typically measure distances as though the Sun (or central star) was fixed and not moving
- In this (non-intertial) reference frame, the orbit you measure is of the secondary moving about the primary
- The semi-major axis of the orbit in this reference frame is $a_{tot} = a_1 + a_2$ (what we need for mass determinations)



$$P^2 = \frac{(a_1 + a_2)^3}{M_1 + M_2}$$

Break

Getting Masses with Kepler's Third Law

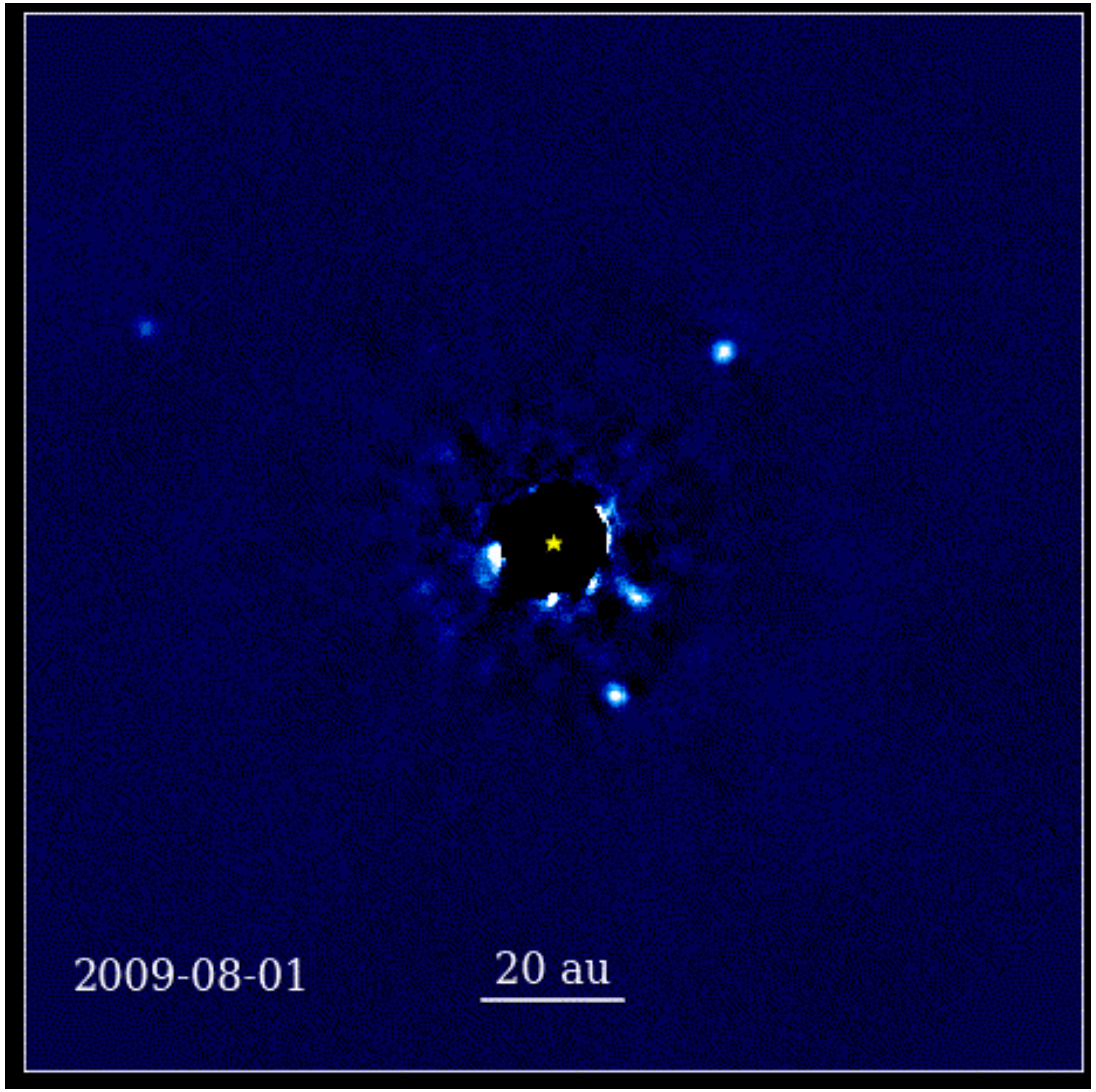
- $P^2 = \frac{a^3}{M_{total}}$ (P in years, a in AU, M_{total} in solar masses)
- If one mass is much larger than the other, we can approximate:

$$M_{total} = M_1 + M_2 \approx M_1$$

- Good cases to use that approximation:

$$\frac{M_{Earth}}{M_{Sun}} = 3x10^{-6}$$

$$\frac{M_{Cassini}}{M_{Saturn}} = 10^{-23}$$



Movie from Jason Wang and Christian Marois

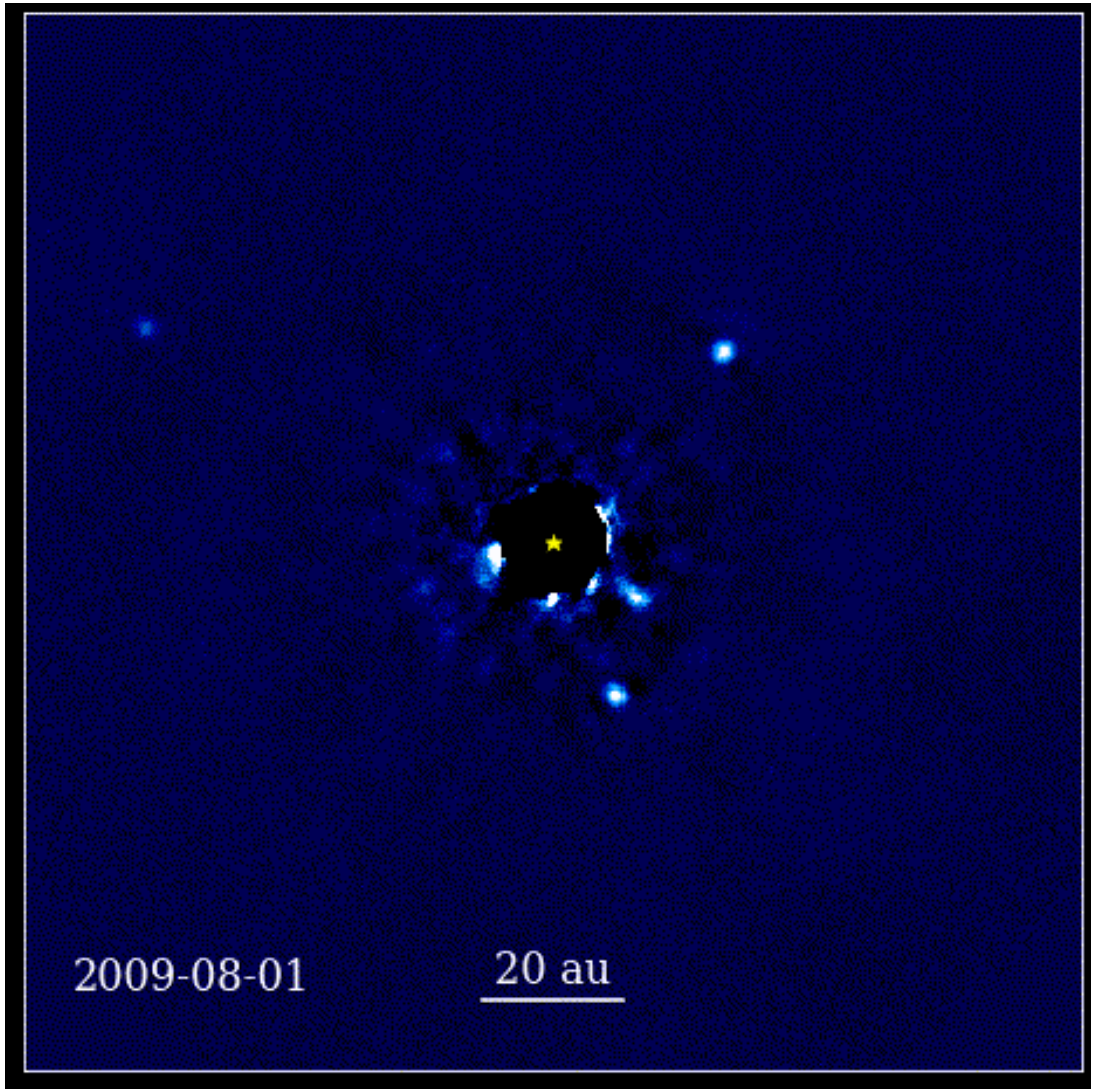
Getting Masses with Kepler's Third Law

- Not so good cases:

$$\frac{M_{Moon}}{M_{Earth}} = 10^{-2}$$

$$\frac{M_{Charon}}{M_{Pluto}} = 10^{-1}$$

- In these cases, can use other data than relative (total) semi-major axis and period:
  - radial velocity of one of the bodies
  - absolute astrometry of one of the bodies

