In [1]:
```
import numpy as np
import matplotlib
%matplotlib inline
import matplotlib.pyplot as plt
```

Noise equation

1. You observe a bright source that produces 1000 photons in a ten second exposure, with negligible sky background. What is the fractional uncertainty and the S/N of your observation?

In [2]:
```
print('fractional uncertainty: ',np.sqrt(1000)/1000)
print('S/N: ',1000/np.sqrt(1000))
```

```
fractional uncertainty:  0.03162277660168379
S/N:  31.622776601683793
```

2. You observe a faint source that produces 1000 photons in a ten minute exposure and, in that same time, the sky produces 500 photons that is measured in addition to the object.

a. How many total photons (sky+background)? What will the uncertainty on that total be?

b. If you assume that you can subtract the sky perfectly (usually you can come close), then the only effect that the background has is in the increased noise. What will be the expected signal to noise of your measurement of the source?

In [3]:
```
s=1000
b=500
print('total: ', s+b)
print('uncertainty: ',np.sqrt(s+b))
```

```
total:  1500
uncertainty:  38.72983346207417
```

In [4]:
```
print('S/N: ', s/np.sqrt(s+b))
```

```
S/N:  25.81988897471611
```

If you are looking at faint, background-limited, stars, how much fainter can you see (i.e., measure at the same S/N) if you have 0.3 arcsec instead of 1.0 arcsec images? Express your answer in magnitudes.

$$S/N = S/\sqrt{BA}$$
$$S_1/\sqrt{BA_1} = S_2/\sqrt{BA_2}$$
$$A_1/A_2 = (0.3/1)^2$$
$$S_1/S_2 = \sqrt{A_2/A_1} = 1/0.3 = 3.333$$

In [5]:
```
sr=1/0.3
dm=-2.5*np.log10(sr)
print('ratio: ',sr)
print('difference in magnitudes: ',dm)
```

```
ratio:  3.3333333333333335
difference in magnitudes:  -1.307196863200844
```

Say you have a system that has a photometric zeropoint of 23.5 in the bandpass where you plan to make observations, and you are observing at a site with 1 arcsec seeing, a background of 21 magnitudes per square arcsec with a detector with readout noise corresponding to 5 electrons/pixel, and a plate scale of 0.5 arcsec/pixel.

a. How long do you need to expose to get a S/N of 100 for a 15th magnitude object?

b. How long do you need to expose to get a S/N of 50 for a 22nd magnitude object?

In [6]:
```
counts = 10**(-0.4*(15-23.5))
print('source cnts/s',counts)
back = 10**(-0.4*(21-23.5))
print('back cnts/s/square arcsec: ',back)
area=np.pi*1**2
print('total background: ',back*area)
sn=counts/np.sqrt(counts+back*area)
print('S/N: ',sn)
print('exptime: ',100**2/sn**2)
```

```
source cnts/s 2511.886431509582
back cnts/s/square arcsec:  10.0
total background:  31.41592653589793
S/N:  49.808217871552586
exptime:  4.030862593636573
```

In [7]:
```
counts=10**(-0.4*(22-23.5))
print('source, background: ',counts,back)

area=np.pi*1**2
sn=counts/np.sqrt(counts+back*area)
print('S/N',sn)
npix=area/(0.5**2)
print(npix)
rn=5
print(npix*rn**2)
print('exptime neglecting rn: ',50**2/sn**2)

sn=50
a=counts**2
b=-sn**2*(counts+back*area)
c=-sn**2*(npix*rn**2)
print('t: ',(-b + np.sqrt(b**2-4*a*c)) / 2 / a)
def snt(t) :
    return counts*t/np.sqrt(counts*t+back*area*t+npix*rn**2)

#for t in np.arange(5000,6000,100) : print(t,snt(t))

#print(counts*600,back*area*600,npix*25**2)
```

```
source, background:  3.981071705534973 10.0
S/N 0.6691396777609151
12.566370614359172
314.1592653589793
exptime neglecting rn:  5583.499003246143
t:  5592.360248186462
```

Propagation of Uncertainties

1. Using the error propagation formula, derive the expression for the uncertainty in the sample mean, $< x >= \sum x_i/N$, for the case where the uncertainty on every x_i is the same, and given by σ.

