1a) [10 pts] Kinetic energy is just $E = \frac{1}{2}mv^2$

Mass is going to be density times volume, $m = \rho V = \rho(\frac{4}{3}\pi R^3) = (3.4g/cm^2)(\frac{4}{3}\pi(5km x\frac{10^5 cm}{km})^3) = 1.78x10^{18}g$

If velocity at infinity is 0, it will land with escape velocity, $v = 11.2km/s = 1.12x10^6 cm/s$

So energy will be: $E = \frac{1}{2}(1.78x10^{18}g)(1.12x10^6 cm/s)^2 = 1.12x10^{30}erg$ about 0.05% of 1 second of the Sun's total energy output. Yowza.

Pressure? Equation 5.18 tells us $P \approx \frac{1}{2}\rho v^2 = \frac{1}{2}(3.4g/cm^2)(1.12x10^6 cm/s)^2 = 2.13x10^{12}dyne/cm^2$

1b) [10 pts] We need the escape velocity from Jupiter:

$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67x10^{-8})(1.90x10^{30})}{(6.99x10^9)}} = 6.02x10^6 cm/s$ about 5 times faster than for Earth

$E = \frac{1}{2}(1.78x10^{18}g)(6.02x10^6 cm/s)^2 = 3.23x10^{31}erg$ About 1% of 1 second of the Sun's total energy output

1c) [10 pts] Mass of this fragment is $m = \rho V = \rho(\frac{4}{3}\pi R^3) = (0.5g/cm^2)(\frac{4}{3}\pi(0.5km x\frac{10^5 cm}{km})^3) = 2.62x10^{14}g$

Which gives an energy of: $E = \frac{1}{2}(2.62x10^{14}g)(6.02x10^6 cm/s)^2 = 4.75x10^{27}erg$

2a) [10 pts] Kinetic energy is just $E = \frac{1}{2}mv^2$

Mass is going to be density times volume, $m = \rho V = \rho(\frac{4}{3}\pi R^3) = (7g/cm^2)(\frac{4}{3}\pi(150mx\frac{10^2cm}{m})^3) = 9.90x10^{13}g$

$$E = \frac{1}{2}(9.90x10^{13}g)(12x10^5cm/s)^2 = 7.13x10^{25}erg$$

2c) [15 pts] An object on a ballistic trajectory will travel the furthest if the launch angle is 45 degrees

$$v_{tot} = 500m/s$$

$$v_x = v_y = v_{tot}\sin 45^\circ = \frac{\sqrt{2}}{2}v_{tot} = 354m/s$$

Equation for distance travelled under constant acceleration: $x(t) = x_0 + v_0t + \frac{1}{2}at^2$

Consider vertical motion, the rock spends total time in the air t_{air}, and has starting and final position equal to 0:

$$0 = 0 + v_yt_{air} - \frac{1}{2}g_Mt_{air}^2$$

Solving for t_{air},

$$t_{air} = \frac{2v_y}{g_M}$$

Surface gravity of the Moon is: $g_M = \frac{GM}{R^2} = \frac{(6.67x10^{-8})(7.35x10^{25})}{(1.74x10^8)^2} = 162cm/s^2$ About 1/6 that of Earth, so that worked (phew)

$t_{air} = \frac{2v_y}{g_M} = \frac{2(3.54x10^4cm/s)}{162cm/s^2} = 437s$ 7 minutes of airtime. Wow.

2c, continued) Now, we do the x-component of velocity to find the total distance between the impact site and the second crater:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (3.54x10^4 cm/s)(437s) + 0 = 1.55x10^7 cm = 155km$$ ...pretty far!

3a) [15 pts] This calls for the Gault scaling relation: $D = 2\rho_m^{0.11}\rho_p^{-1/3}g_p^{-0.22}R^{0.13}E_K^{0.22}(\sin\theta)^{1/3}$ (mks units)

From the problem:

$\rho_m = 3g/cm^3 = 3x10^3 kg/m^3$

$\rho_p = 3.5g/cm^3 = 3.5x10^3 kg/m^3$

$R = \frac{1}{2}1km = 500m$

$\theta = 45°, \sin\theta = \frac{\sqrt{2}}{2}$

And we know the surface gravity of Earth is $g = 9.8m/s^2$

That just leaves the kinetic energy. As before, we start by getting the mass of the meteor:

$M = \rho V = \frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi(500m)^3(3x10^3 kg/m^3) = 1.57x10^{12}kg$

Which then gives a kinetic energy of:

$E_K = \frac{1}{2}Mv^2 = \frac{1}{2}(1.57x10^{12}kg)(15km/s\frac{1000m}{km})^2 = 1.77x10^{20}J$

Phew, got all the numbers, let's plug them in:

$D = 2(3x10^3)^{0.11}(3.5x10^3)^{-1/3}(9.8)^{-0.22}(500)^{0.13}(1.77x10^{20})^{0.22}(\frac{\sqrt{2}}{2})^{1/3} = 1.09x10^4 m = 11km$

3b) [15 pts] Same as before, but we increase the radius from 500m to 5km

Most numbers stay the same, but kinetic energy increases by a factor of 1000:

$$M = \rho V = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi(5000m)^3(3x10^3 kg/m^3) = 1.57x10^{15}kg$$

Which then gives a kinetic energy of:

$$E_K = \frac{1}{2}Mv^2 = \frac{1}{2}(1.57x10^{15}kg)(15km/s\frac{1000m}{km})^2 = 1.77x10^{23}J$$

Phew, got all the numbers, let's plug them in:

$$D = 2(3x10^3)^{0.11}(3.5x10^3)^{-1/3}(9.8)^{-0.22}(5000)^{0.13}(1.77x10^{23})^{0.22}(\frac{\sqrt{2}}{2})^{1/3} = 6.75x10^4 m = 67.5km$$

3c) [15 pts] Same as before, but we replace the Earth with the Moon

From before, g = 1.62 m/s/s

Assume density of the Moon is the same as the density of the Earth

Impact velocity of the two meteors stays the same, as does the mass, so the kinetic energy is the same as well

Ok, first meteor:

$$D = 2(3x10^3)^{0.11}(3.5x10^3)^{-1/3}(1.62)^{-0.22}(500)^{0.13}(1.77x10^{20})^{0.22}(\frac{\sqrt{2}}{2})^{1/3} = 1.62x10^4 m = 16.2km$$

Only about 50% bigger than when it hits Earth

And second, larger meteor:

$$D = 2(3x10^3)^{0.11}(3.5x10^3)^{-1/3}(1.62)^{-0.22}(5000)^{0.13}(1.77x10^{23})^{0.22}(\frac{\sqrt{2}}{2})^{1/3} = 1.00x10^5 m = 100km$$