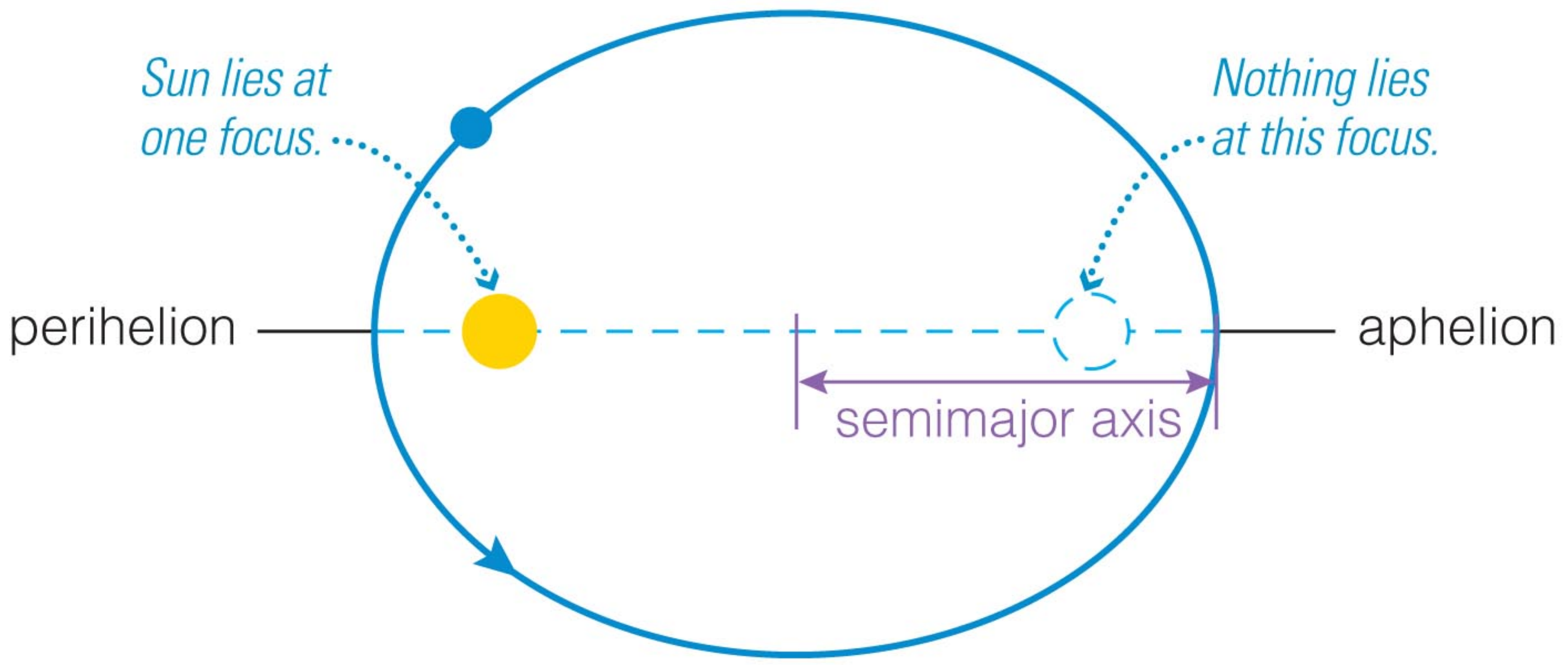


Movie from Jason Wang and Christian Marois

Response Card Question

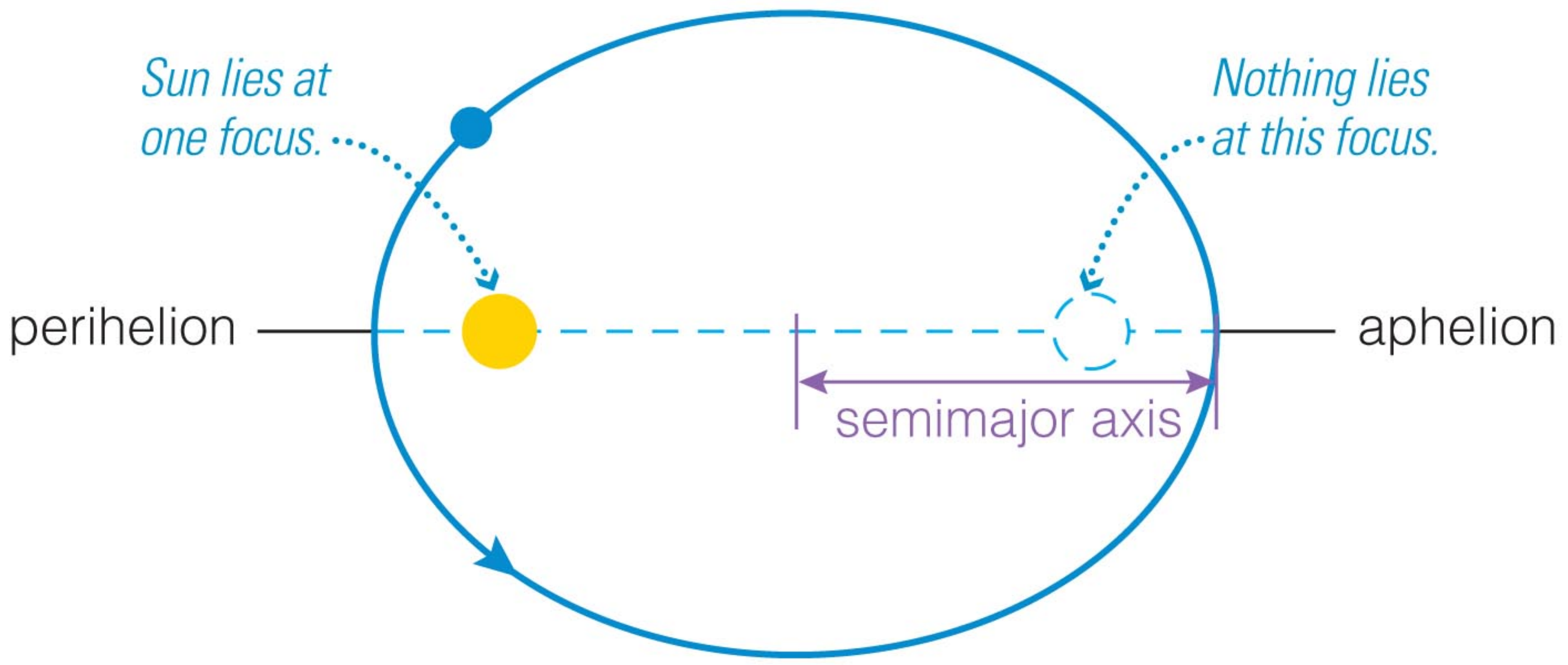
- If an Earth-mass planet had a semi-major axis of 1 AU, but orbited a 4 solar mass star, what would its orbital period be? (hint: $P^2 = \frac{a^3}{M_{total}}$)
  - (A) — 4 years
  - (B) — 2 years
  - (C) — 1 year
  - (D) — 0.5 years
  - (E) — 0.25 years

Orbital elements



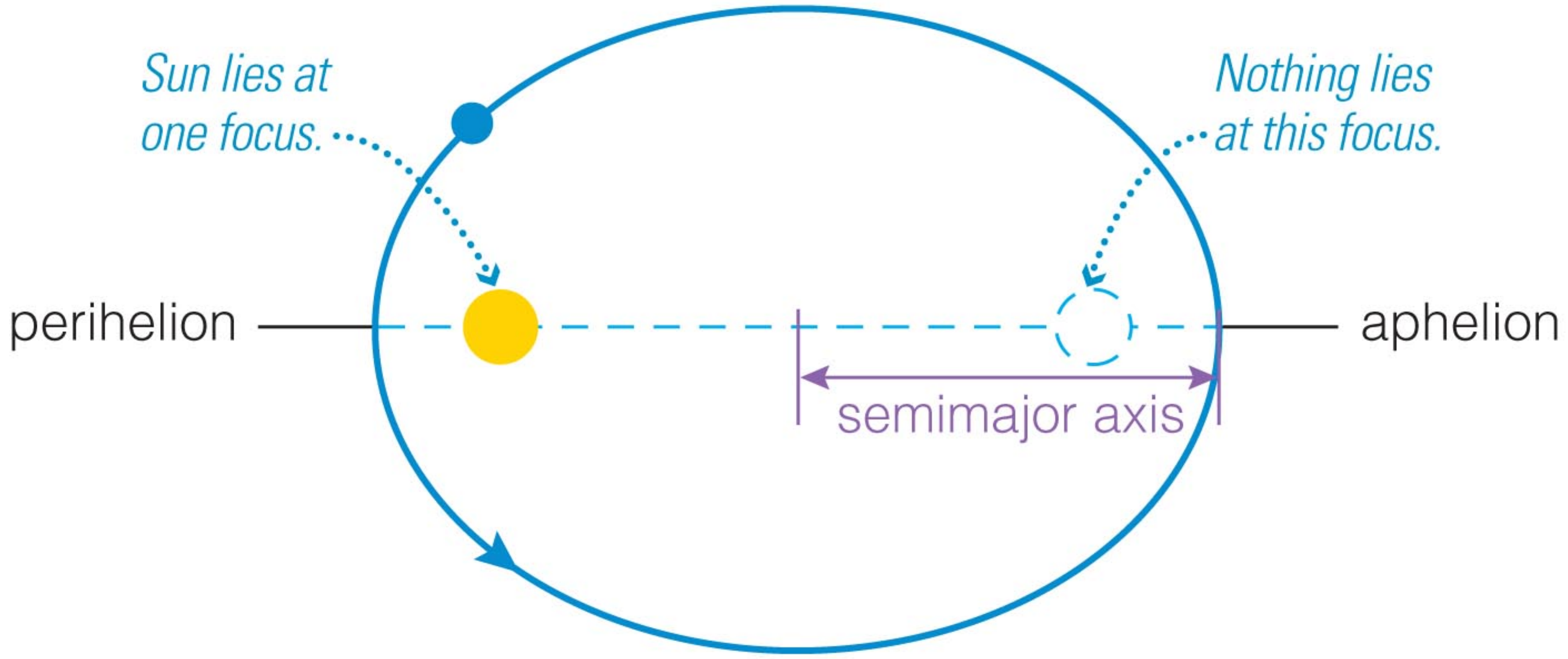
Orbital elements

semi-major axis (a)



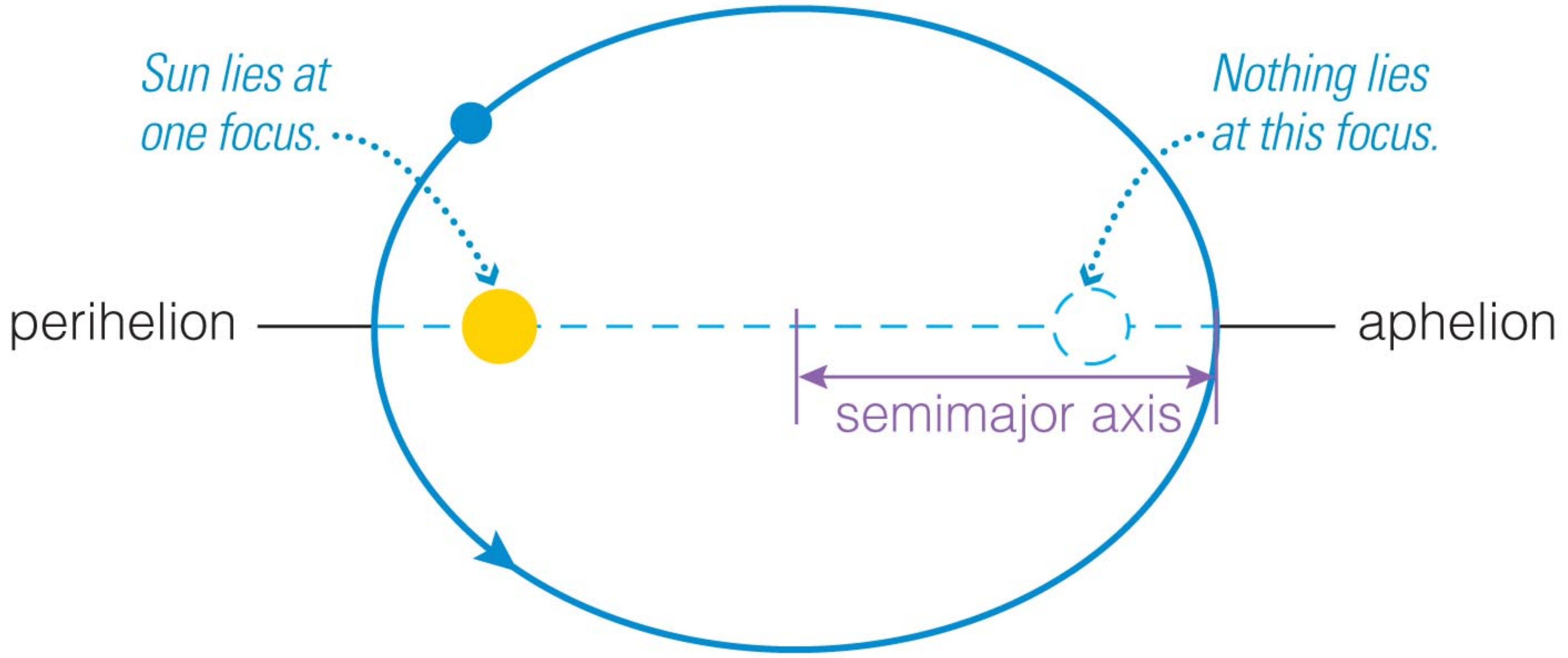
Orbital elements

semi-major axis (a)
eccentricity (e)



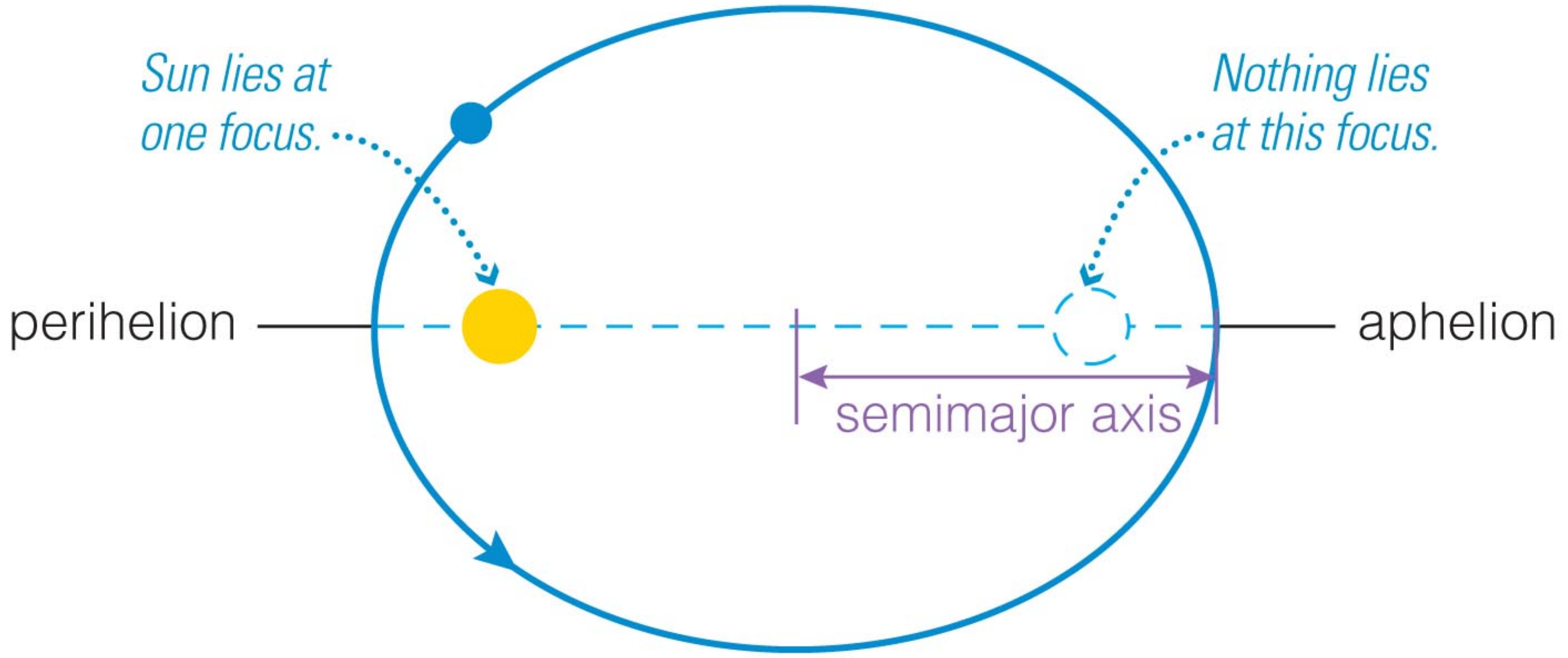
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)



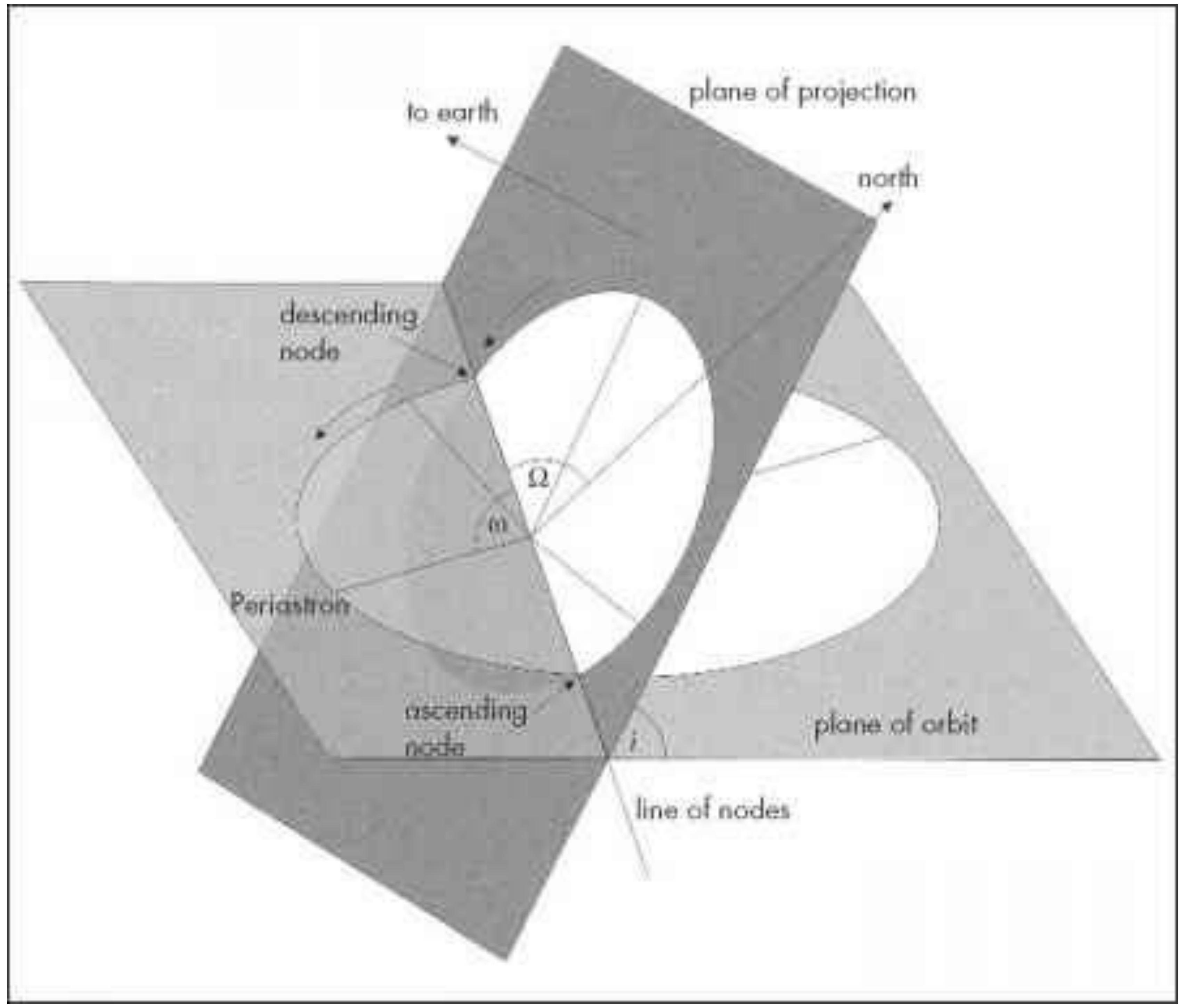
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)



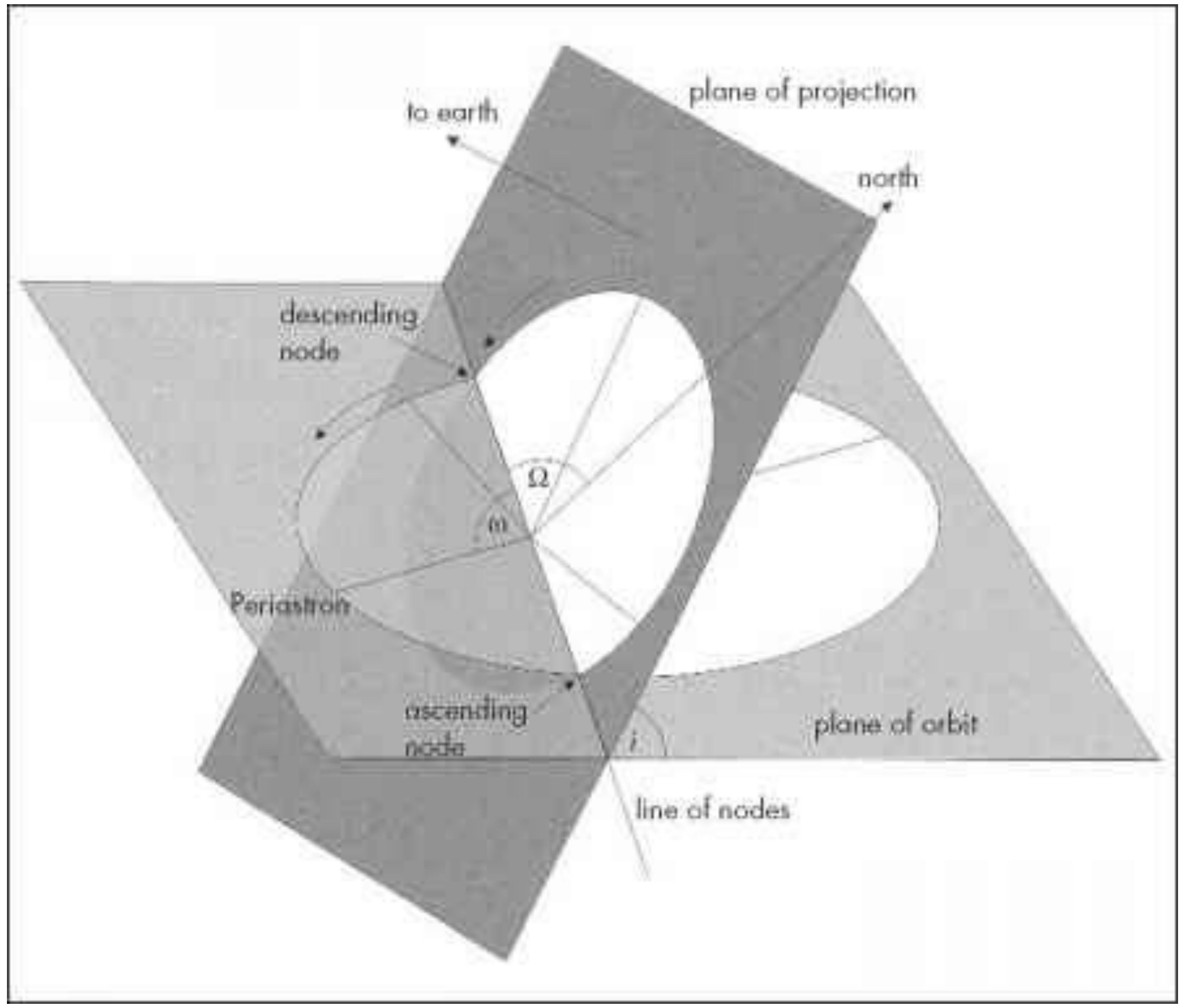
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)



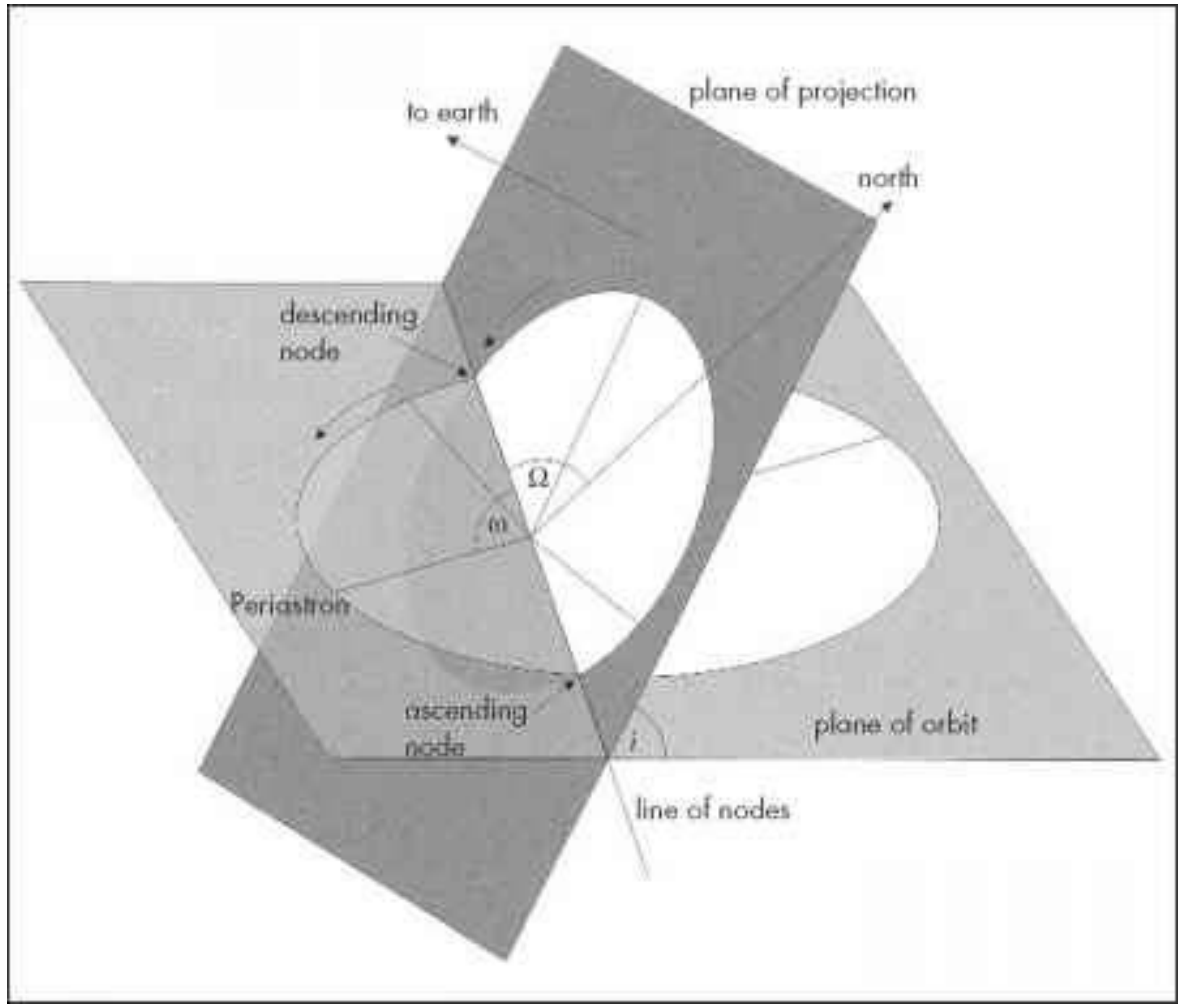
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)