Using error propagation

$$\sigma^2_{<x>} = \sum \frac{\sigma^2}{N^2} = N\frac{\sigma^2}{N^2} = \frac{\sigma^2}{N}$$
$$\sigma_{<x>} = \frac{\sigma}{\sqrt{N}}$$

2. You observe a source that has a measured photon rate, on average, of 10 photons per second, and take a 10 second exposure.

   a. how many counts will you observe on average?

   b. what will be expected standard deviation if you make a series of measurements?

   c. what is the signal-to-noise of each observation?

In [8]:
```
counts=10*10
print('counts on average: ',counts)
var=counts
std=np.sqrt(var)
print('expected standard deviation: ',std)
print('S/N: ',counts/std)
```

```
counts on average:  100
expected standard deviation:  10.0
S/N:  10.0
```

d. If you combine multiple observations, you need to propagate the errors on each individual observation to get an error estimate on your combined quantity. If you average together 10 of the 10s exposures, what is the expected error on this average?

In [9]:
```
nexp=10
totvar=nexp*var
print(np.sqrt(totvar/nexp**2))
print(std/np.sqrt(nexp))
```

```
3.1622776601683795
3.162277660168379
```

3. You take a single observation and count 1000 photons.

   a. what is the expected uncertainty?

   b. what is the expected uncertainty (in magnitudes) if you convert the counts into an instrumental magnitude?

$$m = -2.5 log s$$
$$\sigma_m^2 = \sigma_s^2 \frac{(-2.5 \log e)^2}{s^2}$$
$$\sigma_m = 1.086 \frac{\sigma_s}{s}$$

In [10]:
```
s=1000
print(np.sqrt(s))
m=-2.5*np.log10(s)
print(m)
c=2.5*np.log10(np.exp(1))
print(c*np.sqrt(s)/s)
```

```
31.622776601683793
-7.5
0.03433399345142635
```

4. If you observe a source in the B passband and count 500 photons and observe it in the V passband and count 1000 photons. You can assume no background in either measurement.

a. What is the expected uncertainty in the flux ratio, F(B)/F(V)?

b. What is the expected uncertainty in the color difference (in instrumental magnitudes), b-v?

$$\sigma^2_{B/V} = \sigma^2_B \left(\frac{1}{V}\right)^2 + \sigma^2_V \left(\frac{B}{V^2}\right)^2$$
$$b - v = -2.5 \log \frac{B}{V}$$
$$\sigma_{b-v} = 1.086 \frac{\sigma_{B/V}}{(B/V)}$$

In [11]:
```
b=500
v=1000
boverv=(b/v)
var_b = b
var_v = v
var = var_b/v**2 + var_v*(b/v**2)**2
print(np.sqrt(var))
print(1.086*np.sqrt(var)/boverv)
```

```
0.027386127875258306
0.05948266974506104
```

Splitting and averaging measurements

1.You make 3 identical observations of the same star.

a. How does the uncertainty on the average compare to the uncertainty on an individual observation?

b. Let's say you observe 90, 100, and 110 photons on three identical exposures. What is the mean and its uncertainty?

c. Let's say you take 3 exposure of different exposure times and get: 90 photons in 1 second, 1000 photons in 10 seconds, 11000 photons in 100 seconds. How would you combine these to get the best estimate of the photon rate (photons / sec)? What value do you get?

