- (1) (25 points)
- We start with Kepler's third law, with the dumb units:

$$P^2 = \frac{4\pi^2 a^3}{GM}$$ We'll assume the moon is much less massive than Mars.

- Mars has a 24.5 hour rotational period, and a mass of $6.39x10^{26}g$

$$a = \left(\frac{P^2 GM}{4\pi^2}\right)^{\frac{1}{3}} = \left(\frac{(24.5hr\frac{3600s}{1hr})^2(6.67x10^{-8}(6.39x10^{26}g)}{4\pi^2}\right)^{\frac{1}{3}} = 2.03x10^9 cm$$

- The radius of Mars is $3.39x10^8 cm$, so the moon would have to be at a distance of:

$$d = 2.03x10^9 cm \frac{1R_{Mars}}{3.39x10^8 cm} = 6.00R_{Mars}$$

- (1, continued)

- So the orbital period of Phobos (at 2.76 Martian radii) would be less than a Martian day, while the orbit of Deimos (6.9 Martian radii) would be just a bit more than a Martian day. Deimos would behave like our moon, rising in the east and setting in the west, but would move very slowly across the sky. Phobos would behave more like a small-altitude artificial satellite of Earth, rising in the west and setting in the East, moving quickly across the sky. Our hypothetical synchronous moon would neither rise nor set, and remain above a fixed point on the Martian surface.

- (2) (25 points)
- The ratio of the height of tides (h) should be proportional to the ratio of the tidal forces felt by the Earth for the Moon and the Sun

$$F_T = \frac{2GMmr}{d^3}$$

- Which gives a height ratio:

$$\frac{h_M}{h_S} = \frac{M_M}{M_S}\left(\frac{d_S}{d_M}\right)^3 = \frac{(7.35x10^{25}g)}{(2x10^{33}g)}\left(\frac{1.5x10^{13}cm)}{(3.84x10^{10})}\right)^3 = 2.19$$

- So tides from the moon are about twice as high as tides from the Sun

- (3a) (25 points)
- Radiation pressure is given by:

$$F_{rad} = \frac{LR^2 Q_{PR}}{4cd^2}$$ and the ratio of radiative force to gravitational force is: $\beta = \frac{F_{rad}}{F_G}$

From the figure in the textbook, for 0.1 micron ice at 100K, $\beta = 0.4$, from the Sun

- $$\beta = \frac{3L_{\odot}Q_{PR}}{16\pi GM_{\odot}cR\rho}$$ We're going to need Q_{PR}, so solving for it:

$$Q_{PR} = \frac{16\pi GM_{\odot}cR\rho\beta}{3L_{\odot}} = \frac{16\pi(6.67x10^{-8})(2x10^{33}g)(3x10^{10}cm)(10^{-5}cm)(1\frac{g}{cm^3})(0.4)}{3(3.85x10^{33}\frac{erg}{s})} = 0.070$$

- For the Sun, we just need the luminosity of the Sun and distance of Saturn to the Sun:

$$F_{rad,Sun} = \frac{(3.85x10^{33}\frac{erg}{s})(10^{-5}cm)^2(0.070)}{4(3x10^{10}\frac{cm}{s})(9.58x1.5x10^{13}cm)^2} = 1.08x10^{-17}dynes$$

- (3a, continued)

- For the radiation pressure from Saturn, we also need the luminosity of Saturn in reflected light, which is the Sun's luminosity, multiplied by Saturn's albedo, multiplied by the ratio of areas of Saturn's cross-section and a sphere with radius equal to Saturn's orbital distance:

$$L_{Saturn} = L_{\odot} * a_S * \frac{\pi R_S^2}{4\pi d_S^2} = (3.85x10^{33}\frac{erg}{s})(0.46)\frac{(5.82x10^9 cm)^2}{4(9.54x1.5x10^{13}cm)^2} = 7.33x10^{23}\frac{erg}{s}$$

About 20 billion times less

- The equation for radiation pressure from Saturn to the particle, then is:

$$F_{rad,Saturn} = \frac{(7.33x10^{23}\frac{erg}{s})(10^{-5}cm)^2(0.070)}{4(3x10^{10}\frac{cm}{s})(1.5x5.82x10^9 cm)^2} = 5.61x10^{-19} dynes$$

- So about a factor of 20 smaller radiative force from Saturn compared to the Sun.

- (3b) (25 points)

- So let's calculate the two values of beta:

$$\beta_{Sun} = \frac{3L_\odot Q_{PR}}{16\pi G M_\odot c R\rho} = 0.4 \text{ (We have this from the figure)}$$

$$\beta_{Saturn} = \frac{3L_{Saturn}Q_{PR}}{16\pi G M_{Saturn} c R\rho} = \frac{3(7.33x10^{23}\frac{erg}{s})(0.070)}{16\pi(6.67x10^{-8})(5.68x10^{29}g)(3x10^{10}\frac{cm}{s})(10^{-5}cm)(1\frac{g}{cm^3})} = 2.68x10^{-7}$$

- So beta for the Sun is about a million times larger than the beta from Saturn.

- But what really matters is a third beta: the radiative force from the Sun compared to the gravitational force from Saturn. The gravitational force is given by:

$$F_{G,Saturn} = \frac{GM_{Saturn}m}{d^2_{particle}} = \frac{(6.67x10^{-8})(5.68x10^{29}g)(\frac{4}{3}\pi(10^{-5}cm)^3(1\frac{g}{cm^3})}{(1.5x5.82x10^9cm)^2} = 2.08x10^{-12}dynes$$

- Which gives a final value for beta of:

$$\beta_{real} = \frac{F_{rad,Sun}}{F_{G,Saturn}} = \frac{1.08x10^{-17}dynes}{2.08x10^{-12}dynes} = 5.19x10^{-6}$$

- (3b, continued)

- Which is 10 times larger than the beta from Saturn alone, but still very small. So this ice particle is probably safe in orbit around Saturn over large timescales.