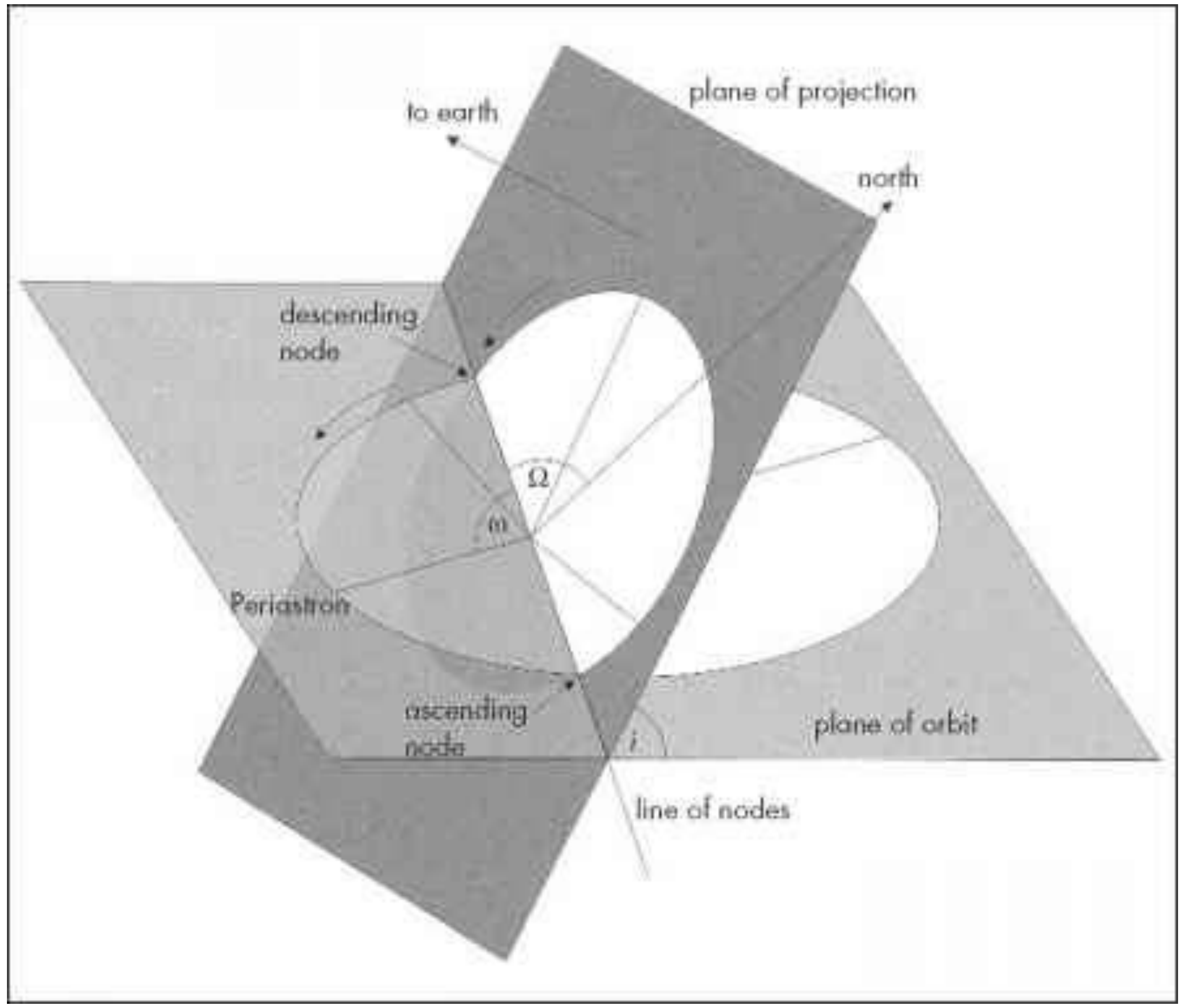
Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)
Position Angle of Nodes (Ω)

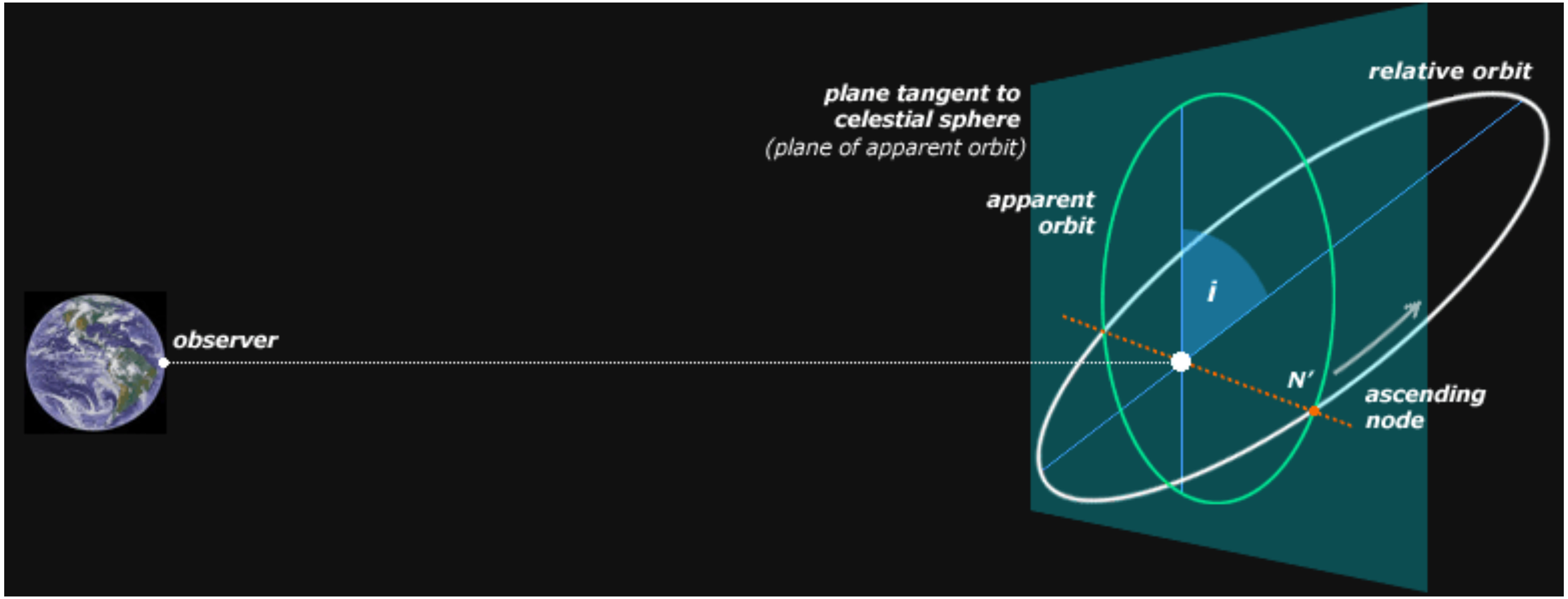


Orbital elements

semi-major axis (a)
eccentricity (e)
Period (P)
Epoch of Periastron Passage (T_0)
Inclination Angle (i)
Position Angle of Nodes (Ω)
Argument of Periastron (ω)

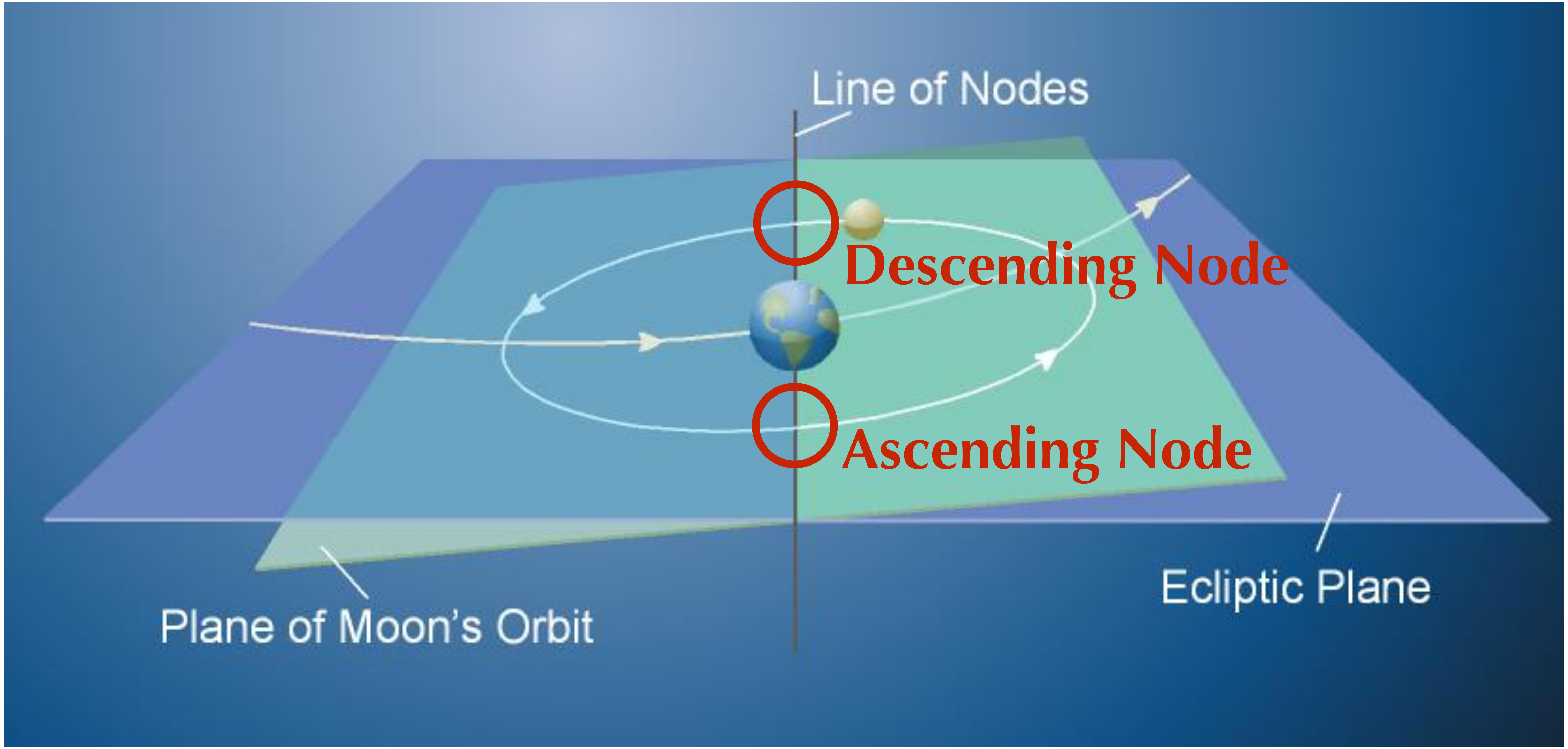


Orbits outside our Solar System



- Inclination angle measured with respect to the plane of the sky
- Position Angle of Nodes given in the plane of the sky, east of north