In [12]:
```
print('1a')
print('uncertainty on mean is down by sqrt(N)')

print('1b')
obs=np.array([90,100,110])
print('mean: ',obs.mean())
n=len(obs)
print('uncertainty on mean: ',np.sqrt(((obs-obs.mean())**2).sum()/n/(n-1)))
print('uncertainy on mean using np.std():',obs.std(ddof=1)/np.sqrt(n))
print(10/np.sqrt(n))

print('1c')
#inconsistent weights
num=90/90 + 100/(1000/10**2) + 110 / (11000/100**2)
den=1./90 + 1./(1000/10**2) + 1. / (11000/100**2)
print('weighted mean with inconsistent weights: ',num/den)

#consistent weights using rate of 100
num=90/100 + 100/(1000/10**2) + 110 / (10000/100**2)
den=1./100 + 1./(1000/10**2) + 1. / (10000/100**2)
print('weighted mean adopting rate of 100:',num/den)

#consistent weights using rate of 110
num=90/110 + 100/(1100/10**2) + 110 / (11000/100**2)
den=1./110 + 1./(1100/10**2) + 1. / (11000/100**2)
print('weighted mean adopting rate of 110',num/den)

# weighted by exposure time
print('weighted mean weighting by exposure time:',(90*1+100*10+110*100)/(1+
```

```
1a
uncertainty on mean is down by sqrt(N)
1b
mean:  100.0
uncertainty on mean:  5.773502691896258
uncertainy on mean using np.std(): 5.773502691896258
5.773502691896258
1c
weighted mean with inconsistent weights:  108.80198019801979
weighted mean adopting rate of 100: 108.91891891891892
weighted mean adopting rate of 110 108.9189189189189
weighted mean weighting by exposure time: 108.91891891891892
```

2. Let's say you observe a star and count 500 photons total in an aperture of diameter 2 arcseconds, in a 10 second exposure. From a different region, you measure 100 photons per square arcsecond coming from the sky. Let' s say readout noise is negligible.

a. What is the S/N of the measurement?

b. Let's say you need to make the measurement to a fractional accuracy of 1 percent. How long do you need to expose?

c. Let's say you are working with a telescope/instrument/detector that gives pixels that correspond to 0.5 arcseconds on a side. If the readout noise is 10 photons/pixel, what is a minimum exposure time to ensure that readout noise is not a significant noise source?

In [13]:
```
tot=500
back=100
s=tot-back*np.pi
print('background per 10s: ',np.pi*back)
sn=s/np.sqrt(tot)
print('2a. S/N in 10s: ',sn)
print('2b. exptime for S/N+100: ',100**2/sn**2*10)
```

```
background per 10s:  314.1592653589793
2a. S/N in 10s:  8.311050312916443
2b. exptime for S/N+100:  1447.7320106757747
```

2c.

$$N = \sqrt{S + BA + N_{pix}\sigma_{rn}^2}$$

S and B scale with exposure time, while readout noise does not. So to avoid readout noise being a significant contributor, need minimum exposure time so that Poisson terms dominate. Here, I choose Poisson to be 10X readout:

In [14]:
```
# want Poisson noise to be greater than readout noise
npix=np.pi*2**2
rn=10
print('total readout noise: ',npix*rn**2)
# divide tot by 10 for counts per second
factor = 10
t = factor*npix*rn**2 / (tot/10)
print('exptime for Poisson to be 10x readout noise: ', t)
```

```
total readout noise:  1256.6370614359173
exptime for Poisson to be 10x readout noise:  251.32741228718342
```

3. Lets say you observe a star at 5 different locations in a given instrument/detector, and measure 1050, 1000, 1080, 920, 950 photons in the different measurements.

a. What is the mean and estimated uncertainty?

b. Should you be concerned about anything? Why or why not?

In [15]:
```
obs=np.array([1050,1000,1080,920,950])
n=len(obs)
std=np.sqrt(obs.mean())
print('mean: ',obs.mean())
print('expected scatter from Poisson: ',np.sqrt(obs.mean()))
print('observed scatter: ',obs.std(ddof=1))
print('uncertainty of the mean: ',obs.std(ddof=1)/np.sqrt(n))
print('chi**2: ',((obs-obs.mean())**2/std**2).sum())
```

```
mean:  1000.0
expected scatter from Poisson:  31.622776601683793
observed scatter:  66.70832032063167
uncertainty of the mean:  29.832867780352593
chi**2:  17.8
```

The χ^2 value is much larger than the number of points and the observed scatter is larger than the expected scatter. So it seems that there is some other source of variation in the counts! Variable star? Flat fielding error? ....?

In [ ]: