1a) [10 points] The dry adiabatic lapse rate is given by

$$\frac{dT}{dz} = \frac{-g}{c_P}$$

Which we know from last homework is about 9.8 K/km. Solving for the specific heat at constant pressure, and putting things in cgs units:

$$c_P = -g/\frac{dT}{dz} = -(980cm/s^2)/(-9.8K/km\frac{1km}{10^5cm}) = 10^7cm^2/s^2/K$$

1b) [10 points] Saturation vapor pressure is given by:

$$P = C_L e^{-L_S/(RT)}$$

Plugging in numbers:

$$P = C_L e^{-L_S/(RT)} = (3x10^7bar))e^{-(5.1x10^{11}erg/mol)/((8.31x10^7erg/K/mol)(280K))} = 9.08x10^{-3}bar$$

1c) [10 points] Water has a mass, in amu, of 2 hydrogens (2x1), and 1 oxygen (16), or 18 amu.

From last homework, Earth is 20% oxygen (16x2) and 80% nitrogen (14x2):

$$\mu = 0.2(32) + 0.8(28) = 28.8$$

So the grams of water per grams of air is given by:

$$w_s = \frac{\mu_{water}}{\mu_{air}}\frac{P}{P_0} = \frac{18}{28.8}\frac{9.08x10^{-3}bar}{1bar} = 5.67x10^{-3}$$

Where P_0 is the pressure of the atmosphere itself, which the problem tells us is 1 bar.

1d) [20 points]

Moist adiabatic lapse rate is given by:

$$\frac{dT}{dz} = -\frac{g}{c_P + L_s dw_s/dT}$$

We know g, we know the specific heat, we know the latent heat (but we'll want to watch units), so we just need to know how the fractional mass of water changes with temperature. But we can get that by differentiating our equation from the last two parts:

$$w_s = \frac{\mu_{water}}{\mu_{air}}\frac{P}{P_0} = \frac{\mu_{water}}{\mu_{air}}\frac{C_L}{P_0}e^{-L_S/(RT)}$$

Following the chain rule:

$$\frac{dw_s}{dT} = \frac{\mu_{water}}{\mu_{air}}\frac{C_L}{P_0}e^{-L_S/(RT)}\left(\frac{L_S}{RT^2}\right) = w_s\left(\frac{L_S}{RT^2}\right)$$

And we have the fractional mass of water from the previous part, R, T, and latent heat.

$$\frac{dw_s}{dT} = w_s\left(\frac{L_S}{RT^2}\right) = \frac{(5.67x10^{-3})(5.1x10^{11} erg/mol)}{(8.31x10^7 erg/K/mol)(280K)^2} = 4.44x10^{-4}K^{-1}$$

And, putting it all together, we just need to convert the latent heat from erg/mol to erg/g, by remembering there are 18 grams in a mole of water (water = 18 amu)

$$\frac{dT}{dz} = -\frac{g}{c_P + L_s dw_s/dT} = \frac{(980 cm/s^2)}{(10^7 cm^2/s^2/K) + (5.1x10^{11} erg/mol \frac{1mol}{18g})(4.44x10^{-4}K^{-1})} = 4.34x10^{-5}K/cm = 4.34K/km$$

The book said this was crude, and that we should have gotten 5-6 K/km. not bad!

2a) [25 points]

Ok, so we should get geostrophic flow if the Rossby number is much less than 1, but we'll settle for less than 1 being approximately geostrophic

$$R = \frac{U}{f_C L}$$

So we need the horizontal wind speed (given in the table), the Coriolis parameter for each planet (at 45 degrees latitude), and the length scale.

Let's set the length scale to the width of the jet stream.