Orbits inside our Solar System



- Inclination angle for objects orbiting the Sun is measured with respect to the ecliptic plane (Earth's orbital plane around the Sun)
- For objects orbiting solar system planets (moons, spacecraft), inclination angle is with respect to the planet's equator
- Position Angle of Nodes given in the plane of the ecliptic, with respect to the vernal point (also where RA=0)

Circular Velocity

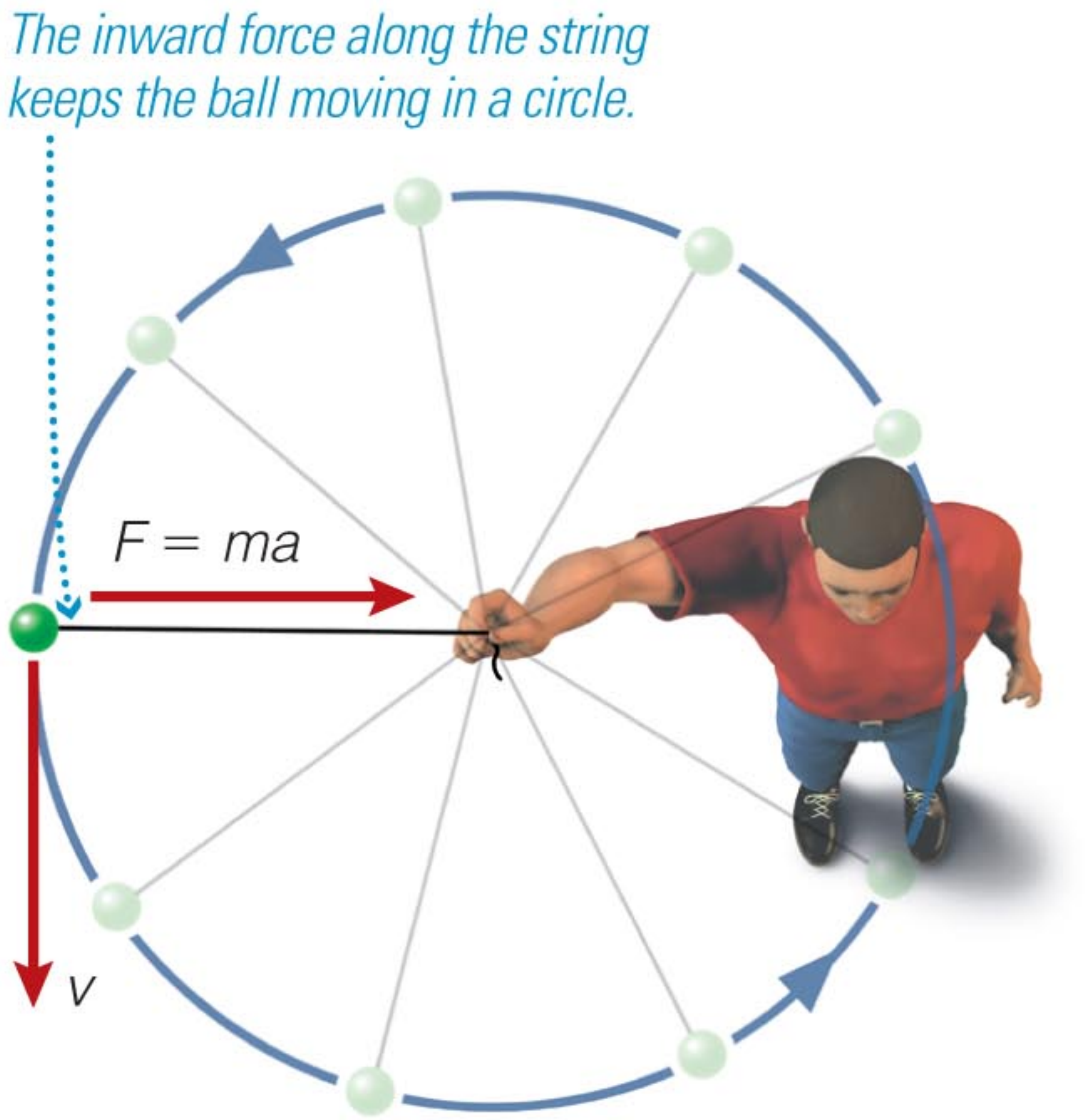
- If we assume a planet is much less massive than the star (a good assumption), then for a circular orbit acceleration has to equal $\frac{v^2}{R}$

$$F = ma = \frac{GMm}{R^2}$$

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$v_c = \sqrt{\frac{GM}{R}}$$

for a circular orbit, R=a: $v_c = \sqrt{\frac{GM}{a}}$

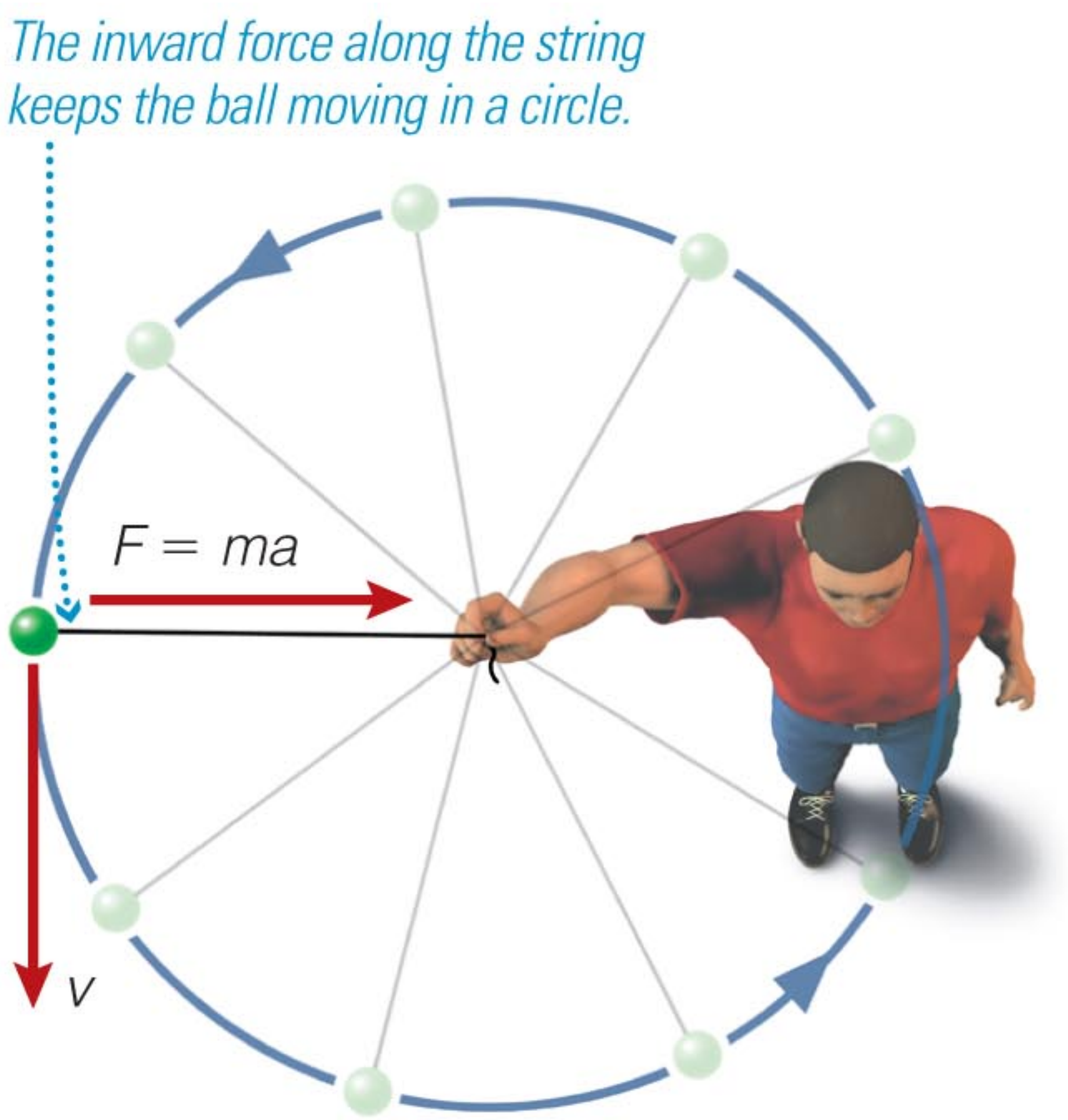


a When you swing a ball on a string, the string exerts a force that pulls the ball inward.

Energy Conservation: Orbits

- An orbiting object has two types of energy: gravitational potential energy and kinetic energy. Total energy (PE + KE) is conserved
- PE (per unit mass of orbiting object): $-\frac{GM_\odot}{r}$
- KE (per unit mass of orbiting object): $\frac{1}{2}v^2$
- For a circular orbit, $v^2 = \frac{GM}{a}$
- Total energy for a circular orbit:

$$E = \frac{1}{2}v^2 - \frac{GM_\odot}{a} = \frac{GM_\odot}{2a} - \frac{GM_\odot}{a} = -\frac{GM_\odot}{2a}$$



a When you swing a ball on a string, the string exerts a force that pulls the ball inward.

vis-viva Equation

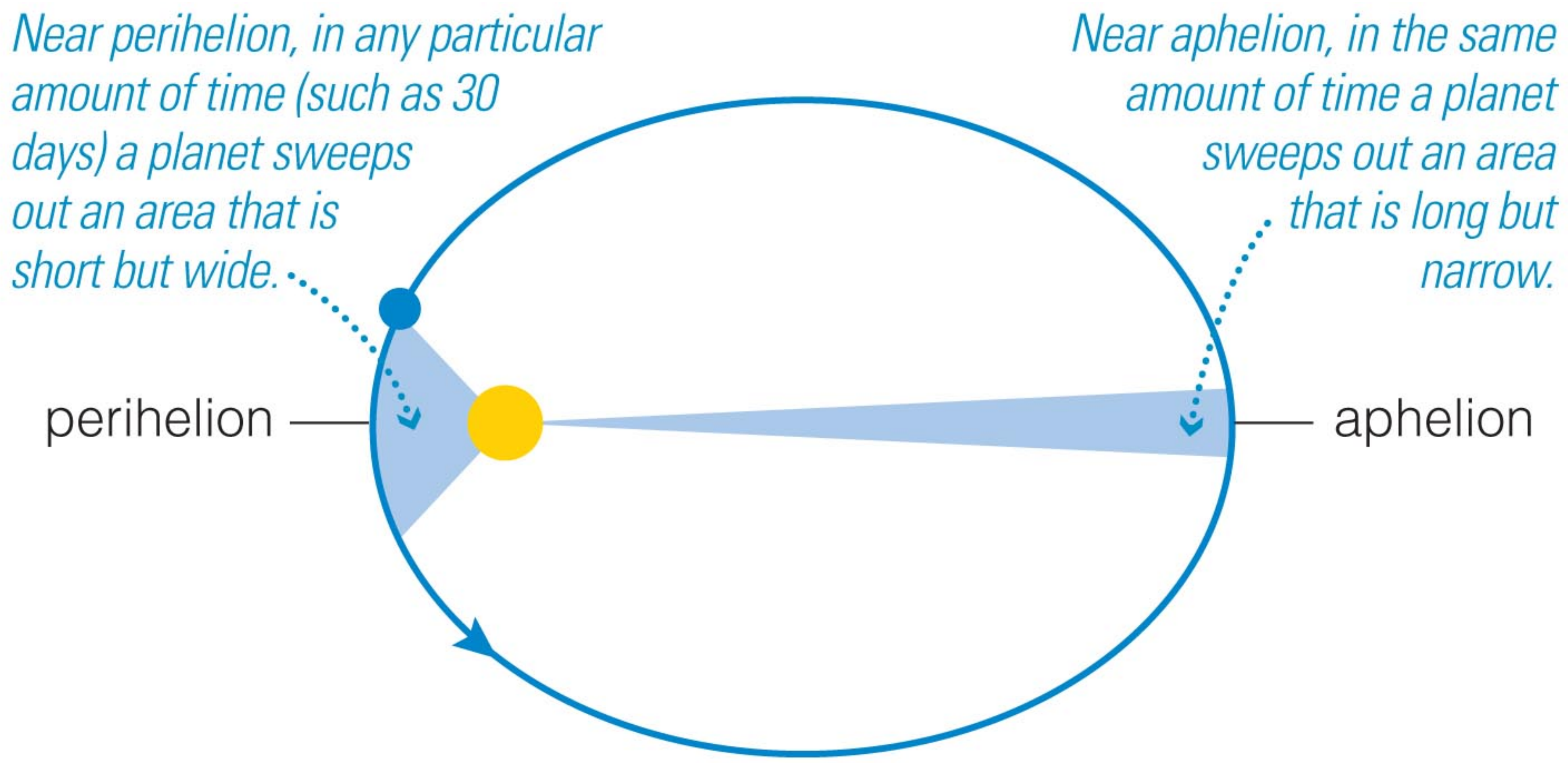
- For an elliptical orbit, velocity changes as a function of distance from the central star (r)

- Conserving energy, and using Kepler's second law, we get:

$$\frac{1}{2}v(r)^2 = GM_{\odot}\left(\frac{1}{r} - \frac{1}{2a}\right)$$

- Or, solving for v, we get the velocity at any point in an elliptical orbit:

$$v(r) = \sqrt{GM_{\odot}\left(\frac{2}{r} - \frac{1}{a}\right)}$$ vis-viva Equation



"Life in the Universe"

Escape Velocity

- Going back to energy conservation:

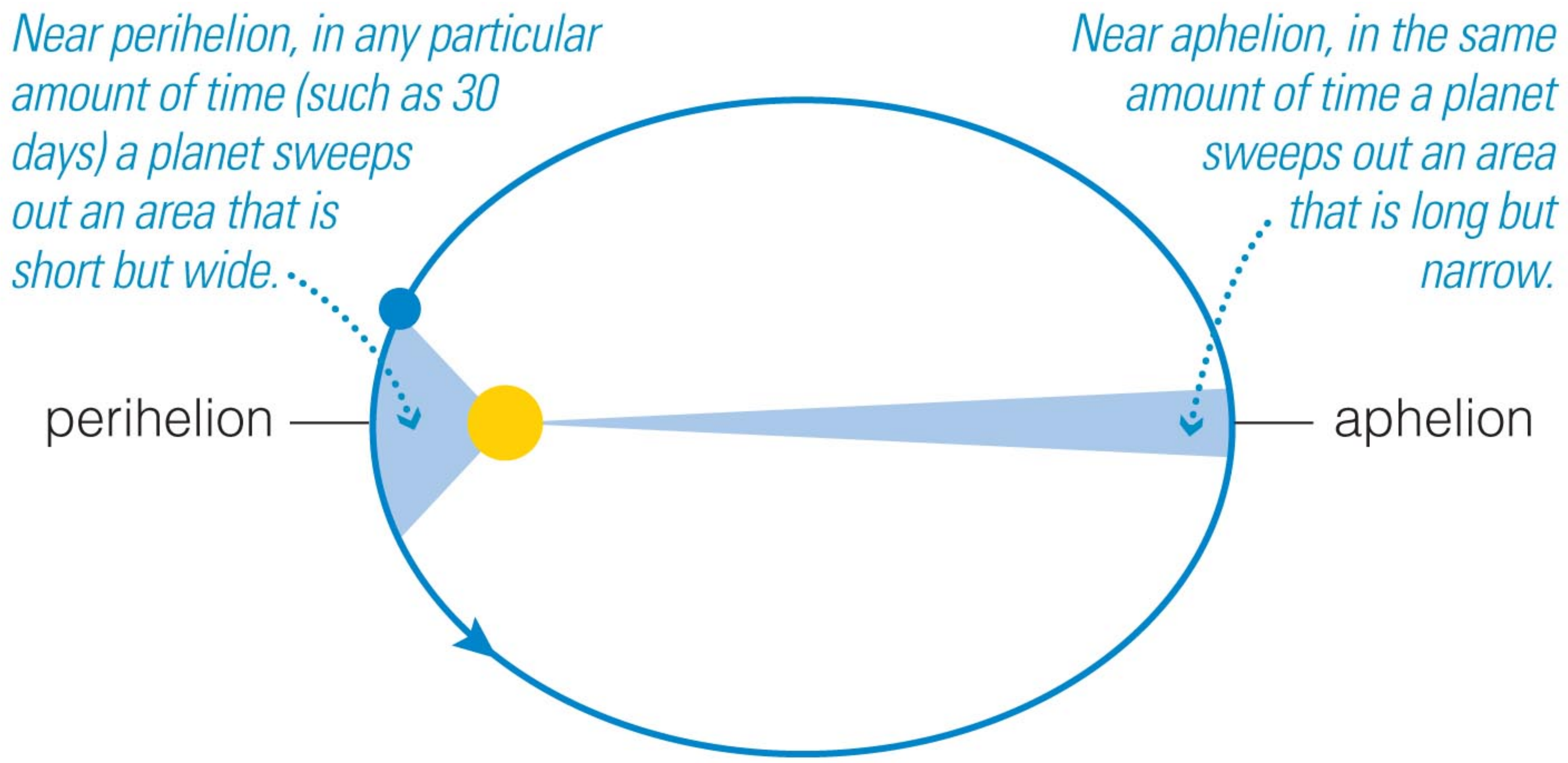
$$E = \text{constant} = \frac{1}{2}mv^2 - \frac{GM_\odot m}{r}$$

- If E<0, the object is **bound**. If E≥0, the object is **unbound.**

- Setting E=0, and solving for v, we get the escape velocity (as a function of distance):

$$v_{esc}(r) = \sqrt{\frac{2GM_\odot}{r}}$$ Escape Velocity

- Note the escape velocity is only $\sqrt{2}$ times larger than the circular velocity of a bound orbit