Coriolis parameter is given by:

$$f_c = 2\Omega \sin\phi = 2\frac{2\pi}{P_{rot}} \sin\phi = \frac{4\pi}{P_{rot}} \sin\phi$$

That gives us: $R = \frac{UP_{rot}}{4\pi \sin\phi L}$

For Earth, U=50 m/s, L = 2000km, and rotational period is 24 hours.

$$R_{Earth} = \frac{UP_{rot}}{4\pi \sin\phi L} = \frac{(50m/s)(24x3600s)}{4\pi \sin 45^{\circ}(2000km\frac{1000m}{1km})} = 0.243$$

Ok, that's not much less than 1, but it's less than 1, so let's say approximately geostrophic. Let's repeat for Mars and Jupiter, getting rotational period from table 1.2 for Mars (1.03 days), and table 1.3 for Jupiter (9 hours, 55 minutes)

$$R_{Mars} = \frac{UP_{rot}}{4\pi \sin\phi L} = \frac{(80m/s)(1.03x24x3600s)}{4\pi \sin 45^{\circ}(2000km\frac{1000m}{1km})} = 0.401$$

Bigger than Earth's, but still less than 1, so let's say still approximately geostrophic. Finally, for Jupiter:

$$R_{Jupiter} = \frac{UP_{rot}}{4\pi \sin\phi L} = \frac{(50m/s)((9x3600 + 55 * 60)s)}{4\pi \sin 45^{\circ}(10000km\frac{1000m}{1km})} = 0.020$$ Ok, that one's clearly much less than 1, so definitely geostrophic.

2b) [25 points]

If we're geostrophic, we can use this equation for the east-west (x) wind speed (u):

$$f_c u \approx -\frac{1}{\rho}\frac{dP}{dy}$$

Solving for the pressure gradient in the north-south direction (y):

$$\frac{dP}{dy} \approx -\rho f_c u$$

From above: $f_c = \frac{4\pi}{P_{rot}} \sin\phi$

The table doesn't give us density, but it does give us temperature and pressure, so we can use the ideal gas law:

$$P = \frac{k_B}{\mu}\rho T$$

Solving for density:

$$\rho = \frac{\mu P}{k_B T}$$

From last homework, $\mu_E = 28.8 m_H$ $\mu_M = 44 m_H$ $\mu_J = 2.2 m_H$

Combining:

$$\frac{dP}{dy} \approx -\frac{4\pi}{P_{rot}} \sin\phi u \frac{\mu P}{k_B T}$$

2b, continued)

$$\frac{dP}{dy} \approx -\frac{4\pi}{P_{rot}} \sin \phi u \frac{\mu P}{k_B T}$$

Then we can start plugging in values, for all three planets:

$$\left(\frac{dP}{dy}\right)_{Earth} \approx -\frac{4\pi}{P_{rot}} \sin \phi u \frac{\mu P}{k_B T} = -\frac{4\pi}{24x3600s} \sin 45^{\circ}(5000cm/s)\frac{(28.8x1.67x10^{-24}g)(0.1barx10^6 dyn/cm^2)}{(1.38x10^{-16}erg/K)(220K)} = -8.15x10^{-5}dyn/cm^3\frac{1mbar}{10^3 dyn/cm^2}\frac{10^5 cm}{1km} = -8.15x10^{-3}mbar/km$$

$$\left(\frac{dP}{dy}\right)_{Mars} \approx -\frac{4\pi}{P_{rot}} \sin \phi u \frac{\mu P}{k_B T} = -\frac{4\pi}{1.03x24x3600s} \sin 45^{\circ}(8000cm/s)\frac{(44x1.67x10^{-24}g)(10^{-3}barx10^6 dyn/cm^2)}{(1.38x10^{-16}erg/K)(160K)} = -2.66x10^{-12}bar/cm\frac{1mbar}{10^3 dyn/cm^2 bar}\frac{10^5 cm}{1km} = -2.66x10^{-4}mbar/km$$

$$\left(\frac{dP}{dy}\right)_{Jupiter} \approx -\frac{4\pi}{P_{rot}} \sin \phi u \frac{\mu P}{k_B T} = -\frac{4\pi}{(9x3600 + 55 * 60)s} \sin 45^{\circ}(5000cm/s)\frac{(2.2x1.67x10^{-24}g)(1barx10^6 dyn/cm^2)}{(1.38x10^{-16}erg/K)(165K)} = -2.01x10^{-10}bar/cm\frac{1mbar}{10^3 dyn/cm^2}\frac{10^5 cm}{1km} = -2.01x10^{-2}mbar/km$$