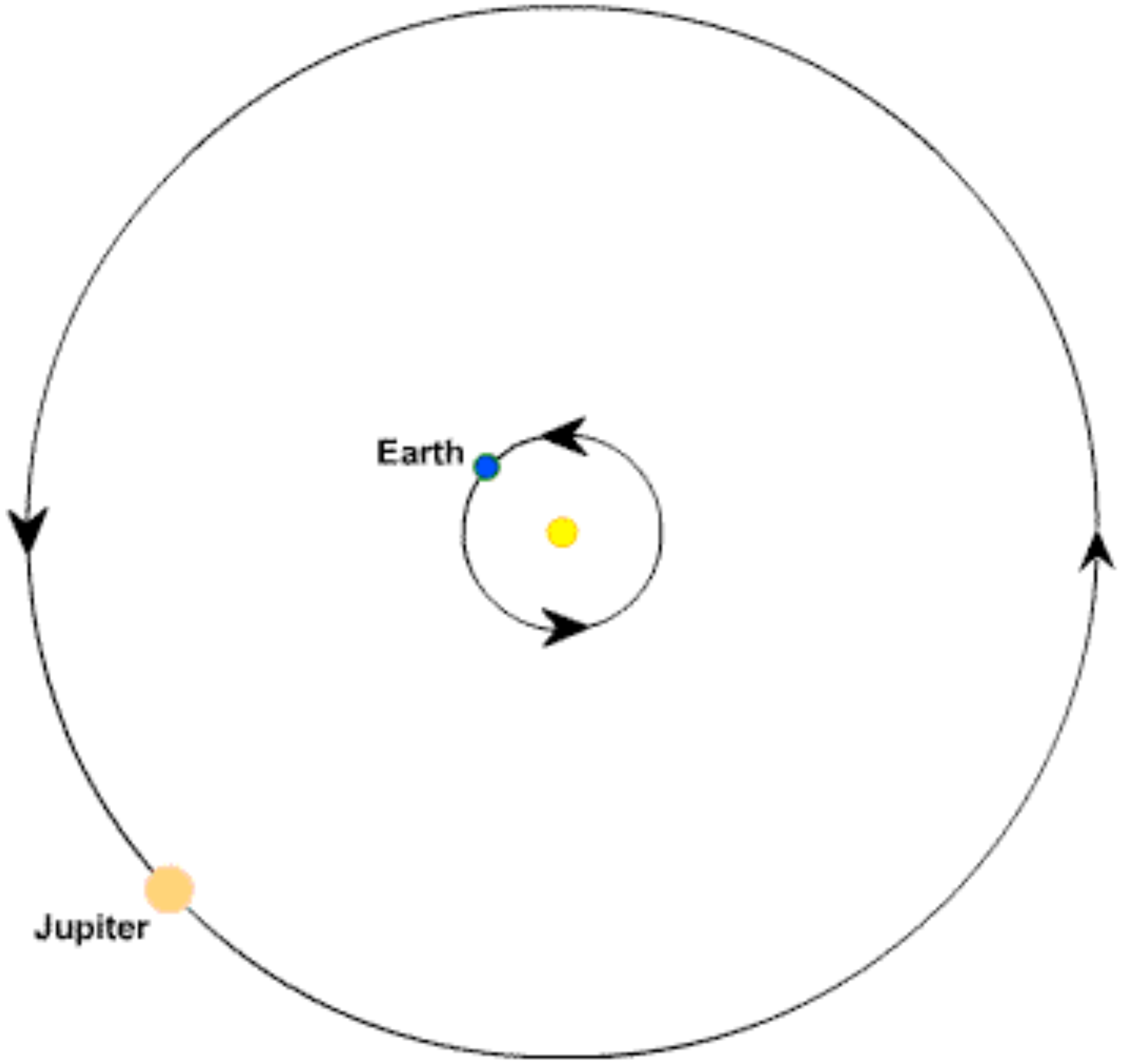


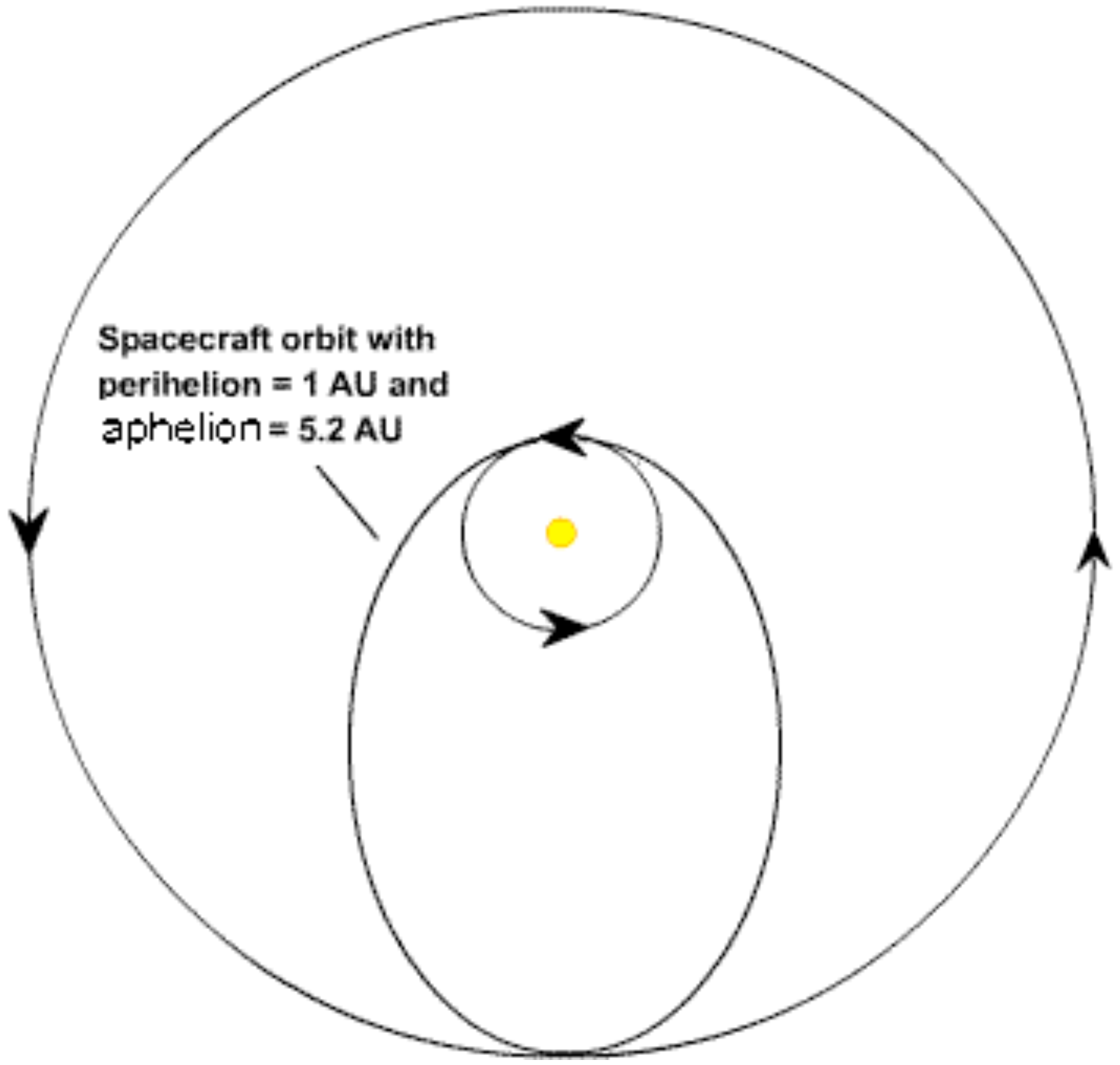
“Life in the Universe”

Hohmann Transfer Orbit

- The most efficient way to change orbits in a short amount of time is a Holman transfer orbit

- For example, when going from Earth to Jupiter, the transfer orbit will have perihelion of 1 AU, and aphelion of 5.2 AU





polaris.iastate.edu/EveningStar

Response Card Question

- The most efficient way to change orbits in a short amount of time is a Holman transfer orbit
- For example, when going from Earth to Jupiter, the transfer orbit will have perihelion of 1 AU, and aphelion of 5.2 AU
- What is the semi-major axis and eccentricity of the transfer orbit?

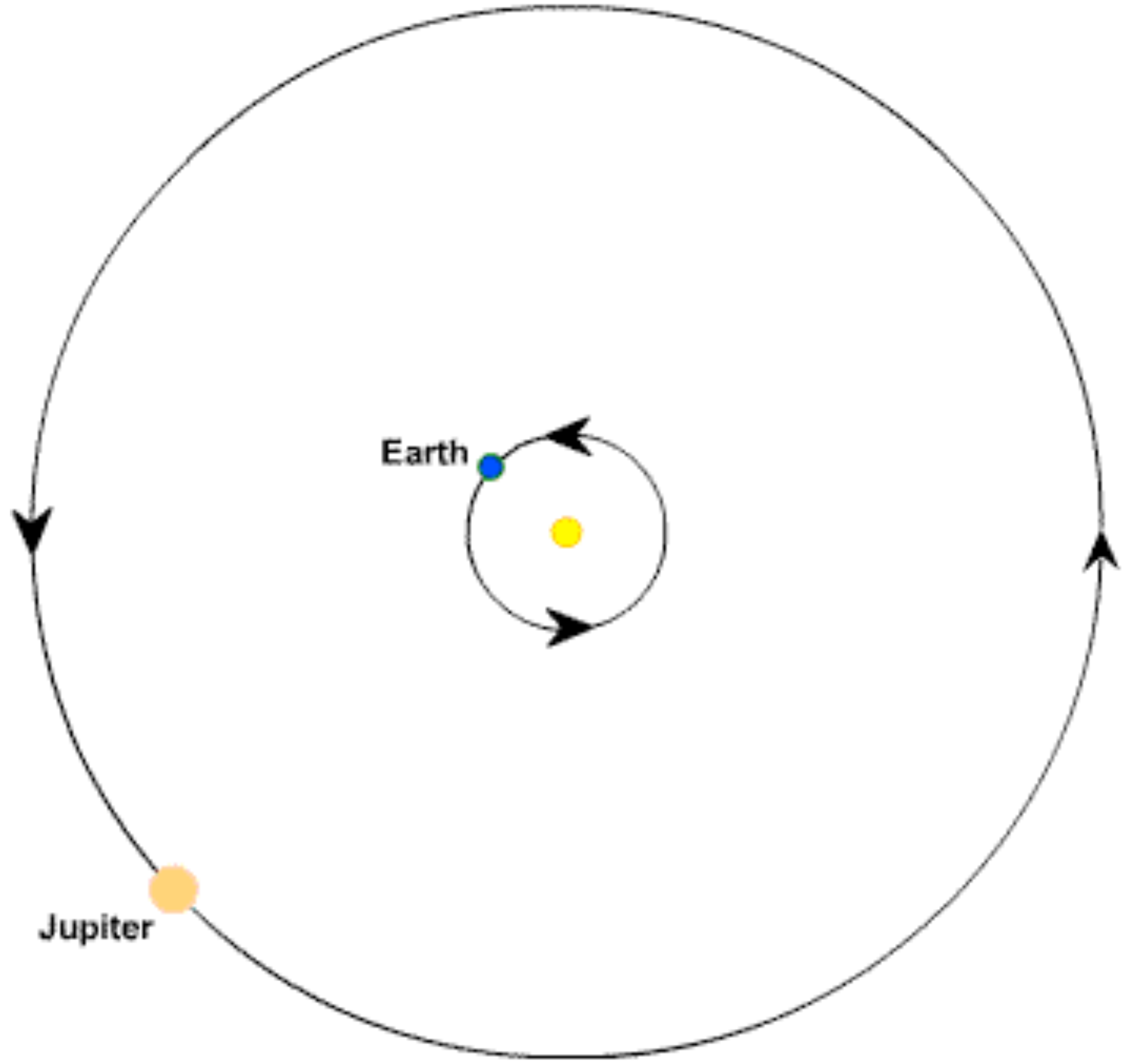
A) a = 3.1 AU, e = 0.67 (=2.1/3.1)

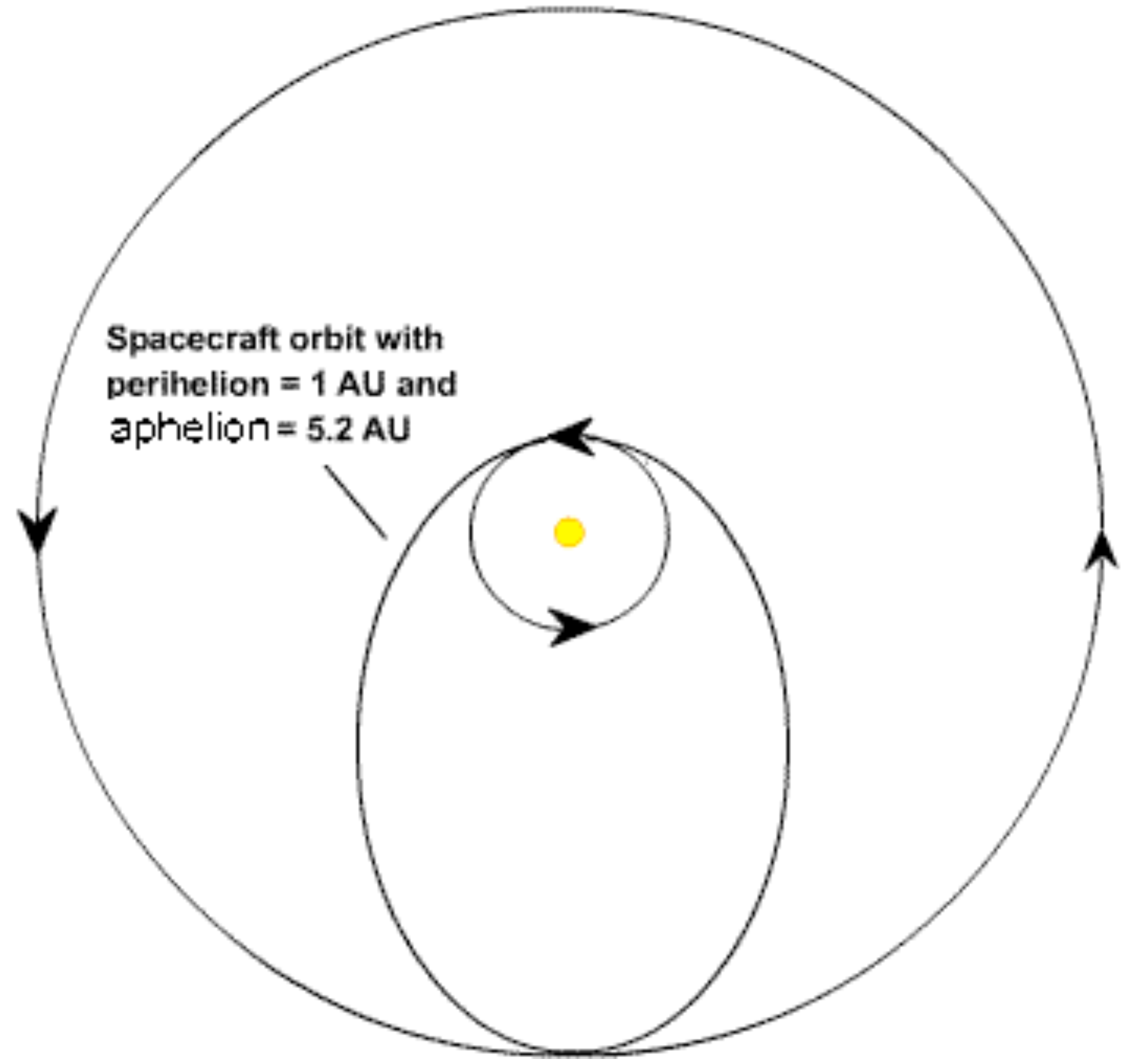
B) a = 5.2 AU, e = 0.67 (=2.1/3.1)

C) a = 1 AU, e = 0.67 (=2.1/3.1)

D) a = 3.1 AU, e = 0.2 (=1/5.2)

E) a = 5.2 AU, e = 0.2 (=1/5.2)





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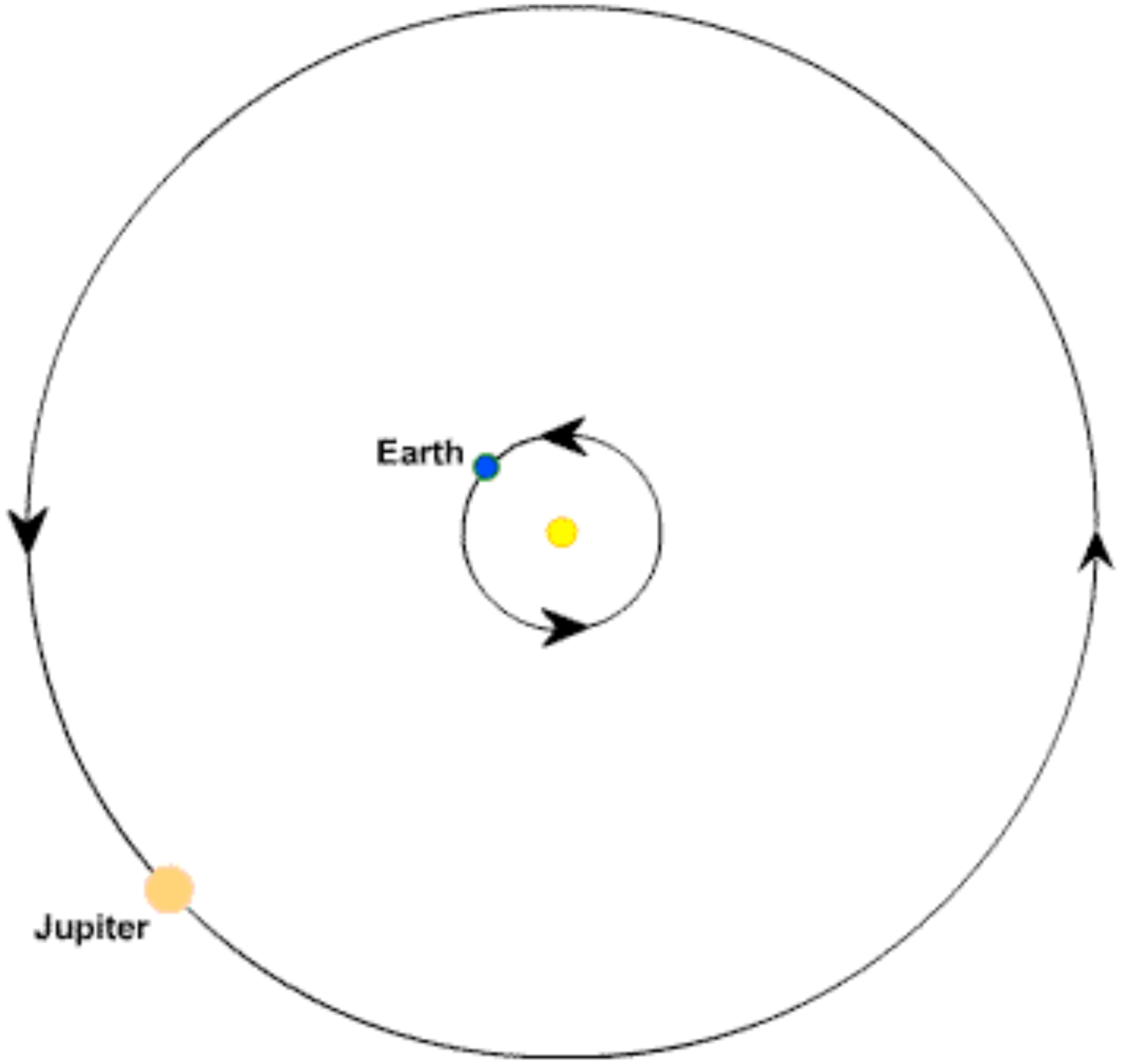
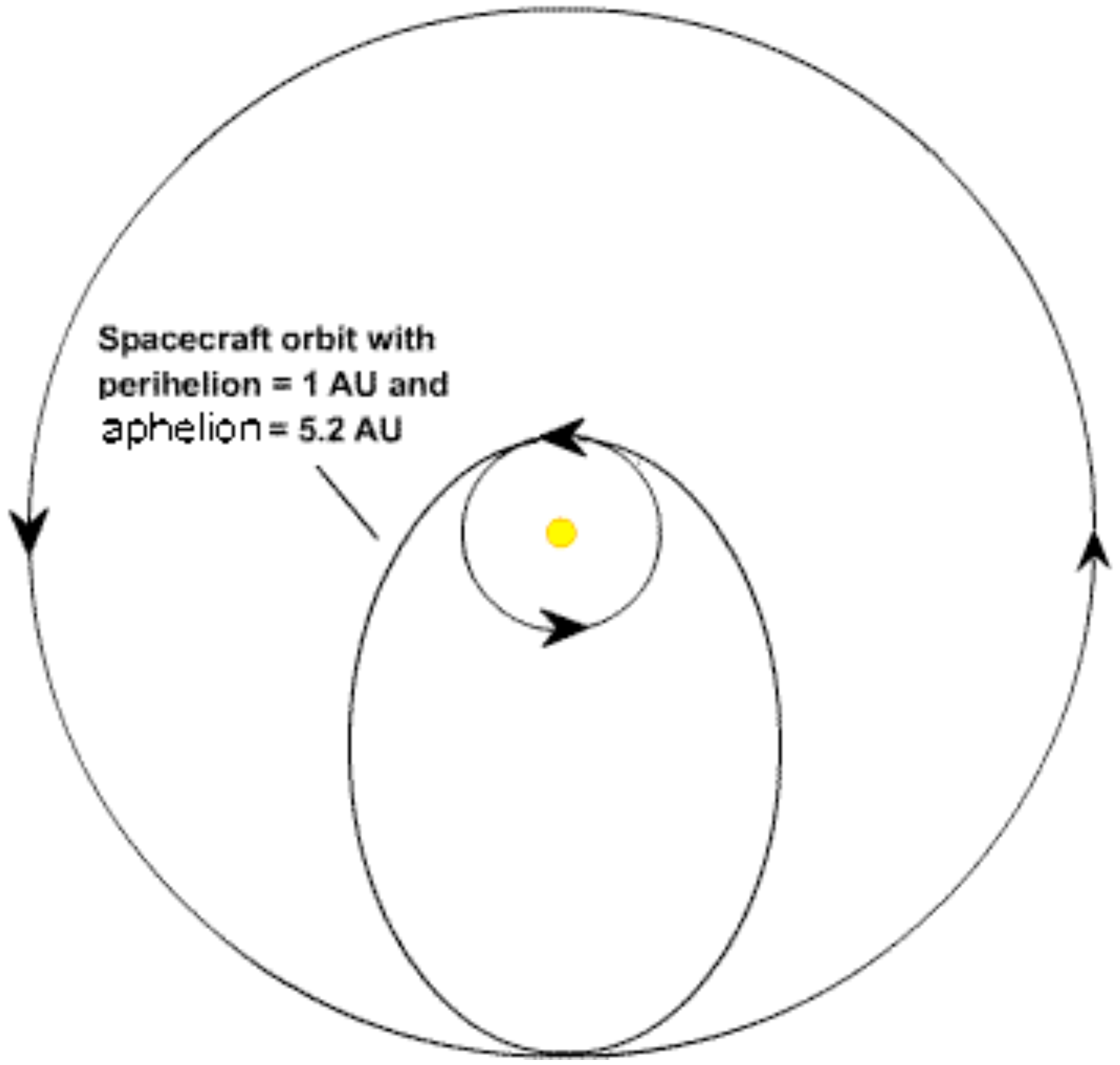
Hohmann Transfer Orbit

- The most efficient way to change orbits in a short amount of time is a Holman transfer orbit
- For example, when going from Earth to Jupiter, the transfer orbit will have perihelion of 1 AU, and aphelion of 5.2 AU
- Perihelion = (1-e) * a = a - ae
- Aphelion = (1+e) * a = a + ae
- 2a = Perihelion + Aphelion = 6.2 AU, so a = 3.1 AU
- 2ae = Aphelion - Perihelion = 4.2 AU, so e = 4.2 / 2 / 3.1 = 0.67





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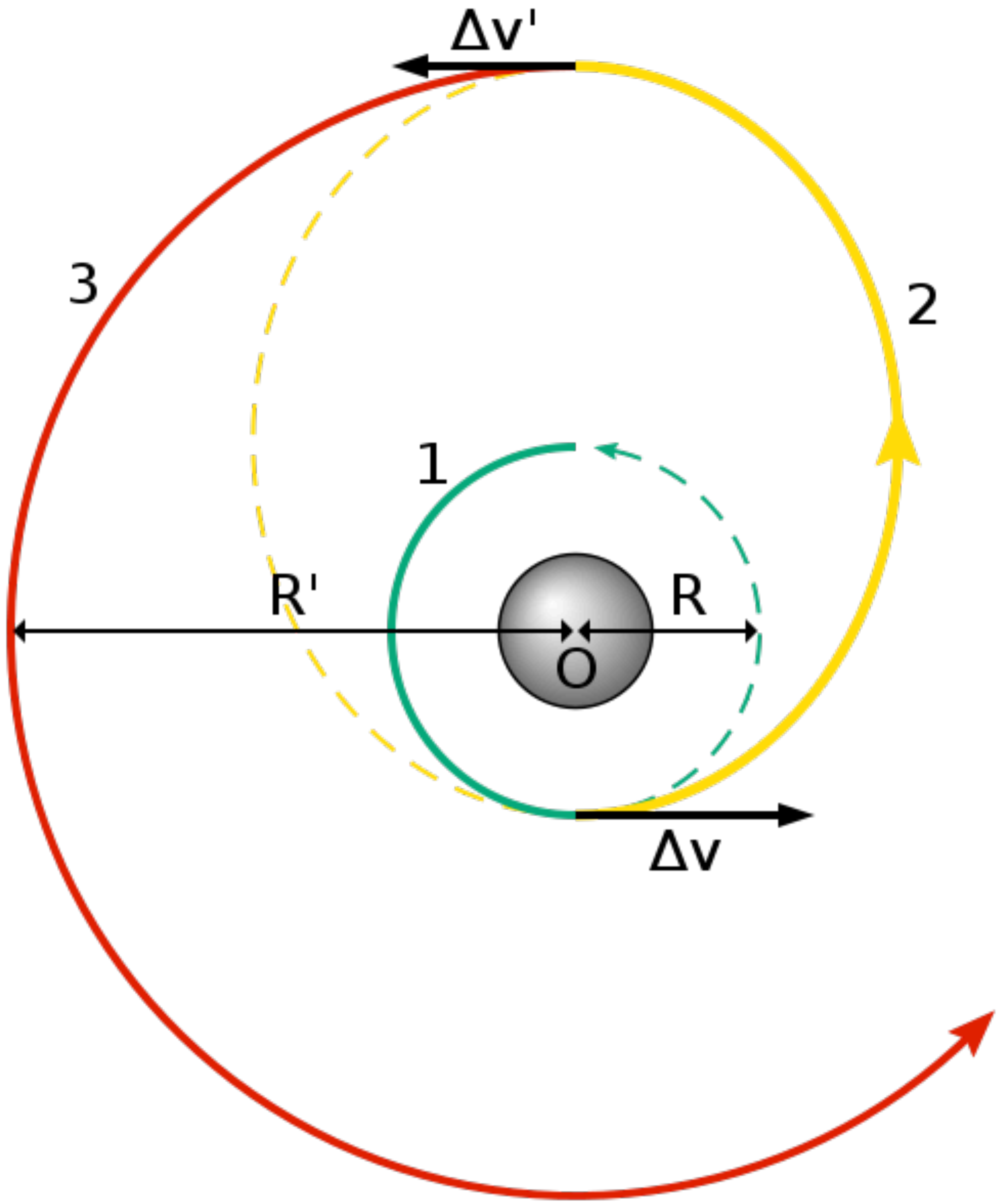
Hohmann Transfer Orbit

- The most efficient way to change orbits in a short amount of time is a Holman transfer orbit
- For example, when going from Earth to Jupiter, the transfer orbit will have perihelion of 1 AU, and aphelion of 5.2 AU
- At 1 AU, Fire spacecraft thrusters to speed up in direction of motion (Δv), the increased velocity means a larger eccentricity and larger semi-major axis
- Once spacecraft reaches 5.2 AU (aphelion of the transfer orbit), Fire spacecraft thrusters to (again) speed up in the direction of motion, to increase semi-major axis and decrease eccentricity
- This is the most time-efficient way to change orbits. More fuel efficient (and slower) paths exist by doing gravitational fly-bys that steal energy from planets.



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For next time

- Find a copy of the textbook (remember: either version of the 2nd edition works — NOT the first edition)
- Reading: de Pater & Lissaeuer Chaper 2, section 2.1.4-2.1.6
- Homework 1 due in 1 week, August 29 at 11:59:59 on Canvas (reminder, late homework loses 10% of possible points each day)